Emanuel Mahler

Scale-Dependent Filtering of High Resolution Digital Terrain Models in the Wavelet Domain

Advisors:
dipl. geogr. Daria Martinoni,
Dr. Bernhard Schneider, Prof. Dr. Robert Weibel

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Department of Geography
University of Zurich

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Abstract

Recently deployed technologies, such as laser scanning or SAR interferometry, can be applied to generate digital terrain models of high spatial resolution. Due to the increasing resolution, approximation techniques for digital terrain models are needed. Surface simplifications are necessary to meet the requirements of traditional applications developed for meso-scale data. Furthermore, scale problems are a very important issue for process modelling. They also need to be dealt with by a representation consisting of multiple resolutions.

The proposed generalisation method consists of a semantic information reduction in a first step and a statistical data reduction in a second step. This investigation is primarily concerned with the controlled reduction of information based on the occurrence of significant landscape structures of a given scale. A mesh filter is subsequently employed for data reduction.

The information reduction is undertaken in the wavelet domain. Space and frequency localisation is a major advantage of a wavelet representation over Fourier or Euclidean bases. The stationary wavelet transform is applied to localise significant landforms of a chosen scale in the digital terrain model. The implemented wavelet coefficient filter is adaptable to the thus detected landscape features at a scale of interest. The resulting approximation preserves significant structures, whereas areas of little relevance for the chosen scale are considerably smoothed. This method is believed to include more semantic information than purely statistical methods such as TIN filters. However, further research is needed in order to develop a comprehensive framework for wavelet-based generalisation of digital terrain models. Furthermore, it is concluded that a landscape feature and its classification into a certain scale is fuzzy. A wavelet-based approach combined with a fuzzy logic approach could probably capture the specific issues of a generalisation based on landform recognition more appropriately.

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Chapter 1

Introduction

1.1 A Changing Perspective

The last few decades have witnessed a variety of rapidly changing technologies for generating digital elevation data. As a consequence the spatial resolution of digital terrain models has increased considerably. The most recent high resolution models derived from laser altimetry contrast starkly with the data sets generated fifteen years ago. As a result a great many new options and applications have become possible. Resolutions of less than two metres with very high vertical accuracy allow to tackle sophisticated problems, such as flood modelling in flat areas, or timber volume estimation in forests. However, more traditional applications may also require digital terrain models with much lower resolutions. Hydrological models, as an example, cannot possibly deal with the vast number of sample points provided by high resolution terrain models. Moreover, handling very large data sets of several hundreds of megabytes is still a problem for most computers. Hence, data reduction, filtering and generalisation is an increasingly important issue.

Any physical process and landform feature is associated with a specific scale and resolution (see Figure 4.2). The advent of very high resolution data is a further challenge for modellers. Such high resolution data needs to be transformed to the required scale, which is largely dependent on the purpose of the model. For multi-purpose terrain models, a representation at multiple scales is desirable. This can be approached by means of multiresolution theory.

Generating a digital terrain model traditionally implied the application of an adequate interpolation method in order to create a continuous surface from often sparse sample points, breaklines and contours. Exact interpolation techniques, such as inverse distance weighting, bivariate cubic or quintic interpolation, Kriging or B-Splines have been widely used to generate continuous surfaces. The focus has predominantly been on the estimation of a surface from scarce information. The often redundant information of high resolution models, in contrast, needs to be simplified in order to obtain models of suitable resolution for conventional applications originally designed for medium or low resolution models. Exact interpolation as a traditionally important step in digital terrain model generation needs to be replaced by the application of a surface approximation containing less information than the raw data. Surface approximations are also referred to as non-exact interpolation. The main objective of both exact and non-exact interpolation is the generation of a continuous surface.

1.2 Research Objective

The objective of this thesis is the recognition of significant structures and the controlled reduction of information in digital terrain models. The primary focus of this thesis is the filtering of high resolution data in the wavelet domain using scale-adaptive and locally adaptive thresholds. The major preservation criterion of the filtering process will be to retain significant landscape forms of a given scale. Fine-scale structures shall be retained, provided they are part of a significant large-scale form. Edge smoothing of relevant large-scale structures is to be avoided, whereas the elimination of noise and small-scale structures in smooth areas shall be permitted. It is thus necessary to detect the predominant features or at least their spatial extent for a given scale.

Instead of detecting significant features, a method shall be found to include known features of importance, such as shorelines or hydrological networks, directly into the set of well-preserved landforms. For instance, shorelines (e.g., of lakes) are often not associated with coarse features but are nevertheless important and should not be simplified too much. Such complementary *a priori* knowledge will be included similarly to important features detected by wavelets.

The statistical data reduction will be performed by the TIN filter which selects important points of the resulting terrain model. The points close enough to the approximated surface will be rejected.

1.3 Wavelets in Terrain Modelling

One of the first fields to employ wavelets was geophysics. In the early 1980s, Morlet (in Hubbard (1996)), working for an oil prospection company, adapted the Fourier transform in order to better analyse seismic signals.

They were rediscovered in geophysics in the early 1990s as a means to aid atmospheric and hydraulic turbulence exploration and de-noising of SAR-images in particular (Foufoula-Georgiou and Kumar (1994)). Wavelets are now successfully applied to many disciplines, from financial mathematics to technical and medical engineering.
The characteristics that make wavelets very attractive for analysing non-stationary or transient signals are time-frequency localisation or scale-space analysis, orthogonality and multiresolution representation. These concepts are explained in chapter 2. These apparent advantages have not been widely exploited by digital terrain modellers so far. Gerstner and Hannappel (2000), Gross et al. (1996), Luca et al. (1996), Gallant and Hutchinson (1996) are notable exceptions.

The wavelet perspective has been adopted to a much greater extent by image processing and image compression researchers. They have extended the wavelet transform successfully to two-dimensional images. Nevertheless, fundamental differences between image processing and terrain modelling restrain the direct transfer of approximation techniques that have been developed for images. Particularly the higher regularity of digital terrain models and the importance of extrema are issues of special emphasis in terrain modelling. In images the occurrence of black pixels next to white pixels is very common, whereas such singularities in a terrain would signify the highest point being a neighbour of the lowest point, for example, on a cliff, which is not often the case.

In this thesis wavelets are used for surface approximation on the one hand and for the estimation of the importance of prevailing landforms on the other hand. The principal focus is on the estimation of significant forms as an approximation criterion rather than on smooth approximation properties. Only orthogonal wavelets of varying support are applied. For visually attractive approximations, bi-orthogonal scaling functions such as splines could be considered. They are described by Stollnitz et al. (1995a), among others. However, the filter proposed in chapter 4 is seen as a preprocessing step for a traditional mesh filter introduced in chapter 3. A conventional mesh filter eliminates redundant points in the grid and possible wavelet-induced artefacts.

1.4 Thesis Organisation

The second chapter will provide the theoretical background to the wavelet transform. For completeness, the mathematical formulations will be given in section 2.2.2. In chapter 3 a few theoretical notions on modelling, its constraints and possible social implications will be considered. Moreover, cartographic generalisation and some types of surface simplification algorithms will be reviewed. Chapter 4 will introduce the conceptual framework of the proposed scale-adaptive wavelet filter, including some thoughts on the perception of landscape features belonging to a specific scale. In chapter 5 the various thresholding options and the applied mesh filter will be discussed. Various types of evaluations will be attempted in chapter 6. Chapter 7 gives a short overview of possible extensions and measures for tackling the encountered deficiencies.

Chapter 2

Theoretical Background

2.1 Interpolation and Approximation

Conventional digital terrain models (DTMs) are derived from points, structure lines and contours. For a more detailed definition see section 3.1.2. As stated in the introduction, terrain modelling implies the application of an adequate interpolation method in order to obtain a continuous surface from the often sparse raw data. High resolution terrain models, in contrast, consist of a vast number of grid points as a result of their small sampling interval. Thus, a problem that is converse to classical precise interpolation has to be tackled. Plausible approximations, also called non-exact interpolations have to be found. This is necessary not only to store and process these data sets efficiently, but also to reduce undesireable detail information for a given scale. Approximation, or non-exact interpolation algorithms will therefore become increasingly important to terrain modellers. This is less revolutionary than it may appear since interpolation and approximation are intimately related. Both methods are applied for the generation of continuous surfaces from incomplete surface representations. The mathematical connections between the two concepts are discussed in detail by Cohen et al. (2000b) and Mallat (1999). Exact interpolators yield surfaces honouring all original data points, whereas a non-exact interpolation (or approximation) does not nessecarily pass through all original points (Laurini and Thompson (1992)).

Mitas and Mitasova (1999) (p. 481) define interpolation in a two-dimensional space as a bivariate function $F(r)$ which passes through all N given points

$$r_i = (x_i, y_i), \qquad i = 1, 2, \ldots, N \tag{2.1}$$

such that

$$F(r_i) = z_i, \qquad i = 1, 2, \ldots, N \tag{2.2}$$

with z_i being the sample values. Approximation functions (or non-precise interpolations) do not fit the original points exactly but find trend surfaces disregarding local variations (Jones (1997), p. 203). Moving averages are typical examples for non-exact interpolators (Laurini and Thompson (1992), p. 263). They work similarly to scaling functions used to derive approximations by the wavelet transform. In this thesis wavelets and the respective scaling functions are applied to derive a simplified surface from the original DTM. The concept of wavelets is shortly reviewed in the following section.

2.2 A Brief Introduction to Wavelets

In this chapter a short explanation of the wavelet transform will be given. At first the focus will be on a qualitative understanding rather than on mathematical details. The more formal section 2.2.2 will complete the theoretical understanding.
The general introduction to wavelets by Hubbard (1996), also available in German (Hubbard (1997)), is highly suggested for people with little mathematical knowledge. On the undergraduate level Nievergelt (1999), Stollnitz et al. (1995a), Triebfürst and Saurer (1999) and Frazier (1999) are a good base to start with. Detailed mathematical discussions are provided by Blatter (1998), Hernandez and Weiss (1996) or Mallat (1999), the latter being a very important exponent of the wavelet community. Another protagonist in the development of wavelets is Daubechies (1992). The considerations in the following section refer to Hubbard (1996) if not stated otherwise.

2.2.1 General Properties of the Wavelet Transform

Fourier transform versus wavelet transform. As Nievergelt (1999) points out, wavelets extend the Fourier analysis. The Fourier transform decomposes a signal into a linear combination of harmonic cosine functions. Predominant frequencies in a signal can thus be extracted and analysed. Looking at a signal from a frequency perspective has a number of advantages over the ordinary representation in the spatial domain which is known from Cartesian co-ordinate systems using the Euclidean basis. Unnecessary or perturbing frequencies can be suppressed in the Fourier domain, which is an important property for many applications such as signal compression, signal transmission or de-noising. Yet, it is not possible to localise the frequencies extracted by Fourier transform because cosine functions have infinite support. They are therefore space-invariant or time-invariant, depending on whether the investigated features appear in space or over time. Considering spatial or temporal distributions of frequencies is equivalent. For analysing non-stationary or transient phenomena with the occurrence of signal changes

at a particular location, such as a terrain, the space-invariant Fourier transform is not suitable.
Instead of infinite sinusoidal waves Morlet and Daubechies used a short waveform expressed by a function with compact support (Hubbard (1996)). Compact support of a wavelet basis means that it is zero everywhere outside a certain interval, for example between -1 and 2 (see Figure 2.1). Daubechies (1992) developed a recursively defined wavelet shown in Figure 2.1. Such a wavelet basis is spatially localised, due to its local support (Mallat (1999), p. 4, Hubbard (1996), p. 45). The Fourier basis vectors are able to detect a narrow range of frequencies in a signal but cannot detect their spatial location. The Euclidean basis vectors used in the Cartesian co-ordinate system, on the other hand, give the exact location of an object in space but no information about its wavelength (Frazier (1999), p. 167). Wavelets have both compact spatial support and also an oscillation. Due to these characteristics they form a basis which is localised both in the spatial and the frequency domain.

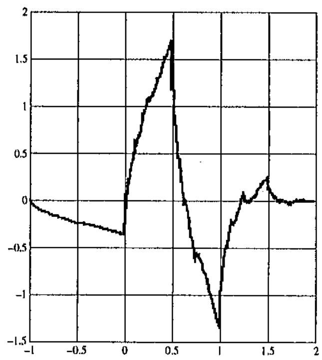
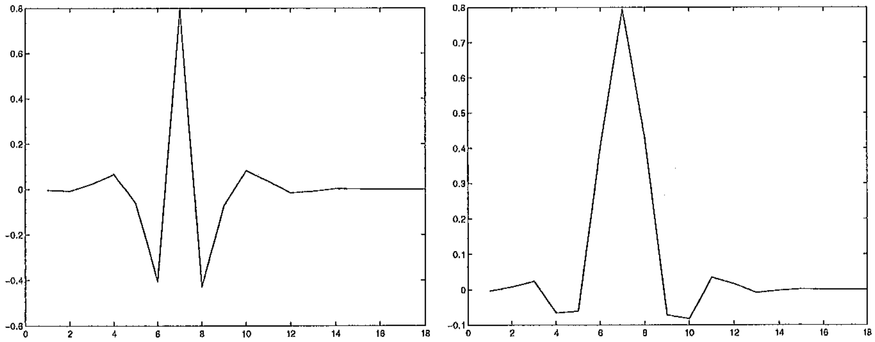


Figure 2.1: The recursively defined Daubechies wavelet applied in this thesis. From: Nievergelt (1999)

Uncertainty principle. The Heisenberg uncertainty principle imposes a limit to any signal representation: it cannot have a precise location and a precise frequency at the same time. Mathematically spoken the product of the uncertainty in space and the uncertainty of the frequency must exceed a minimal value. This also applies to the wavelet transform and expresses the trade-off between localisation in the spatial domain and localisation in the frequency domain (Hubbard (1996), p.49). The localisation in time/space and frequency/scale is best visualised by so-called Heisenberg boxes or time-frequency boxes introduced by Mallat (1999) (see Figure 2.3). To each wavelet coefficient a box in Figure 2.3 is assigned.

Fast wavelet transform. In order to examine a signal at different frequencies or scales it is analysed with dilated (stretched) versions of the original mother wavelet. The *fast wavelet transform* progressively drains the signal of its detail information using *orthogonal wavelets* of increasing support. Very small details are subtracted first and the remaining approximation is subject to detail extraction of the next coarser scale by the next larger wavelet. For the next coarser scale the wavelet is stretched by a factor of 2 and the information content of the approximation is halved. The approximation of each scale can also be computed directly using orthogonal scaling functions ('father' wavelets) instead of subtracting the details obtained by (mother) wavelets (see Figure 2.2). By using orthogonal wavelets (defined in section 2.2.2) the original signal can be split into redundancy-free detail coefficients. This implies that the sum of all levels of detail can reconstruct the original signal. The *fast wavelet decomposition* is concise which means that the original signal can be only reconstructed by using all coefficients.
Orthogonality between different levels of detail is also a principle feature of multiresolution analysis. It will be further discussed in section 2.3.



(a) (b)

Figure 2.2: Coiflet (according to R. Coifman): a) Wavelet function (mother wavelet), b) Scaling function ('father' wavelet).

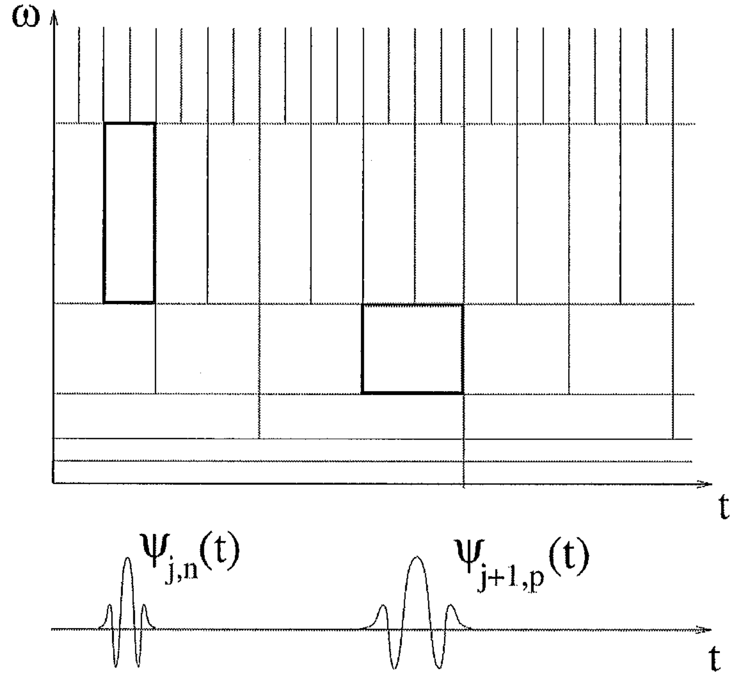


Figure 2.3: Heisenberg boxes. Each box representing a wavelet coefficient has a temporal/spatial (t) uncertainty and an uncertainty of frequency (ω). The more dilated the wavelet, such as $\Psi_{j+1,p}(t)$ the higher the temporal/spatial uncertainty and the better the frequency localisation. From: Mallat (1999)

2.2.2 Mathematical Foundation

General Formulations

As shown in Figure 2.4, the wavelet function ψ, which must have zero mean, is dilated by a scale parameter j and translated by u:

$$\psi_{u,j}(x) = \frac{1}{\sqrt{|j|}}\psi\left(\frac{x-u}{j}\right), \qquad u, j \in \mathbb{R}. \tag{2.3}$$

$\frac{1}{\sqrt{|j|}}$ is responsible for the normalisation of the wavelet transform. This means that the inner product equals 1 for all ψ:

$$\int \psi_{u,j}(x)^2\, dx = 1, \qquad \forall u, j. \tag{2.4}$$

The wavelet transform $W_{u,j}$ is obtained by the inner product between the

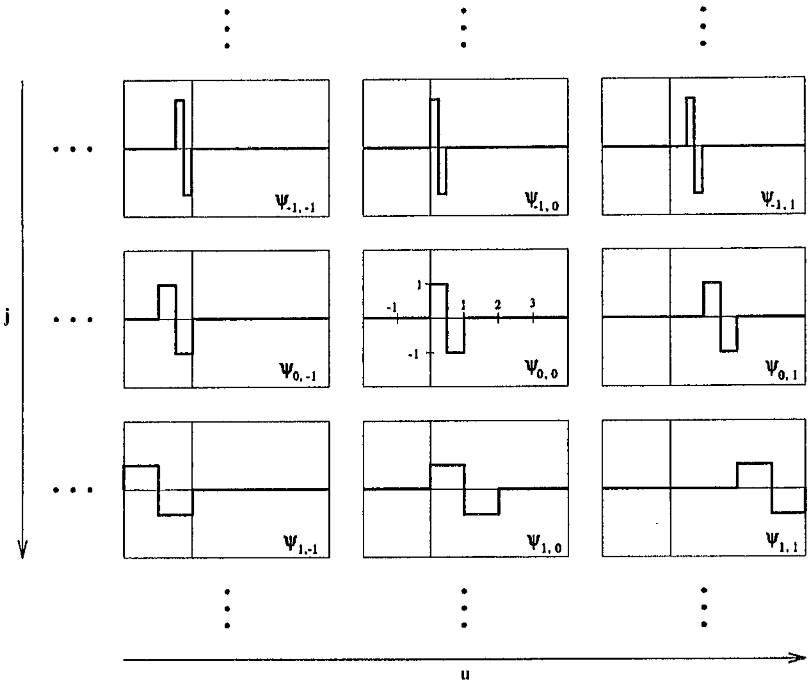


Figure 2.4: A Haar wavelet is dilated by scaling factor j and translated by u. From: Werschlein (1996)

signal $f(x)$ and the dilated and translated wavelet function:

$$W_{u,j} = \int f(x)\psi_{u,j}(x)\,dx, \tag{2.5}$$

which can be interpreted as the correlation or similarity of the two functions (Nievergelt (1999) p.124). The greater a wavelet coefficient the closer the wavelet resembles the signal. This characteristic is useful to recognise structures in the signal with forms similar to the wavelet used.

In the case of a *discrete* signal representation, which is necessary for digital terrain models (DTMs), the discrete wavelet transform is applicable. The standard inner product of two vectors of discrete signal points, such as rows in digital elevation models or discrete wavelets, is defined as

$$\langle f, g \rangle = \sum_u f_u g_u, \qquad \forall u. \tag{2.6}$$

The discrete wavelet coefficients are obtained by (Triebfürst and Saurer (1999)):

$$w_u^j = \langle f, \psi_{u,j} \rangle . \tag{2.7}$$

The signal to be transformed, $f(x)$, can be a cross-section of the DTM (e.g., a row or column of the DTM).

Fast Wavelet Transform

The fast wavelet transform uses *(bi-)orthogonal basis functions.* This is a powerful method to split the entire information directly into scale-dependent detail subspaces and approximation subspaces. It is the most frequently used wavelet transform in signal compression and transmission. Two vectors ψ and ϕ are orthogonal to one another if:

$$\langle \phi, \psi \rangle = 0, \tag{2.8}$$

in analogy to planar geometry using the dot product.
A transformation into concise wavelet coefficients requires wavelet bases which are orthogonal to one another:

$$\psi_{u,j} \perp \psi_{v,j}, \qquad \forall u, v. \tag{2.9}$$

In order to conform with the condition of orthogonality, the fast wavelet transform employs the same mother wavelet ψ as used in equation 2.3, with the difference that it is translated and dilated by defined steps (Mallat (1999)):

$$\psi_{u,j}(x) = \frac{1}{\sqrt{2^j}} \psi \left(\frac{x - 2^j u}{2^j} \right), \qquad j, u \in \mathbb{N}. \tag{2.10}$$

The details of a particular scale j are extracted by the wavelet $\psi_{u,j}$ as in 2.7, which works as a high-pass filter. At the same time the signal is convolved with a so-called *scaling function* ϕ_j working as a low-pass filter and thus extracting an approximation assigned to scale j (Triebfürst and Saurer (1999)):

$$a_u^j = \langle f, \phi_{u,j} \rangle . \tag{2.11}$$

The vector of all a_j is then subjected to both high-pass and low-pass filters at the next coarser scale, $j + 1$. This cascading filter process is depicted in Figure 2.5.
The scaling function, which also can be referred to as *father wavelet*, must be orthogonal to the wavelet. In an often cited capricious essay Strichartz (1993) expresses his utmost dislike for the label *father wavelet*: 'this shows a scandalous misunderstanding of human reproduction; in fact the generation of wavelets more closely resembles the reproductive life style of an amoeba'.
Orthogonality is a very important property of the fast wavelet transform and of multiresolution analysis. Orthogonal vectors allow, as stated in the previous section, not only a loss-less reconstruction of the original signal but also its representation without redundancy. It is therefore a concise

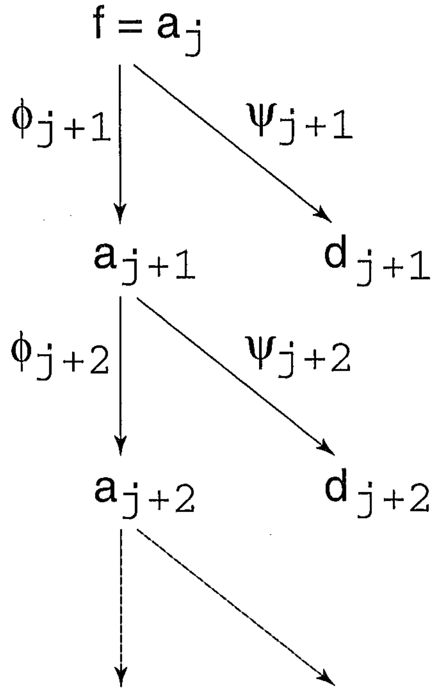


Figure 2.5: Cascading filter process of the fast wavelet transform. The signal a_j is analysed by a wavelet ψ_{j+1}, which extracts detail coefficients d_{j+1} of scale $j+1$, and by a scaling function ϕ_{j+1}, which extracts the approximation a_{j+1} for scale $j+1$. The approximation a_{j+1} is then subjected to the same procedure at the next coarser scale.

representation. Orthogonality of all ψ and ϕ to one another guarantees the reconstruction of the original signal by a simple linear combination. The sum of all detail levels and the approximation of the coarsest scale J exactly reconstructs the original:

$$f = \sum_{j=-\infty}^{J} \sum_{u} w_u^j \psi_{u,j} + \sum_{u} a_u^J \phi_{u,J}. \tag{2.12}$$

Two-dimensional Wavelet Transform

Several types of wavelet transforms have been developed in order to deal with two-dimensional data. This investigation is based on the tensor product wavelet transform. The two-dimensional decomposition using a tensor product wavelet requires the following tensor products between the one-

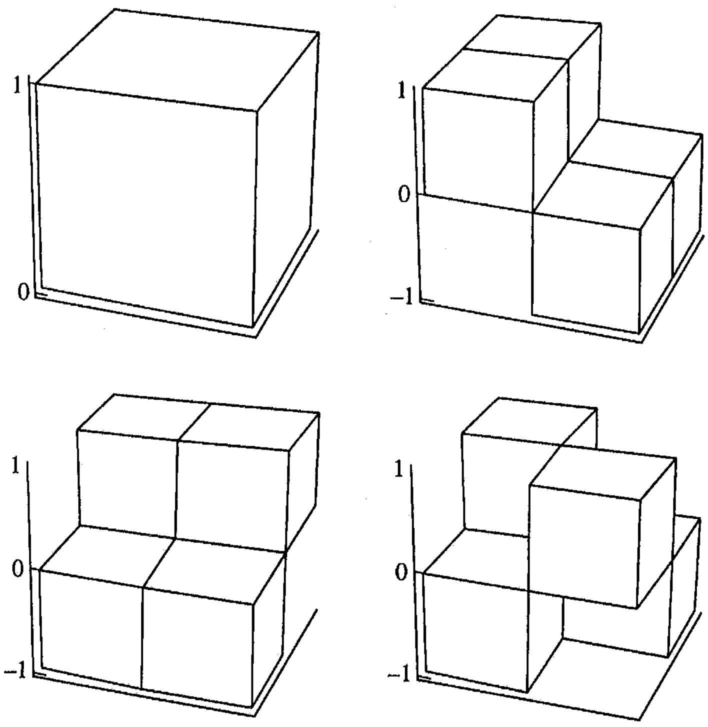


Figure 2.6: Tensor product Haar wavelet. Top left: scaling function; Top right: vertical detail extraction; Bottom left: horizontal detail extraction; Bottom right: diagonal detail extraction. From: Nievergelt (1999).

dimensional wavelet and the one-dimensional scaling function (Nievergelt (1999)):

$$\mathbf{\Phi} = \phi \otimes \phi, \quad \mathbf{\Psi}^{\mathbf{h}} = \phi \otimes \psi, \quad \mathbf{\Psi}^{\mathbf{v}} = \psi \otimes \phi, \quad \mathbf{\Psi}^{\mathbf{d}} = \psi \otimes \psi. \tag{2.13}$$

For each scale an approximative data set and three detail extraction matrices are generated, each of which being one quarter of the size of the coarser approximation. $\mathbf{\Psi}^{\mathbf{h}}$ extracts horizontal changes in the data, $\mathbf{\Psi}^{\mathbf{v}}$ vertical details and $\mathbf{\Psi}^{\mathbf{d}}$ diagonal details respectively. This is shown in Figure 2.6.
For a two-dimensional wavelet transform the resulting wavelet coefficients now require two translational parameters, u and v in order to identify the coefficient at location (u, v). Additionally, for each scale a matrix of horizontal, vertical and diagonal wavelet coefficients are computed, equivalent to equation 2.7 for the one-dimensional case:

$$w_{uv}^{j(h)} = \left\langle f, \psi_{uv}^{j(h)} \right\rangle, \quad w_{uv}^{j(v)} = \left\langle f, \psi_{uv}^{j(v)} \right\rangle, \quad w_{uv}^{j(d)} = \left\langle f, \psi_{uv}^{j(d)} \right\rangle. \tag{2.14}$$

The fast tensor product wavelet transform is implemented by alternating applications of the one-dimensional transform to each row of the two-dimensional data and subsequently to all of the obtained columns. This is equivalent to using the two-dimensional tensor product wavelets, such as those from Figure 2.6.

2.2.3 Stationary Wavelet Transform

The stationary wavelet transform introduced by Nason and Silverman (1995) is very similar to the standard discrete wavelet transform. The wavelets are not translated by $2^j u$ as for the fast wavelet transform but by intervals equal to the sampling distance:

$$\psi_{u,j}(x) = \frac{1}{\sqrt{2^j}}\psi\left(\frac{x-u}{2^j}\right), \qquad j,u \in \mathbb{N}. \tag{2.15}$$

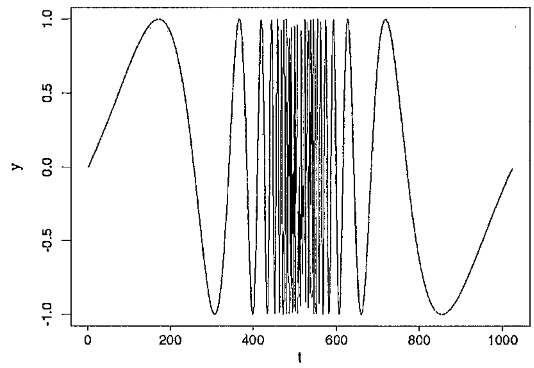
In contrast to the fast wavelet transform the information is not halved with each approximation step. The result is a highly redundant representation of the original signal. For every point in the original signal a stationary wavelet coefficient is computed. For data compression this is not feasible. However, for certain applications, such as feature recognition, the ability of analysing the signal on a point by point basis proves to be very useful. The analysis of signals requires wavelet coefficients which are independent of the choice of the origin. This is the case for the stationary wavelet transform, whereas the fast wavelet transform results in different coefficients if a different starting point in the signal is chosen. The analysis of a chirp signal using both fast and stationary wavelet transform is illustrated in Figure 2.7.

The measurement of gradient, curvature, or some higher order derivatives can be obtained directly from wavelet coefficients. This is a beneficial property of wavelets with specific support described by Beyer and Meier (2001) (see appendix). The stationary wavelet transform applied in chapter 5.3.3 can derive such information for each point of the terrain which results in a more coherent picture.

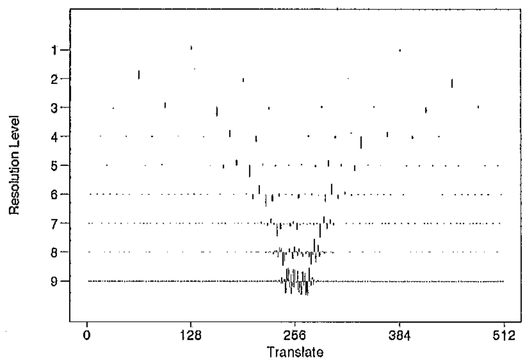
2.2.4 Extensions

Lounsbery et al. (1997) have developed an extension to the two-dimensional wavelet transform, applying wavelets to surfaces of arbitrary topological type. Feng and Jiaoying (1997) further extend this algorithm to allow the application of multiresolution analysis. Jawerth and Sweldens (1994) also sketch an expansion of wavelets to more than two dimensions.

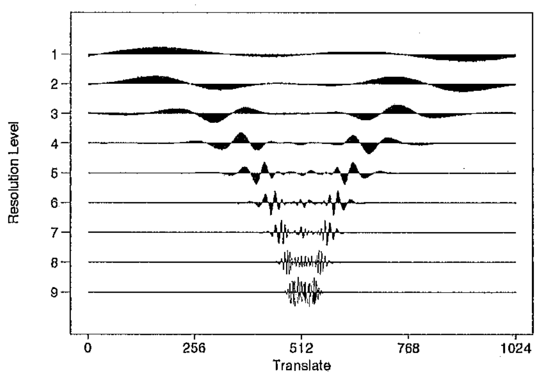
The fast wavelet transform described in this chapter is limited to equally spaced, regular data. It is often applied in practice. Handling irregularly spaced data is addressed by the continuous wavelet transform. Sardy et al. (1999) propose four different techniques to deal with irregularly spaced data



(a) Chirp signal (original)



(b) Fast wavelet coefficients



(c) Stationary wavelet coefficients

Figure 2.7: Comparison of the fast wavelet transform (decimated coefficients) with the stationary wavelet transform (one coefficient for each signal point). From: Nason and Silverman (1995).

whereas Kovac and Silverman (2000) suggest to interpolate the unequally distributed points to a regular grid of a suitable size. An informative paper by Bonneau (1998) introduces the combination of irregular surface meshes represented by hierarchical Delaunay triangulation with a wavelet-like decomposition. Such algorithms could be applied directly to triangulated irregular networks (TINs).

2.3 Multiresolution Theory

The concept of multiresolution has a number of advantages for surface modelling. It is gaining increasing importance not only for computer graphics researchers but also for the GIS community and terrain modellers (De Floriani and Magillo (1999)). Multiresolution analysis offers inherent scalability and simple access to the data at various scales. Scale problems are tackled in a more sophisticated way than just by means of re-sampling the model at arbitrary resolutions. Furthermore, it is often used as a result of computational considerations, for example, for image transmission. In a multiresolution data structure the transition from one approximation step to the next finer one is straight forward. This is a particularly helpful property, for example, for image transmission over the Internet. This allows the resolution of a transmitted picture to be progressively refined.

The fast wavelet transform perfectly complies with multiresolution. Thus, the advantages of multiresolution analysis can be easily exploited by a wavelet transformed model. The structure of a signal, for example a terrain, can be assessed at various scales. This is very important for the analysis of a digital terrain model. It can be used to detect and extract high frequency areas, such as forests, or as a basis for terrain generalisation.

Burt (1984) had described a wavelet-like multiresolution pyramid before the term wavelet was actually known. He applied a Gaussian pyramid as a low-pass filter, equivalent to the scaling function, and a Laplacian pyramid as a high-pass filter in analogy to the wavelet. He could thus detect edges belonging to various scales in a picture. Multiresolution analysis is not only applicable to wavelet transforms but also to hierarchical terrain models consisting of data structures such as quadtrees, which will be introduced in section 3.3.2.

For completeness the mathematical conditions to satisfy the requirements of multiresolution analysis are listed below. For a more detailed, qualitative explanation with simple examples see Hubbard (1996) and for a mathematical definition refer to Mallat (1999).

The space V_0 can be viewed as the space which is generated by the scaling function at scale $j = 0$ together with all its translations. As the scale j increases, the resolution of functions in V_j decreases. A sequence $\{V_j\}_{j \in \mathbb{Z}}$ of closed subspaces of $L^2(\mathbb{R})$ is a multiresolution approximation if the following conditions are satisfied (Mallat (1999), p. 221; Hubbard (1996), p. 145):

$$\forall (j,u) \in \mathbb{Z}^2,\ f(x) \in V_j \Leftrightarrow f(x - 2^j u) \in V_j \quad \text{(Translation)}, \tag{2.16}$$

$$\forall j \in \mathbb{Z},\ V_{j+1} \subset V_j \quad \text{(Nested subspaces)}, \tag{2.17}$$

$$\forall j \in \mathbb{Z},\ f(x) \in V_j \Leftrightarrow f(\frac{x}{2}) \in V_{j+1} \quad \text{(Scaling function)}, \tag{2.18}$$

$$\lim_{j \to +\infty} V_j = \bigcap_{j=-\infty}^{+\infty} V_j = \{0\}, \tag{2.19}$$

$$\lim_{j \to -\infty} V_j = \overline{\left(\bigcup_{j=-\infty}^{+\infty} V_j \right)} = L^2(\mathbb{R}). \tag{2.20}$$

2.19 means that when the scaling function is stretched sufficiently, at infinite resolution, there is no detail information left. The last condition implies that any original signal in $L^2(\mathbb{R})$ can be reconstructed exactly.

Chapter 3

Methods for Surface Representation and Simplification

In this chapter some fundamental theoretical notions and methodological constraints of surface modelling are considered. The often underrated issue of ethical and socio-structural implications of GIS is discussed (see section 3.1.1). Some basic representations and approximation techniques of digital surfaces are then reviewed. The main focus will be on so-called '2.5-dimensional' digital terrain models rather than on general three-dimensional surfaces investigated by computer graphics researchers. Abundant literature on the topic of surface simplification is available. However, some specific issues only applicable to digital terrain models tend not to be taken into account by most methods of computer graphics research.

3.1 Models of Landscapes

Kemp (1996) reviews the importance and several definitions of the term 'data model' in detail. A short definition based on Peuquet and Marble (1990) is given in McDonnell and Kemp (1995): 'In a general sense a data model is an abstraction of the real world which incorporates only those properties thought to be relevant to the application at hand.' The model does mimic reality in order to make it conceivable to the user.

3.1.1 Limitations and Ethical Considerations

If a model is used to represent reality it is inevitable that there are important limitations. To understand the ethical concerns expressed by exponents such as Curry (1998) or Pickles (1995) the controversial idea of complete knowledge and complete control over spatial information by the users of a future global GIS are considered:

> Mirror Worlds?
>
> What are they?
> They are software models of some chunk of reality, some piece of the real world going on outside your window... A Mirror World is some huge institution's moving, true-to-life mirror image trapped inside a computer – here you can see and grasp it whole...
> The picture on your screen represents a real physical layout... Now you see inside a school, courthouse, hospital, or City Hall... Eavesdrop on decision-making in progress...
>
> (Gelernter: Mirror Worlds: Or the day software puts the universe in a shoebox: How it will happen and what it will mean. In: Curry (1998))

To some affluent sections of society this might look like a bright future of simple knowledge transfer and discovery in geographical data mining. Alternatively, it can be seen as a vision of a development towards a horrific 'brave new world', where individuals interact with computer models rather than with the real world. Orwell's '1984' also springs to mind where humans are watched and controlled permanently. Gelernter was one of the victims of Theodore Kaczynski, the Unabomber.
Two issues are felt to be important and have to be taken into account. Firstly, the transformation of parts of the world into information limits the model's ability to express the many elements and activities of reality (Curry (1998)). Roberts and Schein (1995) view all representations of space as artificial abstractions within constructed rules determined by our interpretation of the socio-spatial world. They thus depend on the ideas the scientist has of the world:

> 'In essence, a data model captures the choices made by scientists and others in creating digital representations of phenomena, and thus constrains later analysis, modeling and interpretation'
>
> (Goodchild et al. (1995), p. 10) in Kemp (1996).

It might look as if a model is nothing more than a poor juxtaposition to the real world. In fact it is very important to assess any model-induced constraint and its consequences. The abstraction from reality is the only way to understand reality scientifically. However, any human perception implies some sort of abstraction. The awareness of the deficiencies of the model is seen as a very important aspect of modelling reality. Assuming that a terrain can be represented by a sum of wavelet-shaped components is one has to consider that this might be inadequate. A wavelet which does not resemble a natural terrain is likely to infer artefacts such as structures looking like the wavelet rather than the actual terrain. These effects are visualised in Figure 5.5.

Secondly, we must consider the notion of power being inherent to a representation of space trying to capture reality in the computer. GIS has not only been traditionally affiliated with military institutions (Curry (1998) p.174) but also with geodemographics and geomarketing. They are often seen as an assault on individual privacy (Curry (1998) p.120). Roberts and Schein (1995) suggest that by using GIS and 'by the view from above we establish our own superiority and domination of the scene'. Such considerations must not be underrated. For example, if surface models are used to detect forested areas and control subsidised farming areas one must be aware of issues such as offending the sensibilities of local farmers. Notions such as 'otherisation of the invisible' are concepts discussed by Hooks (1992) and Jackson (1992). They offer a theoretical framework to approach the subject-object dualism and other such social implications.
After this digression we are now going to focus on more specific issues of modelling terrain surfaces.

3.1.2 Data Models for Digital Terrain Models

Digital Elevation Models (DEMs) are representations of the elevation of land surfaces. They are represented as two-dimensional fields, meaning that for each (x, y)-location there exists only one z-value. This is considered adequate for most cases because the exceptions, such as overhangs, caves and natural bridges, are relatively small and rare (Martinoni (2001)). Digital Terrain Models (DTMs), representing the ground surface, can also include topological data (McDonnell and Kemp (1995)). According to Schneider (1998), a digital terrain model is based on sampling points, their connections, an interpolation function and additional semantic information. Jones (1997) describes a rough classification of surface representations that is applicable to terrain models. The major classes are incomplete representations such as point samples (e.g. grids or irregular points) or line samples (e.g. contours and structure lines) and complete representations consisting of contiguous zones such as discontinuous (e.g. raster cells) or continuous

(e.g. TINs or splines) surface patches. The most widely used models are based on grids or on triangulated irregular networks (TINs). For a detailed overview of tessellations please refer to Laurini and Thompson (1992).
As stated by Goodchild et al. (1995) all models have their specific constraints and artefacts. Incomplete grids imply anisotropic behaviour along the sampling directions and are prone to under- or oversampling. TINs, in contrast, have no implicit neighbourhood properties but are usually better adaptable to the original data points. A highly variable surface can be represented by a high number of points in a TIN, whereas a flat terrain is also sufficiently approximated using sparse points.
The applied data model needs to be chosen according to the purpose of the terrain model. As the tensor product wavelet transform is based on regularly distributed points the grid is seen as the most appropriate data structure for this research. In section 4.5.1 a special representation of wavelet coefficients is proposed according to their local relevance. This is useful to assess the local coincidence of various levels of detail.

3.2 Filtering and Generalisation of DTMs

Weibel (1992) identifies two types of generalisations: *cartographic generalisation* for the purpose of visual enhancement and *statistical generalisation* which reduces data according to a form of statistical control. Other authors (as well as Weibel in more recent publications, e.g. Weibel (1995)) term this *model generalisation* or *database generalisation.* Both methods are important in GIS. Moreover, he states that the principle idea of generalisation is to be both structure- and purpose-dependent. Local terrain structure recognition is thus seen as an integral part of generalisation. A conceptualisation similar to cartographic generalisation has been given by McMaster and Shea (1992) (in Weibel and Dutton (1999)) termed 'spatial and attribute transformations'.
The cartographic generalisation processes for terrain generalisation outlined in Weibel (1992) are *selection/elimination, simplification, combination, displacement* and *emphasis.* He proposes a heuristic generalisation method which operates according to all these five processes. Global filtering, as it is used in image processing, and so-called selective filtering, such as the TIN filters described in section 3.3.1, cannot satisfy more than one of these generalisation processes.
Fully automated cartographic generalisation has not yet been achieved by algorithmic approaches. The approximation techniques reviewed in the following sections mainly refer to model generalisation. However, it is believed that some cartographic generalisation processes can be simulated by the filtering in the wavelet domain.

The wavelet-based filter presented in chapter 4 is not thought to be able to mimic all aspects of cartographic generalisation. However, in combination with a subsequently applied TIN filter, such as the one proposed by Heller (1990), a procedure more closely related to cartographic generalisation can be achieved. In particular, the processes of selection/elimination and emphasis of selected landforms are modelled in a more elaborate way than by simply applying a global maximum error.
Analogies between wavelets and vision are described in detail by Hubbard (1996). Similarities are apparent as the visual system also needs to identify both localisation of spatial features and frequencies. A more human-related perception of two-dimensional pictures and landscapes could possibly be helpful to better imitate processes performed by human labour. Operations, such as combination or displacement of features, can take place unintentionally when applying linear filters to a wavelet transform (see section 4.1). An example of the combination of two adjacent peaks is given by Watson and Jones (1993) who report on the occurrence of phantom peaks between two neighbouring maxima. This was a result of the correlation with a low frequency positive wavelet pulse.

3.3 Surface Simplification Methods

A vast amount of literature on the topic of surface simplification is available. The most efficient algorithms prevail not only in digital terrain modelling but also in the fields of computer vision, computer graphics and computational geometry. In this chapter only a very short review is given of the most significant developments concerning terrain modelling. A good survey of the evolution of surface simplification methods in different research communities has been conducted by Heckbert and Garland (1997) and Garland (1999).

3.3.1 Data Reduction

Selective Refinement

Fowler and Little (1979) affirm that a grid must be adjusted to the roughest terrain in the model and is therefore highly redundant in smoother parts. This reflects what Weibel (1992) terms *structure dependence*. Furthermore, they state the very important principle that various purposes demand various resolutions, which is also termed *purpose dependence*. This issue is of increasing importance due to higher resolutions of present day DTMs, as stated in the introduction.
Fowler's and Little's automatic TIN extraction algorithm includes points into an initial set of structurally important points in order to reduce the

maximum error between the grid and the TIN. This procedure is repeated until a specified maximum tolerance is not exceeded anymore. This principle is also referred to as the *hierarchy method* (Lee (1991)) or more commonly *refinement method* (Heckbert and Garland (1997)).
Heller (1990) uses highly efficient algorithms to compute an *adaptive triangular mesh filter* (ATM) based on such a maximum distance criterion. He also shows that the inclusion of breaklines and discontinuities can be realised by forcing triangle edges to pass along such breaklines. Moreover, the initial set of points is chosen more appropriately than it is done by Fowler and Little (1979) and the Delaunay criterion can be maintained.
The starting points of the ATM are significant extrema of cross-sections of the DTM. For example, a local minimum is considered significant, if it is a global minimum in a basin of a given depth. To this initial set the point with the highest vertical distance is included and, if necessary, the triangle edges are swapped in order to conform with the Delaunay criterion. This steps are also compiled in Weibel (1997). By these constraints, Heller's (1990) ATM filter is able to reduce redundancy in DTMs very efficiently. It is therefore applied in this study for data reduction, subsequent to the wavelet filter.

Decimation

Lee (1991) presents the opposite solution to the same problem: instead of starting with an initially minimal set of points his *drop heuristic method* begins with the entire terrain model and excludes the least important point. This point dropping algorithm is repeated until an optimal representation of the terrain at a predefined error tolerance or at a given number of points is achieved. Furthermore, Lee (1991) uses an interesting evaluation method based both on statistical and structural comparisons to assess the quality of the drop heuristic method. The drop heuristic method not only has a smaller standard deviation to the original than refinement methods (excluding Heller's), but is also able to represent structural features such as peaks, pits or passes slightly better. The structural features are extracted by a local operator on a next-neighbourhood basis. The drawback of this method, however, is that it cannot guarantee that the predefined error tolerance is nowhere exceeded (van Kreveld (1997)).
Decimation methods has received more attention in the computer graphics and visualisation community than in terrain modelling (Heckbert and Garland (1997)). For example Schroeder et al. (1992) applied a decimation algorithm to three-dimensional data.

3.3.2 Multiscale Data Structures

Multiresolution TINs

De Floriani et al. (1996) introduce the idea of multiresolution to terrain simplification using hierarchical subdivisions of TINs. Most multiresolution models are based on *hierarchical models*. They consist of a nested subdivision of the terrain model. Traditionally, *regular hierarchical models* were used, such as *quadtrees* or *quaternary triangulations*, as they are shown in Figure 3.1 (De Floriani and Magillo (1999)). Their major problems are the discontinuities between refined and unrefined subdivisions. These are also termed cracks or hanging nodes (Gerstner and Hannappel (2000), Gross et al. (1996)), which will be discussed in the following section.

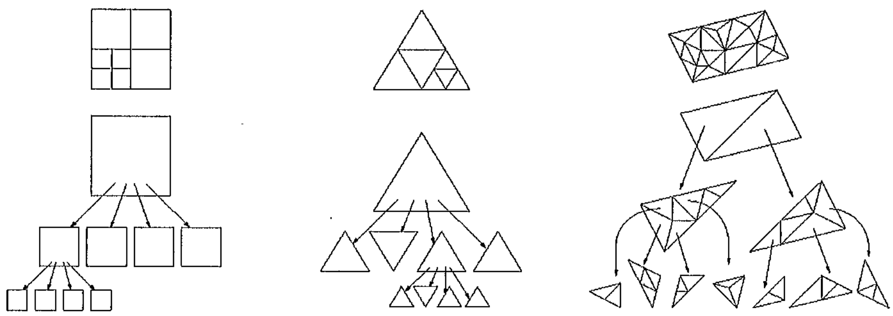


Figure 3.1: Hierarchical models: Quadtree, quaternary triangulation and hierarchical TIN. From: De Floriani and Magillo (1999)

More recently, De Floriani et al. (1998) have proposed the use of multi-triangulations based on refinement or decimation algorithms as a general multiresolution model for TINs. A collection of local updates tackles the problem of various possible re-triangulations. The surface simplification process can be represented by a directed acyclic graph. Each fragment of the TIN is connected to its locally updated versions, which solves the problem of spatial interference between adjacent subdivisions (non-matching edges) (see Figure 3.2). Using such local updates a locally higher accuracy can be achieved. This is an interesting property as in many applications some locally higher level of detail in specific locations is requested. For example areas closer to the observer usually need to be represented more accurately than background information (Magillo et al. (2000)).

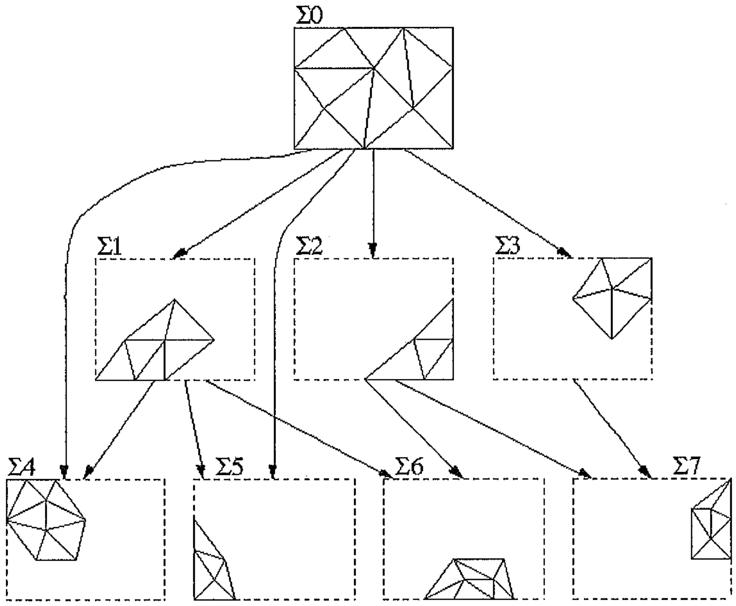


Figure 3.2: A graph representing a multi-triangulation: Each fragment is connected to its local update. From: De Floriani and Magillo (1999)

Regular Hierarchical Models

The vital advantage of hierarchical models is that they satisfy the conditions of multiresolution. The hierarchical subdivisions are typically characterised by a tree structure (Figure 3.1). Irregular hierarchical models are based on TINs whereas regular hierarchical models can be structures such as quadtrees or quaternary triangulations (see Figure 3.1). Regular hierarchical models are recursive partitions of simple geometric forms such as squares or regular triangles. They all require special attention to deal with the problem of discontinuities, so-called cracks between refined and unrefined subdivisions. These phenomena occur when new vertices are inserted on edges belonging to unrefined subdivisions. This issue is often approached by a restriction of the refinement in such a way that adjacent partitions must not differ by more than one level in the refinement operation. The problematic vertices, where cracks appear, are then easily re-triangulated according to a few distinct patterns (Gross et al. (1996), De Floriani et al. (1996)).
Regular hierarchical models are considered to yield approximations of poorer quality than irregular triangulations, yet they are widely used due to their simple handling and their multiresolution characteristics (Heckbert and Garland (1997)). Brigger et al. (1999) developed an improved type of quadtree

for approximation called *centered pyramid.*
Irrespective of the approximation quality they can achieve, quadtrees foster data models which are not only highly efficient but also analogous to the structure of the fast wavelet transform. They thus offer an appropriate tool to tackle the problem of fast information extraction from wavelet coefficients. For example Gerstner and Hannappel (2000) and Gross et al. (1996) use wavelets as a refinement criterion for surface approximation. The next section will focus on their investigations.

3.4 Wavelet-based Approximations

Gerstner and Hannappel (2000) implemented a regular hierarchical triangulation using a binary tree based on isosceles triangles with a right angle at each vertex akin to a quadtree with squares. Each refinement vertex is formed when bisecting a triangle and an error indicator is assigned. By choosing a sufficiently small error threshold ε, a crack exceeding ε forces the adjacent triangle to a further refinement and the discontinuity can thus be eliminated. The fast wavelet transform using pyramidal hat functions coincides very well with the binary tree data structure. A wavelet coefficient can be assigned to each refinement vertex. The wavelet coefficients are set to zero at all vertices where the corresponding error indicator does not exceed ε. An approximation is achieved which is bound by a maximum difference to the original. Due to the simple wavelet basis the error can be derived directly from the wavelet coefficients.
An interesting concept is the locally increased level of detail at peaks, pits and passes. Gerstner and Hannappel (2000) term this 'topography preservation'. This is realised by setting the error indicator at these topographically crucial points to infinity. They claim that by retaining these points accurately the drainage geometry can be preserved. This idea is referred to in section 5.4.

Gross et al. (1996) use a quadtree as underlying data structure of their model. They first apply a non-linear filter (cf. section 4.1) to the wavelet-transformed terrain. Subsequently a quadtree meshing is employed. This also takes advantage of the similarity between quadtrees and fast wavelet transforms. The most important criterion for vertex removal is again the difference to the original surface which can be computed by a single-step inverse wavelet transform.
Similar to Gerstner and Hannappel (2000) they propose a method to locally 'zoom' to a higher resolution, called the *level-of-detail filtering* in the wavelet domain. The basic idea is to generate a locally higher detail level by weighting the wavelet coefficients in the region of interest by some weight-

ing function. This is possible as a result of the localisation properties of wavelets. In the successive non-linear filtering step the coefficients associated with a large weighting factor gain an advantage over the ones outside the region of interest and are retained by a higher probability. This results in a locally more accurate representation of the terrain. This effect is described as a 'magnifying glass'.

A similar concept will be applied in this investigation. However, the region of interest will correspond to terrain features of interest, rather than to the areas close to the observer which is usually the focus in terrain visualisation.

Gallant and Hutchinson (1996) apply the positive wavelet analysis introduced by Watson and Jones (1993) to a digital terrain model. It can thus be subdivided into terrain features of various scales. The analysing wavelet is an elliptical feature with the parameters location, length, width, orientation and height. The thus derived landforms are not only useful for the analysis of scale dependence in topography, but they can also be superimposed to yield an approximated terrain model. The results look promising, yet it is suspected that analysing a more rugged terrain than the smooth rolling hills of the presented example would exhibit the limitations of the proposed method. It has been found that the approximations tend to reflect the form of the wavelet. A wavelet with similar properties as the terrain will also yield good visual results.
The positive wavelet analysis seems to be an appropriate tool for geophysics. Due to the many parameters, however, the implementation is complicated and consequently computationally costly. It also places higher demands on the user and user interface, due to the many parameters.

Chapter 4

A Conceptual Framework for Scale-Dependent Wavelet Filtering

In the first section of this chapter, two common approximation types used for data reduction will be reviewed. They will provide a starting point for the more semantically based information reduction of adaptive filtering. A vital advantatage of wavelets is that they can be localised in the terrain model. Thus, it is possible to detect waveforms of various size in the terrain. Their importance can be assessed by the respective wavelet coefficients which represent the amplitude of the waveforms. Due to the localisation properties waveforms of various scales can be associated with each other. A filtering procedure, which is locally adaptive to significant features of a chosen scale, is described in the following sections.

4.1 Linear and Non-linear Approximation

Significance of wavelet coefficients. As mentioned in section 2.3 multiresolution analysis offers a representation of the terrain at a variety of scales. The fast wavelet transform decomposes a surface into wavelet-shaped components of different wavelengths and amplitudes. Therefore the choice of wavelet has an effect on the quality of the approximation. The scale of the coefficients gives an estimate of the wavelength of the landscape components in the respective scale. It depends on the support of the mother wavelet and the resolution of the DTM. Table 4.1 identifies the approximate size of the principal landforms in the scale classification used here. However, the considerations in section 4.3 suggest that the scales of such landforms can only be represented in a fuzzy way. Thus, table 4.1 can only be used as a rough guideline.

	Feature size (Daubechies 10)	Feature size (Daubechies 4)
Scale 1	10m	4m
Scale 2	20m	8m
Scale 3	40m	16m
Scale 4	80m	32m
Scale 5	160m	64m
Scale 6	320m	128m
Scale 7	640m	256m
Scale 8	1280m	512m

Table 4.1: Rough size estimate of landscape features associated with the scales of the wavelet coefficients computed for this investigation.

Linear approximation. Say a signal f is represented in an orthonormal basis $\mathcal{B} = \{g_m\}_{m \in \mathbb{N}}$. A linear approximation considers only M basis vectors which are chosen *a priori* (Mallat (1999)):

$$f_M = \sum_{m=0}^{M-1} \langle f, g_m \rangle \, g_m. \tag{4.1}$$

If using wavelet bases and scaling functions as an orthonormal basis, usually the wavelet functions associated with high frequencies are neglected. The high frequency features are disregarded, even if they are significant. This results in uniformly smooth approximations of a fixed resolution (see Figure 4.1). The error equals the sum of the squared inner product of the rejected detail levels, which is fast and simple to compute (Mallat (1999), p. 12).

Non-linear approximation. In contrast to the linear approximation, the non-linear approximation consists of the coefficients with the highest impact. The coefficients are 're-ordered' according to their significance (Cohen et al. (2000a)) to minimise the deviation from the original. Using the notation of $\mathcal{B} = \{g_m\}_{m \in \mathbb{N}}$ as orthonormal basis the non-linear approximation is:

$$f_M = \sum_{m \in I_M} \langle f, g_m \rangle \, g_m. \tag{4.2}$$

I_M is the set of basis vectors chosen *a posteriori* with the largest inner product amplitude $|\langle f, g_m \rangle|$ (Mallat (1999)). Figure 4.1 illustrates that features of any frequency are preserved, provided they have a large amplitude. Both Mallat (1999) and Cohen et al. (2000a) state that for data compression non-linear approximations are superior to linear approximations. Even functions with isolated singularities are well approximated with non-linear schemes. In practice, the filter usually applies a threshold to the wavelet coefficients. Say t_k is a fixed value for each scale, such as the mean value of the coefficients of the respective scale, the wavelet coefficients with a higher magnitude than t_k are retained. Wavelet coefficients smaller than t_k are set to 0 in the resulting matrix of coefficients B^k:

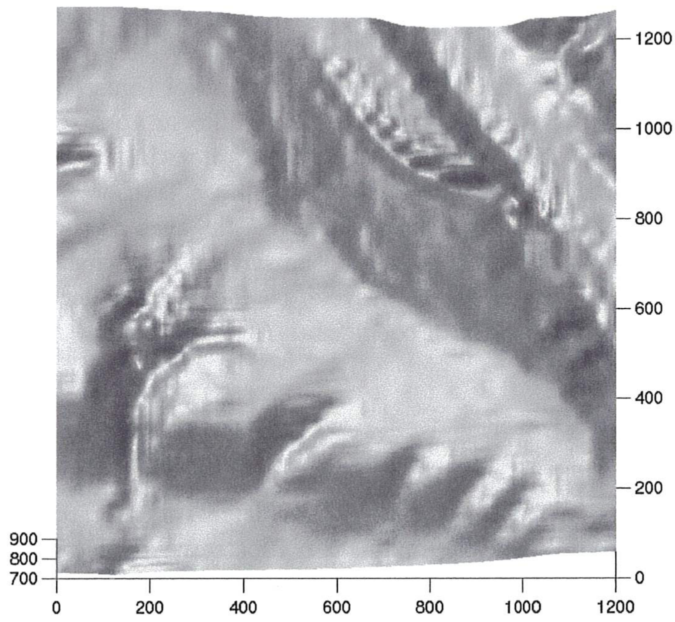
$$B^k = \{b^k_{uv}\} = \begin{cases} w^k_{uv} & : \quad w^k_{uv} > t_k \\ 0 & : \quad \text{otherwise} \end{cases} . \tag{4.3}$$

The reconstruction from the resulting coefficients b^k_{uv} stored in the matrix B^k yields the non-linear approximation illustrated by Figure 4.1. When using the tensor product wavelet transform (see section 2.2.3) this process is applied to the matrices of horizontal ($B^{k(h)}$), vertical ($B^{k(v)}$) and diagonal ($B^{k(d)}$) detail coefficients for each scale.

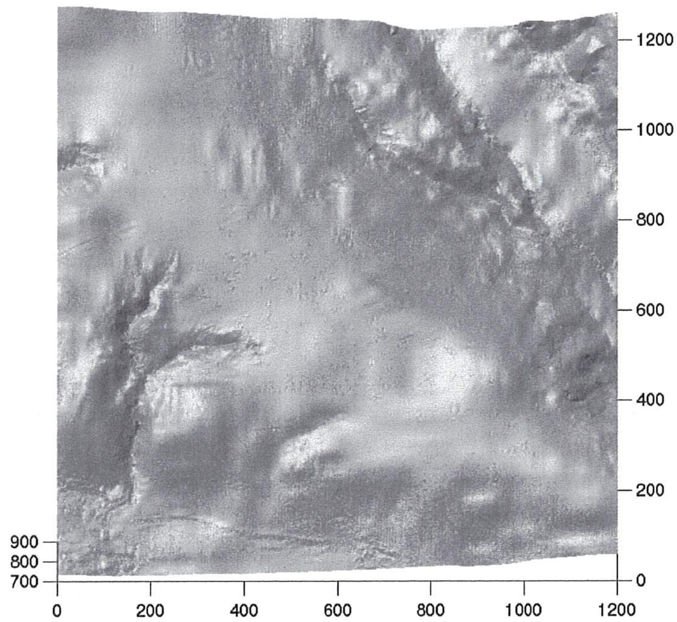
4.2 Objective of Scale-dependent Approximation

In order to enhance and combine the characteristics of linear and non-linear filters the concept of adaptation to local features of the scale of interest is proposed. As mentioned in section 4.1, linear filters smooth the terrain due to the preservation of solely long waves, whereas non-linear approximations represent the surface using all waves exceeding a chosen threshold amplitude. This results in a somewhat irregular landscape where not only large peaks and troughs are well-preserved but also small-scale features.

The aim of adaptive filtering is not data compression as in linear and non-linear approximations, but controlled reduction of information. The objective of the suggested adaptive filter is to retain both form and sharpness of features belonging to the scale of interest. In this investigation a coarse scale of interest has been chosen, such as scale 6 or 7 (see table 4.1). Coarse-scale peaks, valleys and ridges are to be reconstructed with high accuracy. Especially extrema and structure lines that are hydrologically important should not differ much from the original model. However, areas of lesser importance in the chosen scale, such as steady slopes and flat areas, can be smoothed considerably without losing vital information. Dikau (1989) proposes a set of hierarchically ordered components consisting of geometric



(a) Linear approximation



(b) Non-linear approximation

Figure 4.1: Comparison of linear and non-linear approximation. Terrain data: Bundesamt für Landestopographie © (2000) (DV002247).

features which are categorised in terms of differing size and complexity (see Figure 4.2). These features are classified into macro- meso- and micro-scale. If such components are transferred to the wavelet theory they can be seen as the detail information of the varying scales. In such a system, a significant coarse-scale component could trigger the preservation of the finer scale features occurring 'on top' of the coarse-scale components. In terms of generalisation this process is referred to as *selection and elimination.*

	Main type of size order			Type of size order			
	W(m)	A(m^2)	H/D(m)	W(m)	A(m^2)	H/D(m)	
MEGARELIEF	$>10^6$	$>10^{12}$		$>10^6$	$>10^{12}$		Canadien shield
	10^6	10^{12}		10^6	10^{12}		
B			$>10^3$			$>10^3$	
MACRORELIEF				10^5	10^{10}		Mountain area, the Alps, the Rhine Graben
A							
	10^4	10^8	10^3	10^4	10^8	10^3	
B							
MESORELIEF				10^3	10^6	10^2	Valley, moraine, hills, cuesta scarps
A							
	10^2	10^4	10^1	10^2	10^4	10^1	
B							
MICRORELIEF				10^1	10^2	10^0	Gully, ice-wedge, doline, dune, terrace
A							
	10^0	10^0	10^{-1}	10^0	10^0	10^{-1}	
NANORELIEF							Karren, tafoni, erosion rills
	10^{-2}	10^{-4}	$<10^{-1}$	10^{-2}	10^{-4}	$<10^{-1}$	
PICORELIEF	$<10^{-2}$	$<10^{-4}$		$<10^{-2}$	$<10^{-4}$		Glacial striations

W = width of unit A = area of unit H/D = height/depth of unit

Figure 4.2: Orders of magnitude for various landforms. From: Dikau (1989).

Scale as such is thus introduced as a major feature in the selection process of relevant information in a digital terrain model. Fine-scale forms are retained to complete the landscape features in the scale of interest. They would be smoothed considerably by global filtering in the spatial domain. By using the proposed adaptive filter, in a local region of interest the level of detail can be very high, similar to the method proposed by Gross et al. (1996). However, the region of interest is not specified by the proximity to the observer, as it is in Gross et al. (1996), but by significant landforms.

Filtering in the wavelet domain is predetermined as a means to approach this problem. The often mentioned wavelet characteristics of both localisation in space and in the frequency domain allow the detection of relevant fine-scale details in areas where significant features of the coarser scale of interest occur.

4.3 Multiresolution Feature Detection

Weibel (1992) states that methods more closely related to the manual generalisation process than global or selective filtering do necessarily imply the previous detection of important features. Cartographic generalisation selects and emphasises such features. He developed a heuristic approach based on structure lines. Feature detection has thus been an important issue in digital terrain modelling. Local operators, feature classification, surface graphs or concave and convex properties are all prominent examples of feature extraction methods (Martinoni (1997), Hofstetter (2001), Brändli (1997)). Linear features such as drainage and ridge networks derived by surface graphs often are of high significance. The inclusion of such features will be dealt with in section 5.4.

Feature recognition and generalisation seem to be closely related. Because of this, the multiscale representation of the DTM requires a feature detection at multiple scales. The following two issues are of principal relevance for this research, despite their trivial appearance.

Firstly, feature detection is scale-dependent. An important peak of a certain scale can be very insignificant in another. A river appears to have the size of a creek if observed from a large distance. As figure 4.3 suggests, the peak of a distinctive mountain can appear very smooth if observed more closely. It is thus necessary to choose a scale or a range of scales of interest depending on the size (wavelength) of the surface features of interest. For a generalised model only the features of the actual scale of interest are selected whereas smaller forms are eliminated.

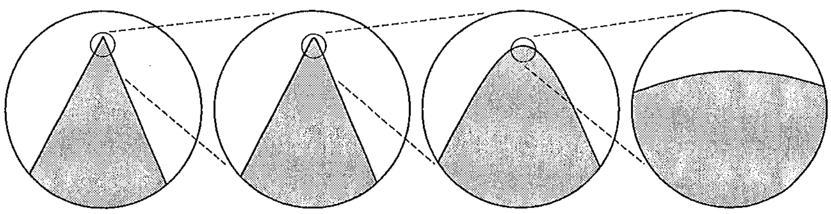


Figure 4.3: The significance of a landscape object is dependent on the scale of observation.

Secondly, the classification of landscape objects is fuzzy. Important for this thesis is mainly the fuzziness of a classification in accordance to scale. On the one hand, the terminology of geomorphological objects is fuzzy and on the other hand it is difficult to assign an exact size to an extended object, such as a mountain. From a signal processing point of view, it is impossible to exactly localise frequencies associated with particular landscape features, nor can one specific wavelength for local objects be found. Watson and Jones (1993) state that most features occur not only over a certain spatial extent but also in a *frequency band*. Most edges of importance, such as ridge lines of large mountains or river lines in major valleys, correspond to various scales indicated by high wavelet coefficients at adjacent levels. Fuzzy classification of terrain attributes derived from a DTM is mentioned by Wilson and Burrough (1999) as a means to classify landforms. MacMillan and Pettapiece (2000) use a number of derivatives of a DTM as basis for fuzzy landform classification. They thus achieve better results than with a set of Boolean rules. Fuzzy classification in physical geography traditionally refers to spatially overlapping objects with continuously varying attributes such as soil types. Landscape features occurring at various scales could be approached using similar techniques. The algorithms described in section 5.3.3 try to accommodate the issue of fuzzy classification. Further research could adapt the fuzzy landform classification algorithms suggested by MacMillan and Pettapiece (2000) in order to deal with fuzzy scale assignment.

Computer vision researchers have found that sharp transitions in images can belong either to edges or to texture. For example, when looking at a brick wall, one may perceive the contours of the wall as edges whereas the bricks define a texture. Alternatively, one may include the contours of each brick in the set of edges and consider the irregular surface of each brick as a texture. Mallat (1999) defines an edge as a sharp variation at a coarse scale of interest whereas texture refers to high variations of finer scales. The discrimination between edges and texture is clearly dependent on the scale of analysis. Edge detection at multiple scales is important for image pattern recognition. It is a prerequisite to distinguish between structures belonging to the scale of interest and (fine-scale) texture. Nievergelt (1999) states that edges are lines which correspond to high wavelet coefficients.

For this investigation edges in the DTM have been defined as structure lines identified by a high curvature. For the structure line detection a wavelet which is proportional to the second derivative has been applied (see section 4.4). The arithmetic mean of the horizontal and the vertical wavelet coefficients is used to detect structurally important areas. Thus, regions of high convexity can be discriminated from regions of high concavity. This is shown in figure 4.4.

As mentioned in section 2.2.3, the stationary wavelet transform, as opposed to the fast wavelet transform, calculates a wavelet coefficient for each signal point. It thus provides each point with a measure of relevance of the landform it is associated with. Relevance, being equivalent to curvature in this investigation (see next section), can be assessed on a series of scales. The stationary wavelet transform is therefore an appropriate tool to support the decision whether a region should be reconstructed with high or low accuracy. The implementation of this technique will be specified in section 5.3.3.

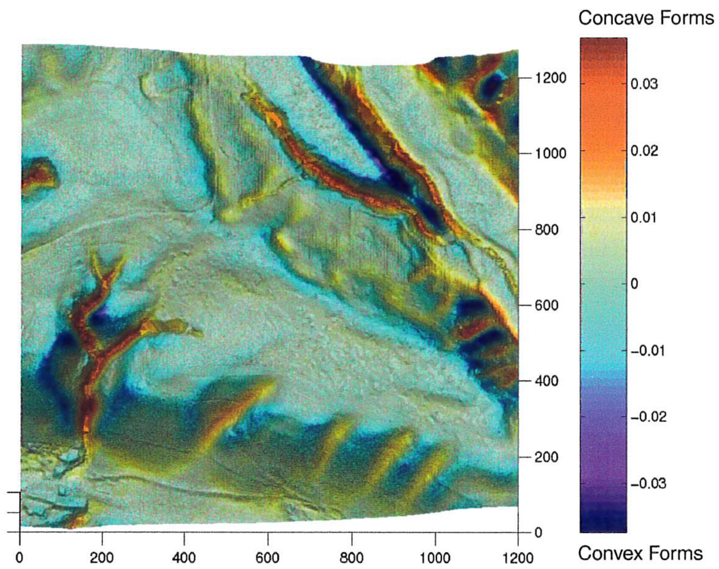
4.4 Curvature as Selection Criterion

Curvature as a criterion for feature detection. Curvature is a more conceivable property of the terrain and has supposedly more significance than other derivatives. Peucker and Douglas (1975), Chen and Guevara (1987) (in Scarlatos (1993)) not only account pits, peaks and passes as surface-specific points but also regard other points of high curvature as very important to terrain representation. These are referred to as break points or break lines, respectively.

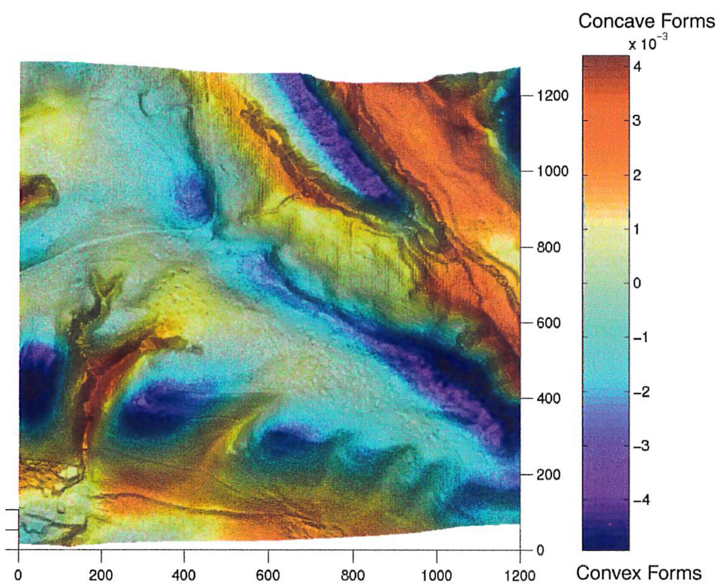
Curvature has been repeatedly used as a criterion to select important points for surface simplification (Heckbert and Garland (1997), Absolo et al. (2000)). In the method presented in this thesis curvature supports the decision process not at the smallest scale, but also at various, selectable scales. Thus, a better informed decision on the significance of various regions for the desired scale is expected to be achieved. Significant regions could then be retained with high accuracy. Wood (1996) suggested a multiscale feature classification based on various resampling steps and calculation of the respective curvatures. To determine curvature at various scales directly from wavelet coefficients, however, is a more efficient method.

Using Wavelets for Curvature measurement Partial derivatives of a signal can be obtained by a wavelet transform with the Daubechies wavelet with twice the support of the desired order derivative (Beyer and Meier (2001)). The 'curvature wavelet', having a support interval of 4 points, is well-localised in the spatial domain. It has two vanishing moments and is proportional to the second derivative. The wavelet coefficient of a certain scale at a given point simply gives an estimate of the curvature at this particular scale. The stationary wavelet transform is not only capable of extracting points of high curvature on a next-neighbour basis as traditional algorithms do, but can also derive structure lines of larger scales.

The curvature of different scales is displayed as elevation values in Figure 5.8. High positive curvatures occur in valleys and are thus represented as peaks. Conversely, low (negative) curvatures correspond to convex forms (i.e. ridges), represented here as valleys. Additionally, a hypsometric shading is applied. The stationary wavelet transform has only been used to derive partial curvatures in the north-south and east-west direction. The simple sum of the two values can give an estimate of the mean curvature according to Mitasova (1992) (in Wood (1996)), illustrated in Figures 4.4 and 5.8.



(a) Curvature of scale 5



(b) Curvature of scale 7

Figure 4.4: Curvature of scales 5 (features of approximately 60m) and 7 (features of approximately 260m) plotted onto the shaded terrain. Terrain data: Bundesamt für Landestopographie © (2000) (DV002247).

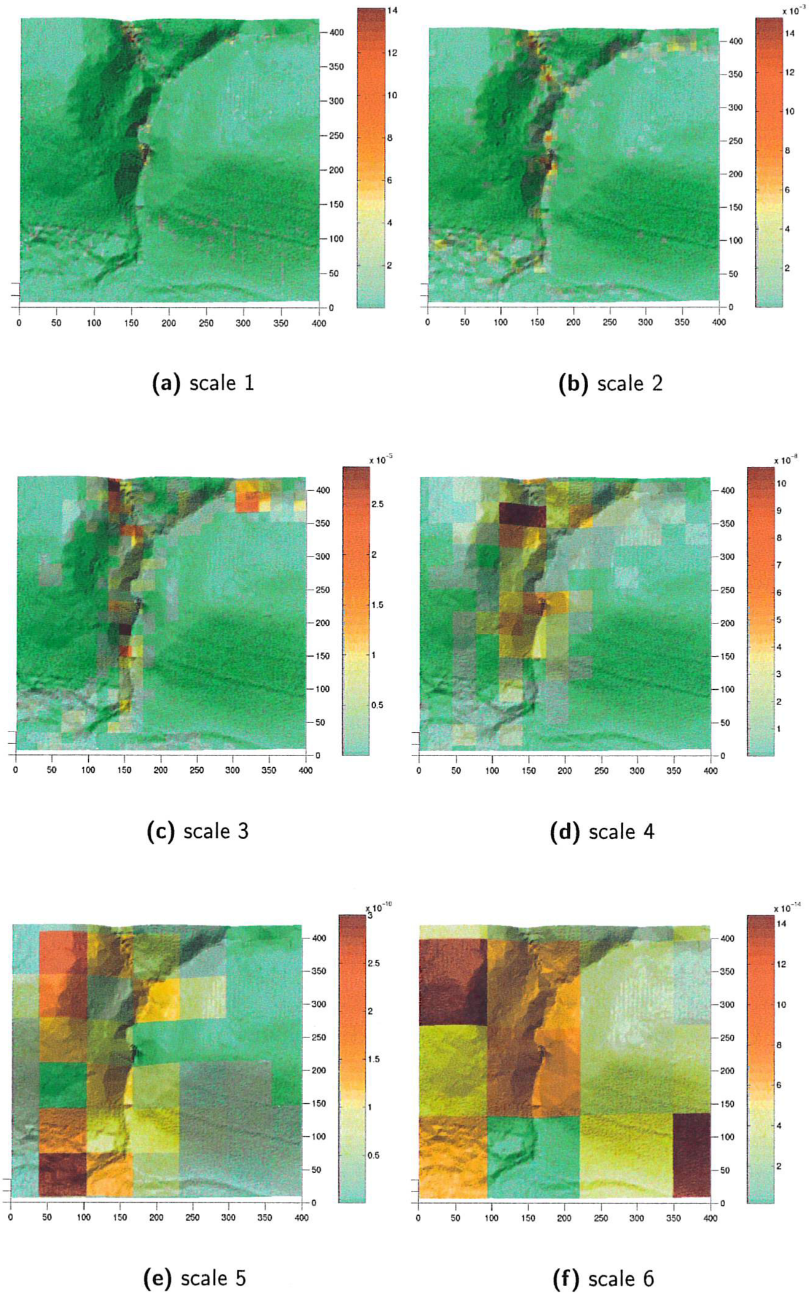
4.5 Localisation of Wavelet Coefficients in the Spatial Domain

In the following section a different representation termed 'stationary wavelet representation' of the fast wavelet coefficients is proposed. With such a representation scale-dependent filtering and the comparison with stationary wavelet coefficients, which are computed for each point, is possible. Section 4.5.2 discusses the problem of precisely localising the region of influence of a wavelet coefficient.

4.5.1 Coefficient Representation Model

With each convolution in the decomposition of a signal the number of detail coefficients is halved. This is illustrated in Figure 2.3 where the number of Heisenberg boxes, relating to coefficients, is clearly dependent on the scale ω. The number of wavelet coefficients representing a two-dimensional signal at scale k shrinks to $(s/2^k)^2$, where s denotes the length of the lateral edges of the original terrain.
The coefficient matrices of various scales cannot be directly compared due to their different size, nor can the coefficients of the stationary and the fast wavelet transform. A new representation of wavelet coefficients related to their localisation in space domain has to be found. This is necessary because wavelet coefficients of small scales have to be filtered according to the presence of significant large-scale coefficients at the respective location. The decimated wavelet coefficients are spread out to the entire size of the original DTM. They thus exhibit the spatial range which they affect most. A simple expansion of the length of the coefficient vector by factor 2^k brings it back to the original length of the signal. This representation can be seen as the two-dimensional equivalent of the Heisenberg boxes introduced in section 2.2.1 and Figure 2.3 for the one-dimensional case. Figure 4.5 depicts a series of scales. The elevation of the boxes corresponds to the energy of the wavelet coefficients at the respective scale.

This expansion process is implemented by a separate inverse transform of the wavelet coefficients of each scale with a type of 'neutral wavelet' [1 0]. The reconstruction of a signal can be written as a matrix multiplication of the wavelet with the vector of coefficients (Stollnitz et al. (1995b), Beyer and Meier (2001)). The neutral element of matrix multiplication, the identity matrix, is simulated by the 'wavelet' [1 0]. Hence, the coefficients remain the same but are placed at the location where they in fact have an impact on the reconstructed signal. This is referred to as the stationary wavelet representation. However, for the precise localisation in the spatial domain, a small 'phase shift' has to be considered (cf. next section).



(a) scale 1 **(b)** scale 2

(c) scale 3 **(d)** scale 4

(e) scale 5 **(f)** scale 6

Figure 4.5: Two-dimensional equivalent to the Heisenberg boxes introduced in Figure 2.3. Each square represents a wavelet coefficient. Terrain data: Bundesamt für Landestopographie © (2000) (DV002247).

4.5.2 Precise Localisation of the Region of Influence

The area which is influenced by a wavelet coefficient is not symmetrical to the point in space where it has been placed by the algorithm described in the previous section. When using the stationary wavelet transform to extract derivatives directly from wavelet coefficients, a translation in the spatial domain or 'phase shift' according to Beyer and Meier (2001) has to be taken into account (see appendix). However, when a fast wavelet transformation is used and expanded to a stationary wavelet representation, as descirbed in the last section, it has been found that the correction differs slightly. The formulae proposed by Beyer and Meier (2001) were adapted to this procedure.

The Heisenberg uncertainty principle proclaims the dichotomy between localisation in space and localisation in the frequency domain. Thus, a wavelet has a spatial extent, as all waves. In order to identify the region of influence of a wavelet coefficient a square around its assigned location in the space domain is evaluated. The square of size 2^k by 2^k (where k denotes the scale) around the calculated point can be interpreted as the area where the wavelet coefficient most affects the reconstructed signal. However, as Figure 4.6 shows, wavelets of a higher order regularity and bigger support, extend to a much larger area than such a square. Particularly at coarse scales the neighbouring wavelets are overlapping considerably.

Hence, one has to take into account that several wavelet coefficients need to be considered for exact reconstruction of the elevation at a specific point. Gross et al. (1996) consider the wavelet coefficients in a so-called four-neighbourhood. The bigger the support of the wavelet the less precisely it can be localised in space and the smoother the approximation.

Two methods have been examined for transforming the 'stationary representation' back to the regular wavelet representation. In a first method, only the most central coefficient of the region of influence is taken into account. Other overlapping wavelets are disregarded. However, the highest deviations of the reconstructed model from the original terrain model occur at crucial locations such as ridges and valleys which are vital to the terrain structure.

For wavelets of small support the issue of overlapping regions of influence is of little importance. But since mainly Daubechies wavelets with a support interval of 10 raster points are used here, the effect of overlapping wavelets needs to be considered. The problem is solved by the second method of transforming the 'stationary representation' back to the regular wavelet representation. After the filtering, which will be introduced in sections 5.3.2 and 5.3.3, a fast wavelet coefficient at a certain point is rejected only if all coefficients in a 2^k by 2^k neighbourhood have been rejected by the filtering method. Thus more coefficients are retained and the vital areas, such as local extrema, can be preserved much more accurately.

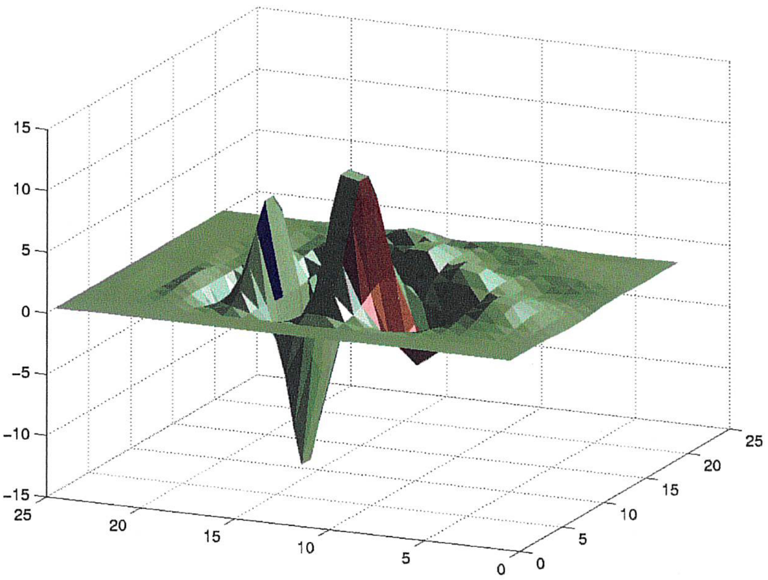


Figure 4.6: Localisation of a wavelet. Red: localisation according to section 4.5.1. Blue: the cell for which the 5. order derivative is proportional to the wavelet coefficient. It is computed according to the algorithms by Beyer and Meier (2001).

4.6 Filtering Procedure

In this section the single steps of the proposed filtering procedure are compiled. Generally speaking, the wavelet filter serves for simplification of the terrain structure using frequency information as a criterion. Subsequently, the ATM Filter, as described in section 3.3.1, is applied for data reduction.

The localisation in both spatial and frequency domain is a principal advantage of a signal representation in the wavelet domain. It is also a vital prerequisite for scale-dependent feature detection in a DTM. The terrain generalisation in this research is based on the detection of important features at a chosen scale. The scale-dependent filter is designed for controlled information reduction. However, the number of grid points after the inverse wavelet transform remains the same. The simplified surface is represented with the same grid resolution. Since the surface has been smoothed, it is an even more redundant representation than the initial terrain model. The actual data reduction is thus achieved by the ATM filter proposed by Heller (1990). With the application of a very small vertical tolerance, such as 10cm, the information content can be preserved, whereas the amount of data is reduced considerably. Redundant points are eliminated by such an ATM filter.

The proposed adaptive filtering procedure in the wavelet domain consists of the following sequential steps:

1. Wavelet transform of the DTM. The representation in the wavelet domain allows to exploit the beneficial property of localisation in both the spatial and the frequency domain.

2. Transformation of the regular wavelet representation to the stationary wavelet representation. This is necessary in order to compare the coefficients of different scales at the same location (see section 4.5.1). Additionally, the regular coefficients can be weighted by curvature (represented by stationary wavelet coefficients), as described in section 5.3.3.

3. Thresholding of the wavelet coefficients according to section 5.3.

4. Transformation of the resulting matrices back to the representation of the regular wavelet transform.

5. The inverse wavelet transformation results in a simplified DTM of the same size as the original data set.

6. The data reduction is performed in the space domain using Heller's (1990) ATM filter.

Point removal due to small wavelet coefficients (Gross et al. (1996)) has not been implemented. Wavelet coefficients are valid for a region rather than just for a single point. Due to their overlap wavelets can also extinguish each other in a way that no significant structure in the spatial domain is apparent. They are thus not seen as a feasible filter criterion for data reduction. The mentioned problems are avoided by filtering in the spatial domain.

Chapter 5

Implementation

5.1 Test Data Sets and Technical Environment

5.1.1 SAR Interferometry and Laser Altimetry Data

In this investigation a high resolution DTM with 2m grid resolution is used. The test site in the Emmental (Switzerland) has been recorded on behalf of the Swiss Federal Office of Topography, L+T. It has a size of approximately 18km by 5km. A partition of 1024 by 1024 raster points has been chosen, which represents a 2048m by 2048m square. However, due to a distortion of the wavelet coefficients on the margins as a result of edge effects, only 600 by 600 points are displayed.

Three types of digital elevation models were provided by the Federal Office of Topography.
First, a digital surface model (DSM, including vegetation and houses) generated by synthetic aperture radar (SAR) interferometry has been assessed. A slight noise is remarkable in Figure 5.1. Its magnitude, however, is too small to affect the detection of significant landscape features and it could be eliminated by disregarding the first few scales of a wavelet transform. In forested areas only very few points of the SAR-derived surface model belong to the actual terrain. It is therefore difficult to give an estimate of the digital terrain model by analysing the surface model generated by SAR interferometry. The model has not been investigated further because the main focus of this research is the generalisation of DTMs according to important geomorphological terrain features.

Second, a surface model produced with laser altimetry has been considered (upper picture in Figure 5.2). Thanks to a smaller aperture angle a considerable amount of sampling points in forested regions belong to the ground

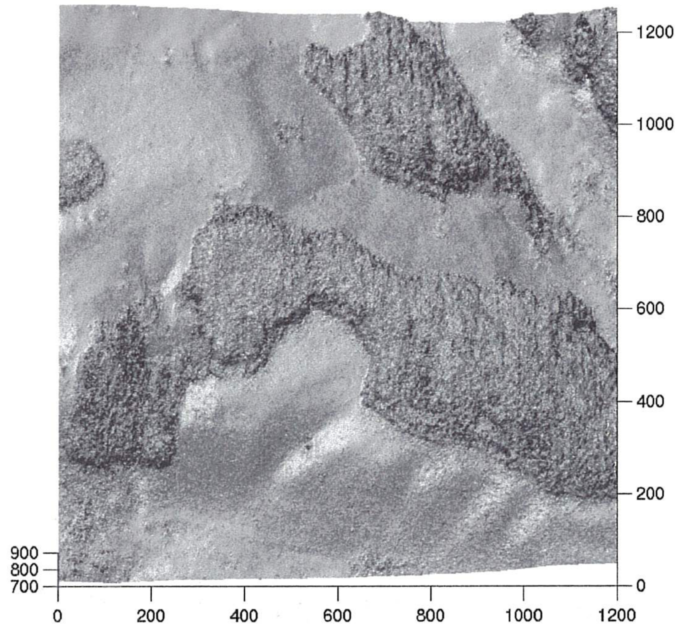
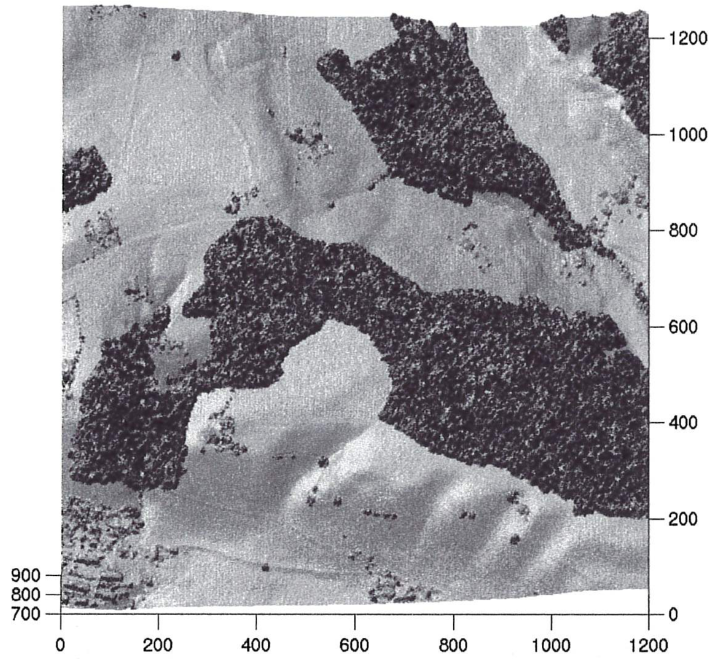


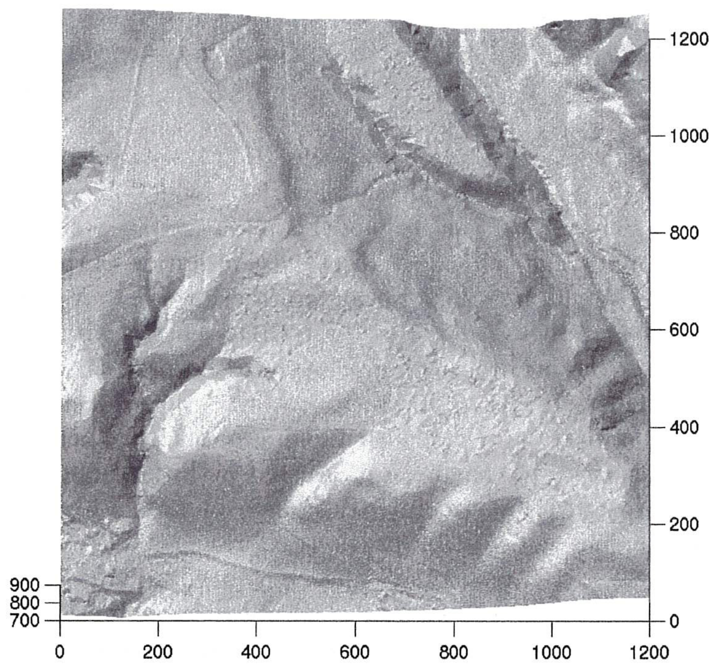
Figure 5.1: Digital surface model generated by SAR interferometry. Terrain data: Bundesamt für Landestopographie © (2000) (DV002247).

(Kraus and Pfeifer (1998)). Only in very dense forest the laser beam cannot penetrate the canopy.

The third model is the digital terrain model (DTM, without vegetation and houses) derived from the laser altimetry surface model. The extraction of the terrain model from the surface model is performed by assigning to each point a probability that it belongs to the vegetation or to the ground, respectively. The points with high negative deviation from an average trend surface are more likely to belong to the terrain, whereas the points which are higher than the average are more likely to be part of the vegetation (Kraus and Pfeifer (1998)). The terrain model is subsequently derived by interpolation between the points with a high probability of belonging to the ground. In this case this has been conducted by linear interpolation over triangles resulting in visible irregularities (lower picture of Figure 5.2). The thus derived terrain model is, in 'previously forested' areas, not as smooth as in other areas. Such triangular artefacts can be an impediment to the structural analysis by local operators, as described in chapter 6. This model has been used most in this investigation. The statistical measures of all three models are displayed in table 5.1



(a) Digital surface model



(b) Digital terrain model

Figure 5.2: Digital surface (a) and terrain (b) model generated by laser altimetry.. Terrain data: Bundesamt für Landestopographie © (2000) (DV002247).

DTM	min. height	max height	range	mean	std. dev.
SAR	675.93	985.21	309.28	803.269	58.2031
Laser-DSM	676.33	984.44	308.11	802.5972	57.5617
Laser-DTM	676.17	961.15	284.98	796.7896	55.3456

Table 5.1: Statistical measures of the three models. All units are [m].

5.1.2 Terrain Identification and Artefact Elimination Using Wavelets

Terrain Identification. A wavelet transform could support ground point extraction methods due to the decomposition of the signal into orthogonal, complementary frequencies. The discrimination between frequencies associated with the forest and frequencies associated with the average trend surface has been attempted using wavelets. A high-frequency part and a low-frequency part, which represents the trend surface, can be identified by dividing the signal into its major frequency fractions. The summed frequencies from scale one to four (see table 4.1) correspond to the oscillation of the forest, whereas the scales higher than four represent the smooth average corresponding to the large geomorphological forms. However, forested breaklines of a terrain are difficult to extract from a surface model, because the locally higher curvature is partly reflected by the coefficients at finer scales. At such locations the high frequencies are also part of the terrain. Figure 5.3 shows an early attempt of ground point extraction with wavelets.

Artefacts. Directional artefacts of fine scale and magnitude looking like ripple marks are noticeable in Figure 5.2. Such fine-scale structures pose considerable difficulties to traditional data reduction methods, such as ATM filtering with a small vertical tolerance (see section 3.3.1). These difficulties are reflected by the fact that the filtering of the original data set with 10 cm tolerance produces a TIN with 31% of all points. The problem can be approached by a tensor product wavelet transform and selective elimination of only the horizontal details at scale one and two with the amplitude of this regular, horizontal variation. By such a procedure the model was smoothed to an extent that the subsequent TIN filtering with 10 cm only yielded 18% of the original amount of points. A slight additional smoothing of the vertical and diagonal details even reduced the extracted points to 12% (Figure 5.4).

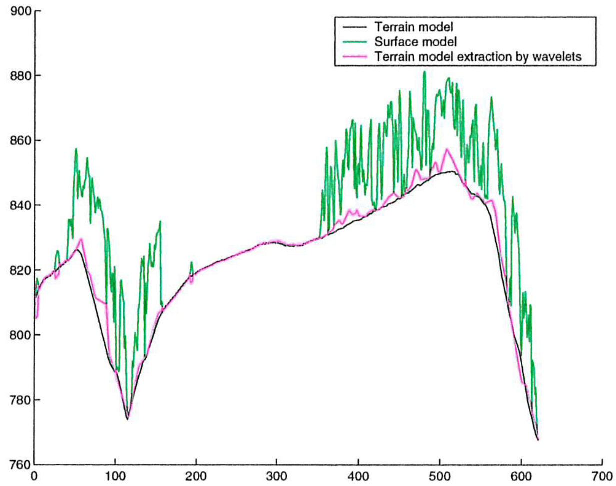


Figure 5.3: Extraction of the terrain model from the surface model.

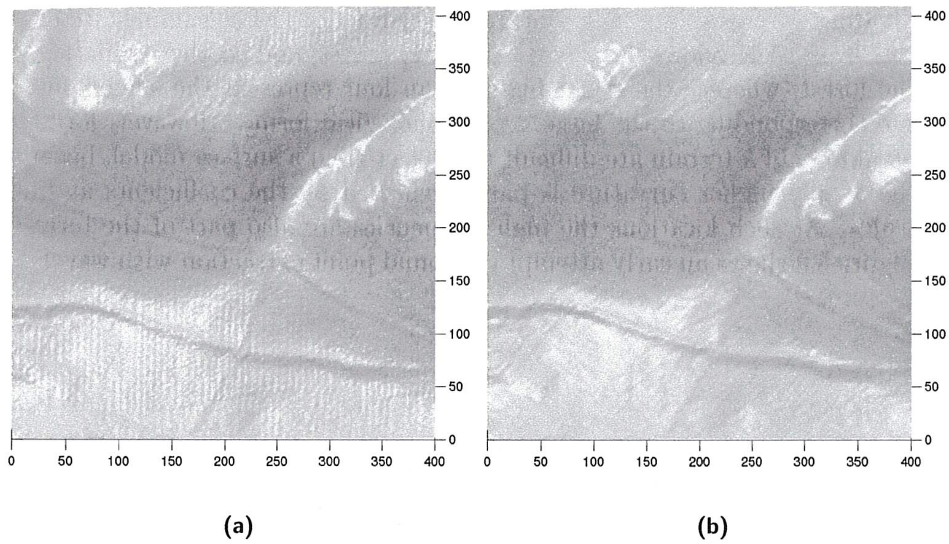


Figure 5.4: Elimination of the directional artefact as a preprocessing step for the ATM filter: a) Original DTM, b) smoothed DTM. Terrain data: Bundesamt für Landestopographie © (2000) (DV002247).

5.1.3 Technical Environment

This investigation is largely based on ©MATLAB, version 5.3.11. Matrix operations are remarkably fast in ©MATLAB, which is very important for computationally complex algorithms such as the wavelet transform. Moreover a vast number of mathematical functions are available, among which also a complete wavelet package with various types of transforms. According to the ©MATLAB function reference on the Internet, even the stationary wavelet transform will be included in the newest ©MATLAB version. For this research it has been implemented using Daria Martinoni's fast tensor product wavelet transform as a basis (Martinoni (2001)).

A further advantage of ©MATLAB is the capability of visualising meshes and surfaces in a simple way. The surface visualisations shown in this thesis were mostly generated using ©MATLAB.

5.2 Selection of a Wavelet

As already mentioned in section 1.3, wavelets are used for two different purposes in this investigation. On the one hand, they are employed for detecting significant landscape structures at different scales with the stationary wavelet transform, as described in section 5.3.3. On the other hand, for the actual approximation, the decomposition of the terrain into wavelet coefficients is realised by the fast wavelet transform.

Wavelets with good approximation characteristics must first of all be smooth, meaning that they ought to have a sufficiently high order of regularity. The order of regularity is equivalent to the number of continuous derivatives, and also to the number of vanishing moments of the wavelet. The Daubechies wavelet of twice the support of the desired regularity provides coefficients being proportional to the respective derivative (see appendix). This is an important property for curvature estimation, described in section 4.4 and applied in section 5.3.3.

Symmetry, orthogonality, compact support and smoothness are the major requirements for approximation. They cannot all be met at the same time by a wavelet. In particular, a compromise has to be found between smoothness/regularity and compact support. Figure 5.5 displays a series of nonlinear approximations using Daubechies wavelets with increasing support. The root mean square error has been the smallest for the approximation with a Daubechies wavelet of support interval of 10 raster points (5 vanishing moments). A further increase of the support results in a very smoothly undulating landscape. Flat areas tend to be influenced by neighbouring peaks. Very smooth wavelets with a support interval higher than 12 exaggerate the dominating forms to an extent that artificial hills appear next to distinct valleys.

The above stated reasons lead to the choice of a Daubechies wavelet with support interval of 10 points for the surface approximation. Nevertheless, a more adequate wavelet may offer some potential for improving the surface representation. This could be achieved by using bi-orthogonal splines, described by Stollnitz et al. (1995b), or a bi-orthogonal pseudo coiflet, also called Mexican hat wavelet, proposed by Reissell (1996). Also continuous wavelet transforms with two-dimensional wavelets, as applied by Gallant and Hutchinson (1996), deserve further attention.

For detecting significant structures a Daubechies wavelet of support 4 is used. The main advantage of this wavelet is its order of regularity of two, which allows to measure curvature directly from the stationary wavelet coefficients. This is discussed in section 4.4.

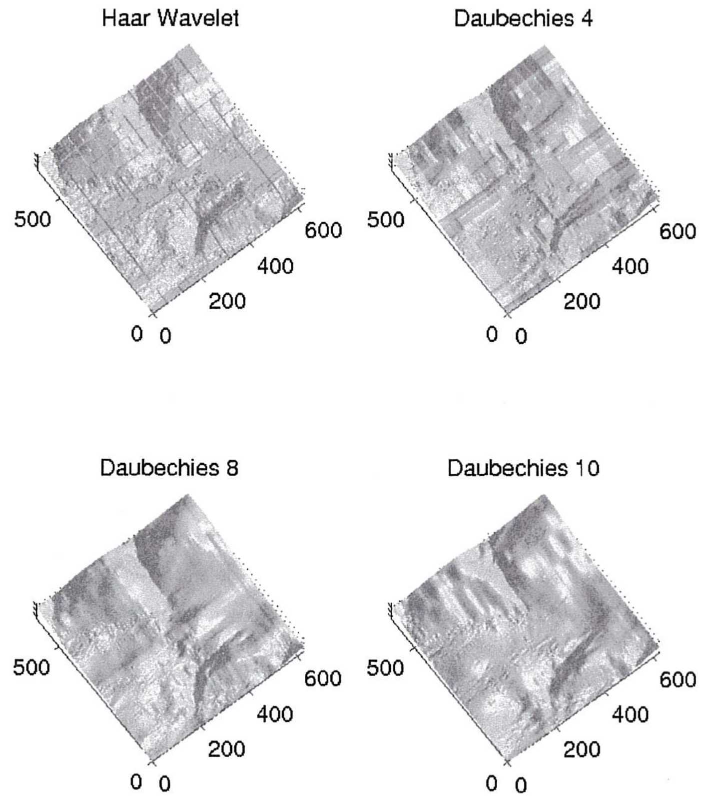


Figure 5.5: Non-linear approximations using Daubechies wavelets having support interval 2 (equivalent to the Haar wavelet), 4, 8, and 10. The smooth wavelet with support 10 is used for the terrain approximation. Wavelets generally produce approximation with properties similar to the applied wavelet. Terrain data: Bundesamt für Landestopographie © (2000) (DV002247).

5.3 Thresholding Wavelet Coefficients

5.3.1 Introductory Remarks

There are many options and combinations of thresholding algorithms. Several possibilities have been tested and some will be reviewed here. In section 5.3.2, a cascade frequency preservation algorithm, depending on all scales, will be compared to the dependence on the mere arbitrative scale. In section 5.3.3, the idea of weighting fine-scale coefficients by coarse-scale stationary wavelet coefficients will be introduced as a fuzzy decision concept taking into account the influence of more than just one parameter.

In this chapter the term *arbitrative scale* will be used repeatedly. It denotes the scales at which significant features determine the representation of details belonging to such significant structures.
The term *significant structure* refers to a landform which is represented by a high wavelet coefficient. This is equivalent to a feature with a large amplitude at the wavelength examined by the respective wavelet. The threshold between significant and insignificant is, in this investigation, the simple mean of all wavelet coefficients disregarding the extreme values on the margin of the coefficient matrix.

5.3.2 Cascading Scale-adaptive Thresholding

First, a threshold is applied directly to the wavelet coefficients of the fast wavelet transform. This is the standard procedure of the non-linear wavelet coefficient filter introduced in chapter 4.1 (equation 4.3).
Subsequently, an arbitrative scale, which can be freely chosen, is identified according to the desired level of generalisation. Significant structures at the arbitrative scale are identified by the remaining coefficients ($b_{uv}^k \neq 0$ in equation 4.3). The finer scale coefficients at the locations of the significant structures are subjected to further filtering. The noticeable, large rectangular yellow areas in the upper picture of Figure 5.6 represent the significant areas at scale 6 (see table 4.1), which, in this case, is a valley. Outside these areas, a regular linear approximation is employed, retaining only the coefficients between the largest scale and the scale of interest. Within the significant region two different procedures are implemented.

Method 1. The first method simply filters non-linearly at all scales lower (finer) than the arbitrative scale a, but only in significant regions. The significance of a coefficient is identified by threshold t_a in the arbitrative scale a:

$$\{d_{ij}^k\} = \begin{cases} w_{ij}^k & : \quad w_{ij}^a > t_a \wedge w_{ij}^k > t_k \\ 0 & : \quad \text{otherwise} \end{cases} , \tag{5.1}$$

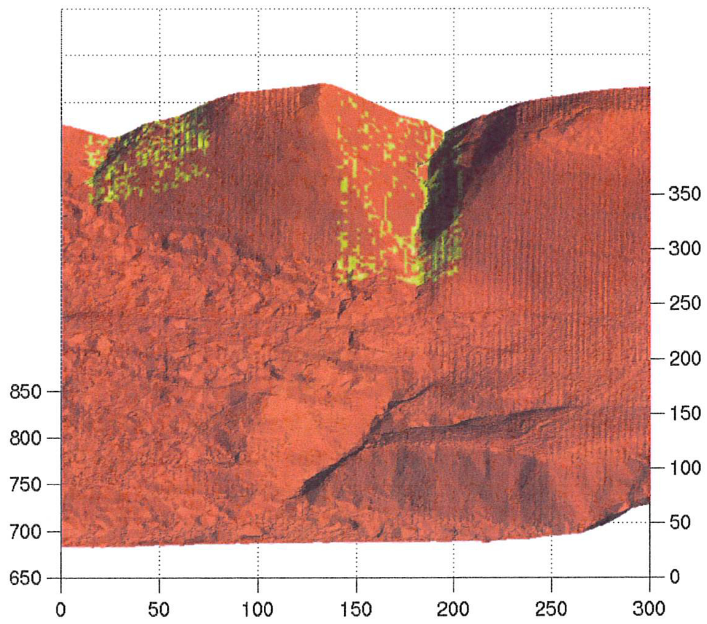
where t_k denotes the threshold at scale k. In practice, the appliance of a range of arbitrative scales has been implemented. If a feature that is relevant to at least one of the arbitrative scales is detected, the significant finer scale coefficients at the respective location are retained.

Method 2. The second option, using scale dependency for more than just the chosen arbitrative scale has been found to be more closely related to multiresolution analysis. Each finer scale coefficient is recursively dependent on the coefficients of the next coarser scale. If a feature is significant at all scales it will be reconstructed very accurately using coefficients of every scale. However, if some landscape form does not pertain to all scales, the coefficients at its location are blocked on the finer scales. Insignificant medium scale coefficients can prevent small structures from being preserved. This happens for example with large, rolling hills without considerable edges or, as can be seen in the lower picture of Figure 5.6, at planar slopes of valleys. Only in the actual channel line were all coefficients used for the reconstruction. The recursive assessment of the coefficients is performed by a cascade algorithm stepping from one scale to the next finer one. At each step the coefficient is only retained if the corresponding coefficient at the same location on the next coarser scale has also been retained. Otherwise it is set to zero. Using b_{ij}^k as the notation for the previously non-linearly filtered matrix of wavelet coefficients located at (i, j) (equation 4.3) the cascade algorithm processes the coefficient matrices from the coarsest arbitrative scale K down to the finest scale:

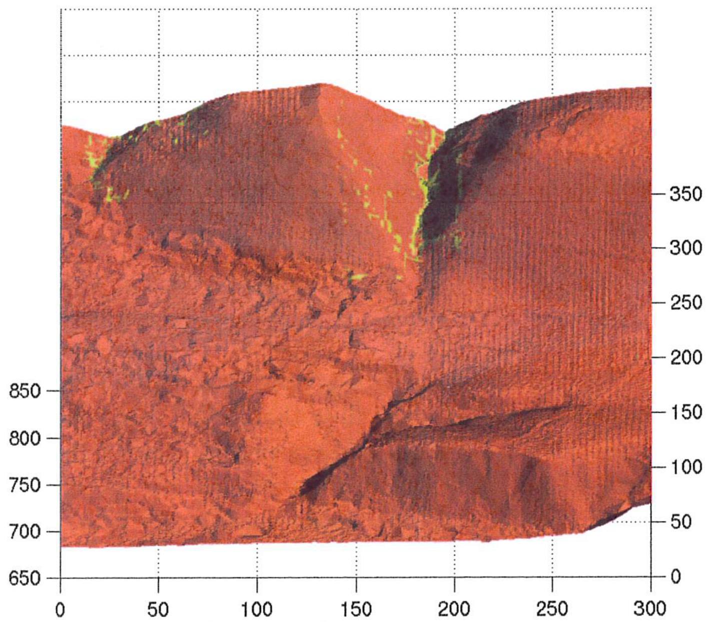
$$E^k = \{e_{ij}^k\} = \begin{cases} b_{ij}^k & : \quad b_{ij}^{k+1} \neq 0, \\ 0 & : \quad \text{otherwise,} \end{cases} \tag{5.2}$$

for $k = K, K-1, \ldots, 1$. The result is a notably generalised terrain model with edges of importance on all scales finer than K being retained. Figure 5.7 shows an example using scale 6 and 7 (corresponding to wavelengths around 320 to 640 metres) as the arbitrative frequencies. The channel line of the valley has been reconstructed, by visual assessment, to a reasonable extent.

Results. Three major difficulties constrain the quality of approximations obtained by the described methods. First, at crucial locations, such as extrema and breaklines, the difference to the original is small compared to

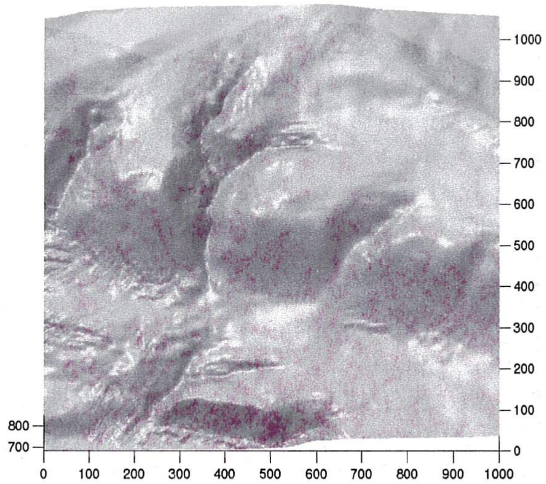


(a) Method 1

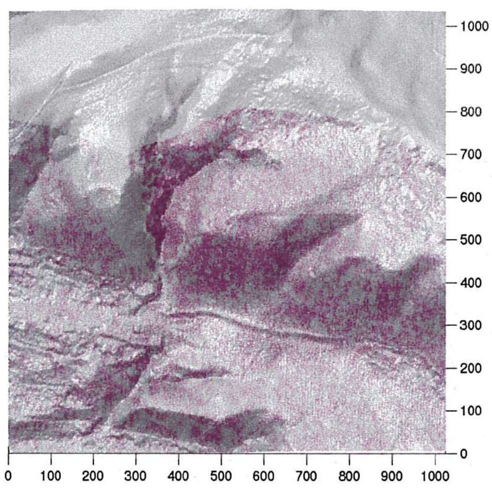


(b) Method 2

Figure 5.6: Cascading scale-adaptive filtering according to two different methods. The retained wavelet coefficients are yellow. Terrain data: Bundesamt für Landestopographie © (2000) (DV002247).



(a) Cascading adaptive filter



(b) Original

Figure 5.7: Reconstruction of the cascading scale-adaptive filter according to method 2 compared with the original. Terrain data: Bundesamt für Landestopographie © (2000) (DV002247).

other locations, but it is not zero. This difference occurs due to the spatial extent of the Daubechies wavelet with a support interval of 10 points, which is used for reasons of smoothness. A wavelet coefficient can still influence a point in the valley line although its coefficient is located at a distance of up to 10^k (with k denoting the scale) raster points. This problem is partially solved in the next section by weighting the coefficients with a wavelet that has a more compact support and by retaining all coefficients which can possibly affect the respective location.
Second, a further obstruction to exact reconstruction in crucial areas are small medium scale coefficients preventing the preservation of small structures even if they are part of an important large scale form. Equation 5.2 defines a crisp condition which allows no fine scale feature to survive if any coefficient at an intermediate scale does not meet the threshold requirement. This is, for instance, the case if a creek has carved a small notch or ravine into a large U-shaped glacially induced valley. Such problems can also be avoided by the weighting concept introduced in the next section.
Third, the transitions between smoothed and precisely reconstructed areas are somewhat abrupt, which reduces the visual quality of the picture. The sudden change on the boundaries between the precise and approximated regions is illustrated in Figure 5.7. The ATM filter by Heller (1990), being applied subsequently to the filtering process in the frequency domain, can reduce this effect to some extent. However, this issue is also approached by weighting with stationary wavelet coefficients, as described in the following section.

5.3.3 Weighting by Curvature Values

Better results have been achieved by weighting the wavelet coefficients by curvature values than by applying a rigid threshold as discussed in the last section. The gradual change of the curvature at coarser scales depicted in Figure 4.4 leads to a seamless transition between exactly reconstructed and approximated areas.

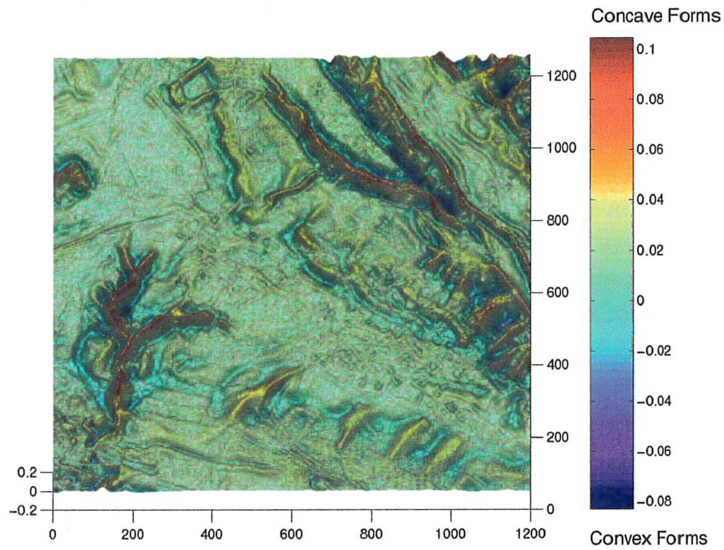
Curvature can be measured directly from stationary wavelet coefficients using the Daubechies wavelet with support 4 (see chapters 2.2.3 and 4.4 and appendix). The stationary wavelet transform can give curvature estimates at various scales, which is an important characteristic if the scale-related importance of a feature needs to be determined. The basic idea is that curvatures of one or several predefined arbitrative scales are used for weighting all finer scale coefficients. At all locations (i, j) a threshold is then imposed on the product of the curvature c_{ij}^a at arbitrative scale a and the wavelet coefficient w_{ij}^k at scale k. The arithmetic mean t_k of the weighted wavelet coefficients $|c_{ij}^a w_{ij}^k|$ has been chosen as a threshold whereby the coefficients on the edge of the DTM have been disregarded. Hence, the distortion of the mean by the high marginal coefficients can be avoided. The threshold t_K is applied, in accordance to equation 4.3:

$$\{g_{ij}^k\} = \begin{cases} c_{ij}^a & : \quad c_{ij}^k w_{ij}^k > t_k \\ 0 & : \quad \text{otherwise} \end{cases} . \tag{5.3}$$

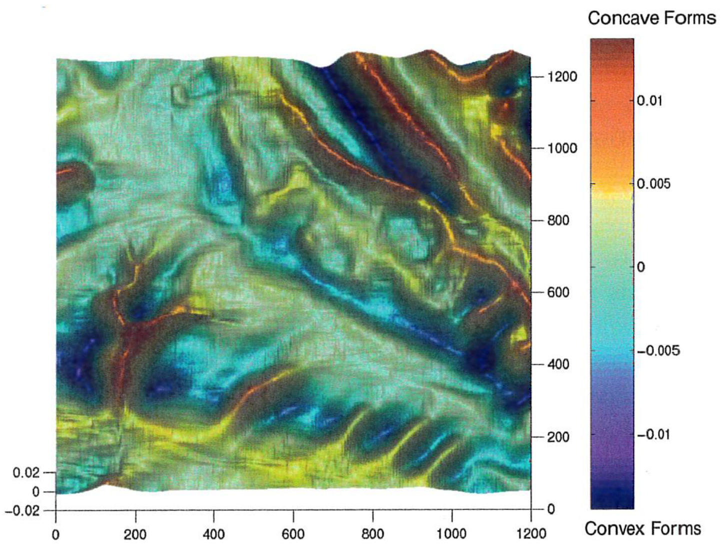
As mentioned in section 4.3 the scale or frequency of a feature cannot be clearly identified. Any terrain object occurs over a range of scales and likewise over a spatial range. It is therefore advisable to predetermine a range of scales of interest rather than just a single scale. The product of the curvature values at the chosen arbitrative scales is used as a weight instead of a single curvature. The fuzzy classification of features can be dealt with by combining their significance in several scales. Wilson and Burrough (1999) advocate the application of fuzzy classification to digital elevation models for forest mapping. The combination of various parameter values can give a better estimate of a class membership than using crisp criteria to assign an object to a certain class. Figure 5.8 depicts the weight matrices referring to curvature values of scale 4 and 6, which corresponds to a feature size of 32m and 128m, and figure 5.9 the product of scale 5, 6, and 7, respectively. The representation of curvatures as a height field might appear somewhat unusual but the well-localised extrema suggest that applying a threshold to identify significant areas might prove to be a reasonable method. In significant areas all but the very small coefficients exceed the threshold value. The reconstruction in significant areas is therefore almost identical to the original.

The stationary representation of wavelet coefficients, as introduced in chapter 4.5, allows to perform weighting and thresholding by fast matrix operations. Yet, this representation needs to be converted back to the regular notation of wavelet coefficients in order to apply the inverse wavelet transform. Two different methods have been tested to deal with the problem of overlapping wavelets. The best results have been obtained by retaining all coefficients that could possibly affect the features of interest. For a detailed discussion of this problem refer to section 4.5.

The results of the presented method are visualised in Figures 6.2, 6.3 and 6.4. The vertical deviation from the original data is illustrated by the colour plotted onto the approximated terrain. Features in the desired scale of interest are reconstructed with particular accuracy whereas smaller forms are smoothed out unless they are part of a form at the scale of interest. Singular arbitrative scales have been used to demonstrate the gradually changing size of the precisely reconstructed features. At scale 3 even small channels and ridges are well-preserved and only the irregular structures originating from the initially forested areas are smoothed. At scale 7, for comparison, small



(a) Curvature at scale 4



(b) Curvature at scale 6

Figure 5.8: Curvature at scales 4 (28m) and 6 (128m) plotted as elevation and as colour information. Terrain data: Bundesamt für Landestopographie © (2000) (DV002247).

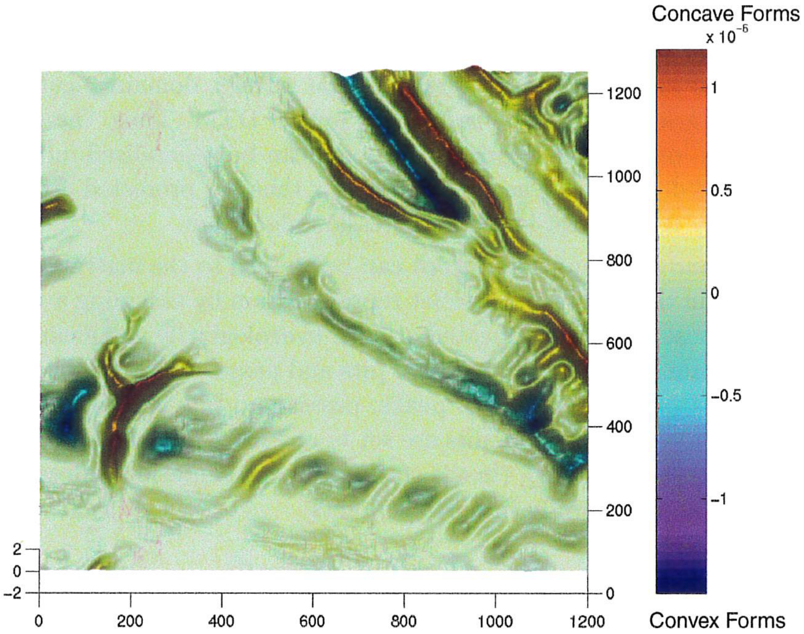


Figure 5.9: Combination of scales 5, 6 and 7: only the landforms which are significant at all three scales appear. Terrain data: Bundesamt für Landestopographie © (2000) (DV002247).

creeks and edges are considerably less prominent and the approximation differs up to 3 metres from the original.

5.4 Inclusion of Known Structures

In this section the inclusion of *a priori* knowledge about significant features, for example hydrological networks or shorelines, is discussed. In order to preserve the drainage properties of a DTM Gerstner and Hannappel (2000) suggest including all critical points such as pits, peaks and passes into the generalised model. To assure that these points are retained in the final approximation they set their error indicators (see section 3.4) to infinity. The error thus exceeds the applied threshold and these critical points are represented accurately. This is equivalent to a local level of detail control suggested by Gross et al. (1996) or a 'local query of a multi-triangulation' according to Magillo et al. (2000). For surface visualisation, the area close to the observer needs to be represented to a higher level of detail. In digital terrain modelling, however, a local level of detail control is more useful to preserve important geomorphological features, such as hydrological networks or coastlines, which otherwise would be smoothed. Beyer (2000) proposes to transform known structure lines into the wavelet domain together with the regular grid model. The result is a hybrid DTM. Thus, the structure line information and the regular grid model can be generalised individually to the desired extent. The breaklines can thus be protected from being smoothed by the wavelet-based compression algorithm.

Gerstner's and Hannappel's concept can be applied to the matrix of wavelet coefficients in the stationary wavelet representation by assigning a very high weight to those coefficients located at such hydrologically significant points. The significant points, such as peaks, pits and passes, have been derived by a local operator. A matrix $\mathcal{H}$ of peaks, passes and pits is computed, where the structurally important points are set to one in an initial matrix of zeros. The wavelet coefficients w_{ij}^k (in the stationary representation) are set to very high values at structurally important points,

$$\{w_{ij}^k\} = \begin{cases} w_{ij}^k & : \mathcal{H}_{ij} = 0 \\ \infty & : \mathcal{H}_{ij} = 1 \end{cases}, \tag{5.4}$$

in order to exceed the subsequently applied threshold. In the high resolution DTM used in this study, local operators employed for the recognition of peaks, passes and pits described by Brändli (1997) detect many points with little importance to a meso scale (features larger than 40m) drainage network. Most of the extracted points lie in areas where the terrain model has been derived from the surface model by elimination of forests and houses. They can therefore be interpreted as artefacts. If the hydrological network is available as *a priori* knowledge or if it was previously extracted by more robust algorithms, it can be included directly in the same way as the pits, peaks and passes. By such a procedure it is ensured that the hydrological network is identical to the one in the original model. Furthermore, flat areas, such as known lakes or sea surfaces, which are often not represented very well by a wavelet approximation, can be represented with a high level of detail in order to protect the original form. Wavelet induced artefacts such as oscillations on the water surface can thus be avoided and the subsequent ATM filter (cf. section 5.5) can be applied using a very small error tolerance. Furthermore, shorelines, which are often a problem for approximations (Bonneau (1998)), can be preserved accurately.

With this method hydrological correctness can be introduced to the desired degree of generalisation. However, in many cases the adaptive non-linear wavelet filter produces near approximations since significant channels are often accompanied by significant landscape features such as a network of interlocking valleys and ridges, according to Werner (1988). The inclusion of these drainage properties, as mentioned above, is more important for flat areas with important rivers.

5.5 Data Reduction

Gross et al. (1996) use the information of the wavelet coefficients, among other criteria, to decide whether a vertex of a DTM shall be retained in the final mesh. Using such an approach several constraints need to be considered. A major problem is that the wavelet coefficients cannot be exactly localised at a specific vertex. They have an influence on several points which implies that for the removal of a vertex several coefficients have to be taken into account. In order to avoid such problems, a TIN filter in the spatial domain has been chosen for reducing the number of points in the filtered DTM. The two-stage approach consisting of information reduction in the wavelet domain and data reduction in the spatial domain is described in section 4.6.

The adaptive triangular mesh (ATM) filter introduced by Heller (1990) is a very efficient algorithm designed for retaining significant points of a DTM. A vertical tolerance is used as the criterion, whether a point shall be included in the final mesh or not (see section 3.3.1). The applied ATM filter selects important extrema, which is a major objective of statistical generalisation. However, when using a small vertical tolerance the result can be affected by noisy raw data and small irregularities of the surface. Particularly in regions where the terrain model was extracted from the surface model, the prevailing artefacts exceed the filter tolerance. A preprocessing in the wavelet domain eliminates such fine-scale artefacts to a degree that the number of points selected by the ATM filtering are considerably reduced (see section 5.1.2).

The scale-adaptive wavelet filter creates very smooth surfaces outside significant regions. Thus, only few points need to be preserved in smoothed areas, whereas many vertices are needed to represent the precisely reconstructed significant landforms. The number of points retained by the ATM filter proposed by Heller (1990) is shown in table 5.2. They can be used as an estimate of the size of the smoothed area. However, it is also dependent on the occurrence of the above described artefacts. But the fact that the number of retained points decreases with increasing arbitrative scale suggests that coarse arbitrative scales result in a higher degree of generalisation. The result of the ATM filter with 10 cm vertical tolerance is illustrated in Figure 5.10

Arbitrative Scale	TIN points [%]
Scale 3	15.3
Scale 4	12.7
Scale 5	11.9
Scale 6	11.2
Scale 7	11.0
Scale 8	10.7

Table 5.2: Percentage of remaining points after the application of Heller's (1990) ATM filter with 10 cm vertical tolerance.

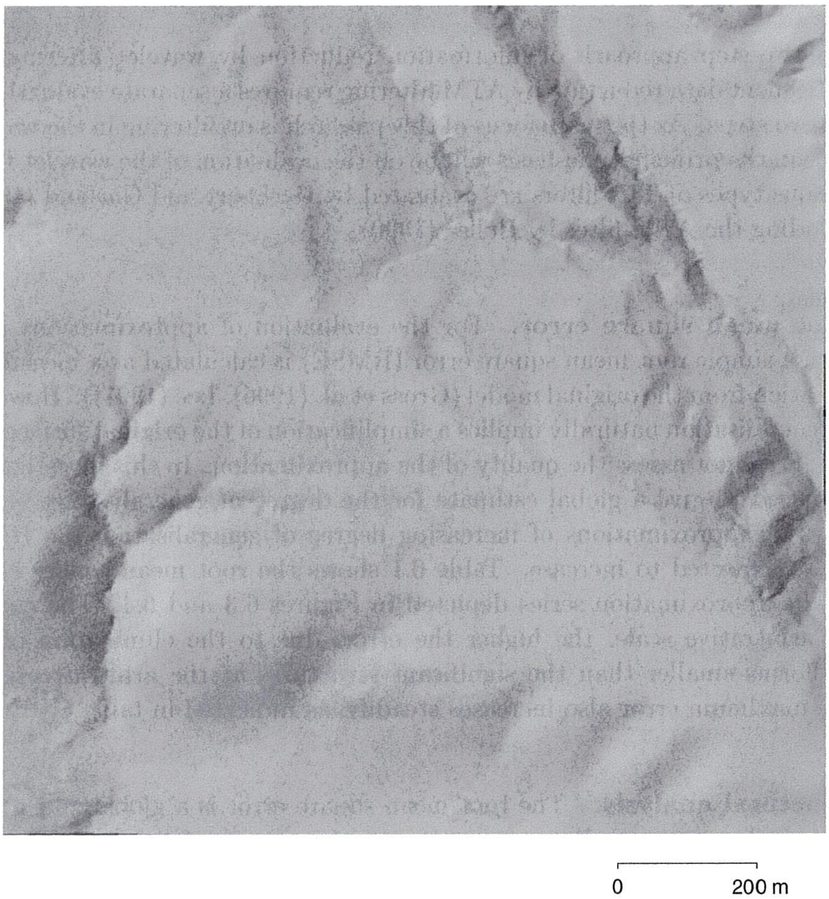


Figure 5.10: ATM filter for scale 5. The filter tolerance is 10 cm. Terrain data: Bundesamt für Landestopographie © (2000) (DV002247).

Chapter 6

Evaluation

The two-step approach of information reduction by wavelet filtering and subsequent data reduction by ATM filtering requires a separate evaluation of the two steps. As the main focus of this research is on filtering in the wavelet domain the principal emphasis will be on the evaluation of the wavelet filter. Various types of TIN filters are evaluated by Heckbert and Garland (1997), including the ATM filter by Heller (1990).

Root mean square error. For the evaluation of approximations most often a simple root mean square error (RMSE) is calculated as a measure of deviation from the original model (Gross et al. (1996), Lee (1991)). However, as generalisation naturally implies a simplification of the original surface, the RMSE cannot assess the quality of the approximation. In this investigation it is used to give a global estimate for the degree of generalisation. For a series of approximations of increasing degree of generalisation the RMSE is also expected to increase. Table 6.1 shows the root mean square errors for the approximation series depicted in Figures 6.3 and 6.4. The coarser the arbitrative scale, the higher the error, due to the elimination of the landforms smaller than the significant structures at the arbitrative scale. The maximum error also increases steadily, as indicated in table 6.1.

Structural analysis. The root mean square error is a global and purely statistical measure and cannot capture many aspects of the approximative quality. Lee (1991) suggests a structural analysis of both the original and the approximated terrain comparing the number of peaks, pits and passes before and after applying the approximation algorithm. This is a valuable assessment of a digital terrain model simplification. The issue of hydrological consistency is approached by such a comparison of structural features. However, it is difficult to determine which structure points are significant to

Arbitrative Scale	RMS Error [m]	Maximum Error [m]
Scale 3	0.0498	0.5200
Scale 4	0.0614	0.9565
Scale 5	0.0787	1.3814
Scale 6	0.1107	2.4978
Scale 7	0.1318	3.1847
Scale 8	0.2839	5.1929

Table 6.1: Accuracy estimation of the wavelet filter by root mean square error and maximum error.

the coarse-scale catchment area. Particularly in high resolution DTMs the occurrence of very small peaks and pits is likely. In the employed DTM, the feature extraction algorithm suggested by Lee (1991) results in an excessive amount of peaks, pits and passes due to noise and irregularities. This is an artefact of the DTM data described in section 5.1.1. Such artificial structures reduce the significance of the quality assessment using structural analysis. Table 6.2 shows that the number of structure points does not decrease monotonously. This happens as a result of the uneven distribution of the artefacts. Particularly the significant structures of scale 7 and 8 seem to be covered by such small irregularities. Therefore the artefacts are especially well preserved in an approximation using scale 7 or 8 as the arbitrative scale. This is reflected by the high number of structure points for the arbitrative scales 7 and 8 in table 6.2. The amount of suitably restructed areas also depends on the proportion of the surface which is associated with significant landforms of the respective scale.

Arbitrative Scale	Peaks	Pits	Passes
Original	3690	2509	17873
Scale 3	1008	919	5882
Scale 4	663	632	3579
Scale 5	553	560	2979
Scale 6	510	526	2793
Scale 7	585	601	3317
Scale 8	608	563	3334

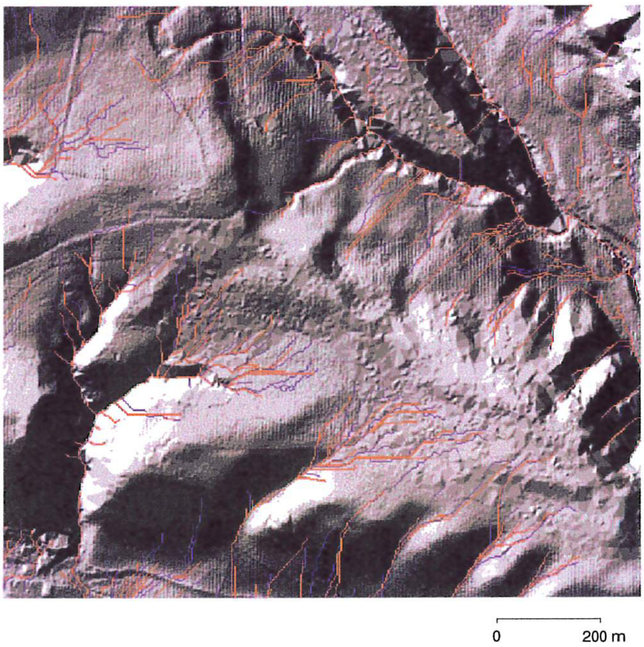
Table 6.2: Comparison of the number of peaks, pits and passes of a section of 600 × 600 points.

In order to adapt the structural analysis to the issue of scale-dependent approximation, the structure points of high relevance in the scale of interest must be detected. The comparison of peaks, pits and passes in the scale of interest has a higher significance for the filter quality. For example, Heller (1990) proposed an algorithm to detect significant maxima of a certain size.

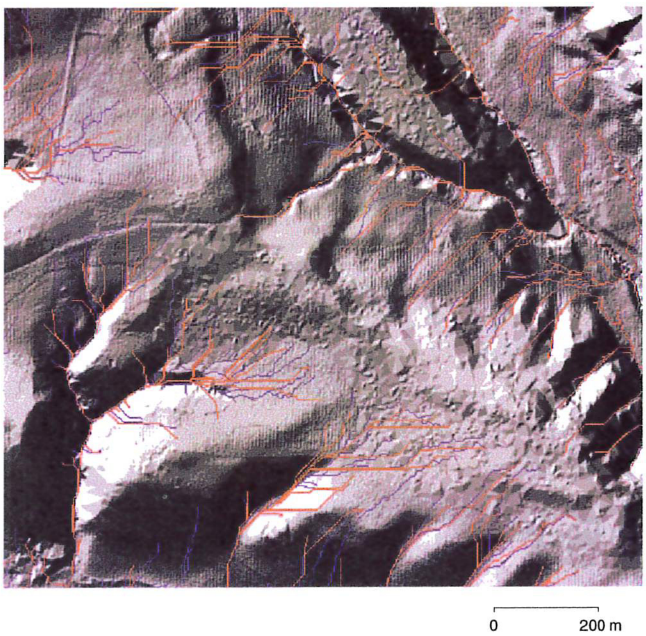
Comparison of hydrological networks. A method to ensure that important catchment areas and the respective channel lines are preserved has been discussed in section 5.4. Such linear features can also be used to provide a quality estimation if they are not used for the approximation in the first place. Figure 6.1 shows the channel lines with a flow accumulation according to Jenson and Domingue (1988) of the runoff generated by more than 300 cells. The catchment areas are largely preserved at both arbitrative scales. In the northerly flat area dominated by artificial triangles a changing runoff is not very surprising.

Point selection by the ATM filter. The number of remaining points after the ATM filter can also be used as a means to evaluate the previously applied wavelet filter. A higher degree of generalisation should also yield smoother areas and larger parts of the model being smoothed. Therefore less points exceed the threshold of the ATM filter and a higher proportion of points is discarded. The relation between retained points and arbitrative scale is shown in table 5.2. A vertical tolerance of 10 cm has been applied to all scales which is less adequate for larger scales but shows that the number of points is monotonously decreasing from one approximation to the next coarser. When using an increasing tolerance for approximations with larger arbitrative scales, the reduction intervals are expected to be more regular.

Visual assessment. Nevertheless, it is believed that visual evaluation of the results of a generalisation process is the most adequate approach. A visualisation technique has been developed whereby the vertical deviation from the original model is plotted as colour information onto the shaded terrain. Hence, the differences can be regarded in correspondence to the surface forms where they occur. Figures 6.3 and 6.4 show a series of approximations using scale 3 to 8 as arbitrative scales. The coarser scales exhibit a limitation of the tensor product wavelet transform. Along diagonal structure lines the favoured directions appear. Such artefacts are concealed by contour plots. A visual assessment using hillshading information is believed to give the best impression of the quality of the approximation.

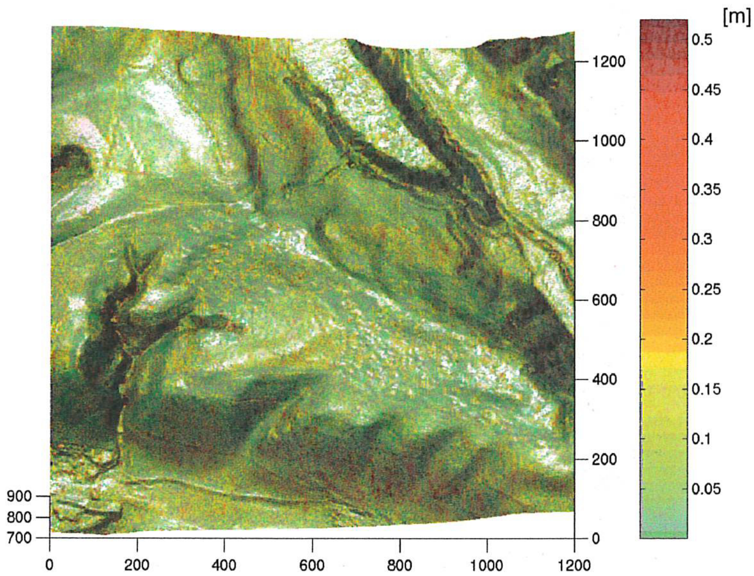


(a) Arbitrative scale 4

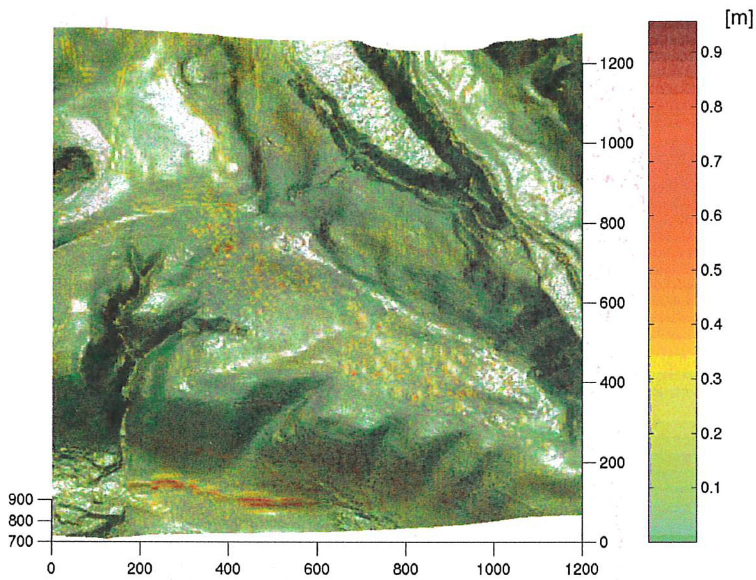


(b) Arbitrative scale 8

Figure 6.1: Comparison of flow accumulations of a) the original (blue) and the generalised model (red) with arbitrative scale 4 (features of approximately 80m wavlelength), b) the original (blue) and the generalised model (red) with arbitrative scale 8 (features of approximately 900m wavlelength). Terrain data: Bundesamt für Landestopographie © (2000) (DV002247).

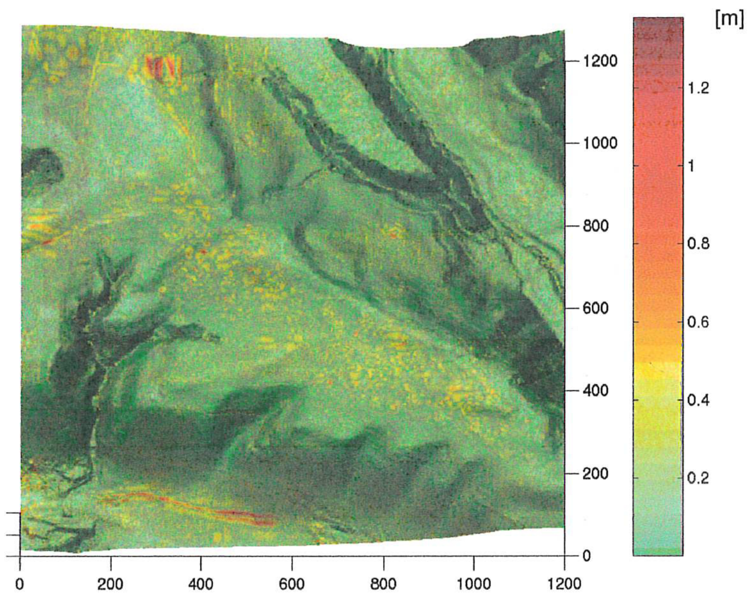


(a) Arbitrative scale 3

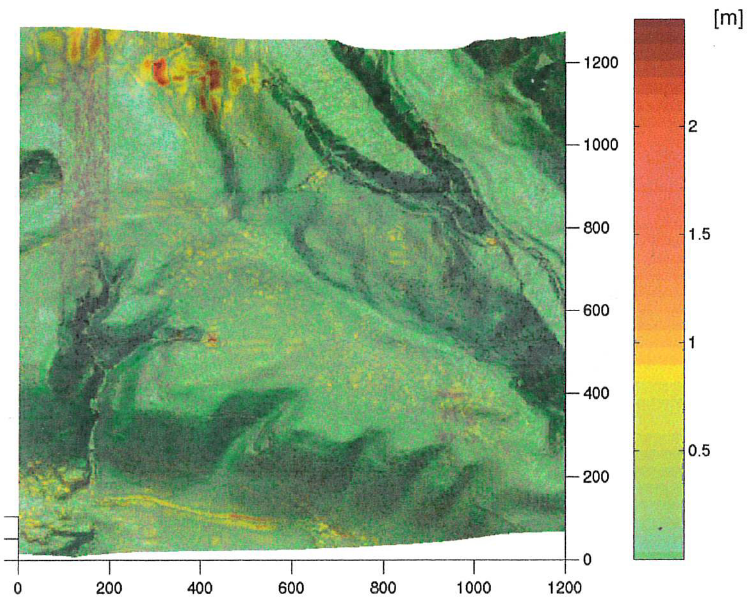


(b) Arbitrative scale 4

Figure 6.2: Comparison of approximated models with arbitrative scales 3 and 4. The colour information illustrates the deviation to the original. The unit is [m]. Terrain data: Bundesamt für Landestopographie © (2000) (DV002247).

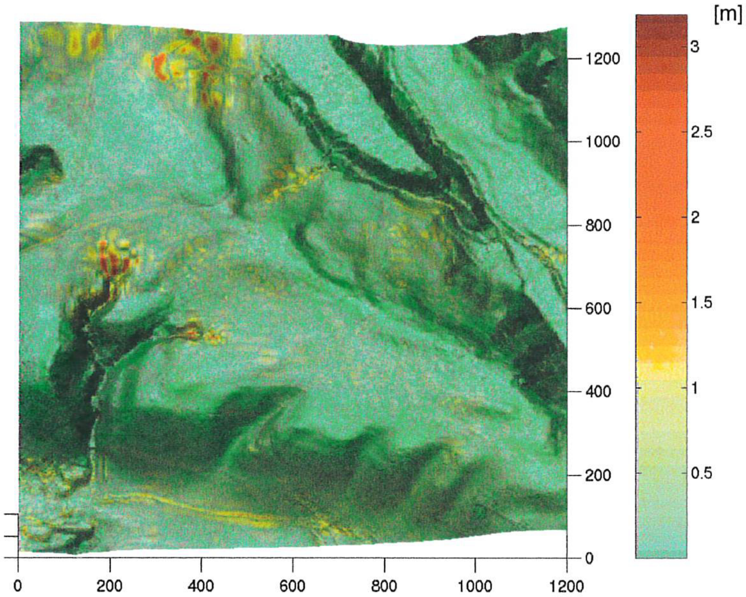


(a) Arbitrative scale 5

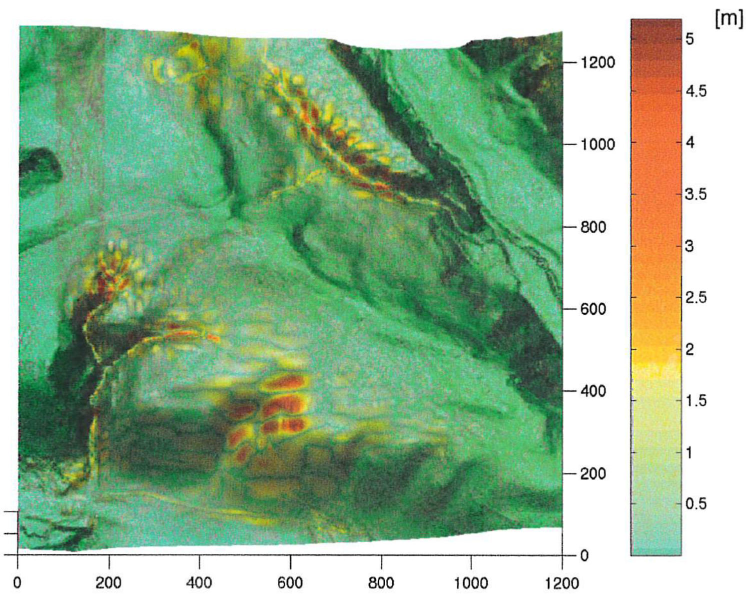


(b) Arbitrative scale 6

Figure 6.3: Comparison of approximated models with arbitrative scales 5 and 6. The colour information illustrates the deviation to the original. The unit is [m]. Terrain data: Bundesamt für Landestopographie © (2000) (DV002247).



(a) Arbitrative scale 7



(b) Arbitrative scale 8

Figure 6.4: Comparison of approximated models with arbitrative scales 7 and 8. The colour information illustrates the deviation to the original. The unit is [m]. Terrain data: Bundesamt für Landestopographie © (2000) (DV002247).

Chapter 7

Conclusion

7.1 Summary

To approach the increasing importance of surface generalisation for high-resolution DTMs, a framework consisting of two steps is suggested. The first step involves the reduction of information in the wavelet domain whereas the second one reduces the number of points in order to obtain a TIN. A wavelet coefficient filter, which is adaptive to locally significant features, expressed by curvature values at different scales, has been implemented. Features of a scale of interest are detected by the stationary wavelet transform or they can be assumed to exist as *a priori* knowledge. Such significant regions are retained with high accuracy. Other areas are smoothely represented by the approximation at the scale of interest. The results have been assessed mainly by visual comparison of the approximations and their local deviation from the original. A structural analysis according to Lee (1991) and a comparison of the hydrological networks has also been performed. The data reduction is conducted by a so-called ATM filter by Heller (1990) using a vertical tolerance as a filter criterion. The result is believed to be more closely related to the generalisation processes of selection/elimination and emphasis than methods filtering only in the spatial domain.

7.2 Future Research

The results of this thesis and the experiences made suggest that the wavelet transform is an adequate tool for analysing and generalising digital terrain models.
However, in order to find an appropriate representation of a terrain surface by wavelets further investigation is needed. First of all, the search of suit-

able wavelets is a primary focus. Bi-orthogonal wavelets, such as compactly supported pseudo-coiflets proposed by Reissell (1996) have been considered to have good approximation properties for the types of surfaces used here (i.e. terrain).

A more suitable representation could also be achieved by introducing an additional degree of freedom. A rotational parameter could help to detect features more accurately than wavelets restricted to horizontal, vertical and diagonal directions. The positive wavelet transform applied by Gallant and Hutchinson (1996) yielded promising results. A wavelet looking like a proper terrain feature with the parameters location, length, width, orientation and height can obviously represent a landscape better than a tensor product wavelet with an inherent directional bias. In this study, such directional artefacts were eliminated by the subsequent application of a traditional spatial domain TIN filter. Nevertheless, it is strongly believed that the wavelet should be similar to the transformed terrain in order to minimise the generation of artefacts. These artefacts often lead to a poor performance of the ATM filter.

Representing a terrain by components of some specific form, such as wavelets, is an abstraction from reality. It is very important to take into account any constraints and deficiencies of the applied model. This is also applicable for modelling terrain using wavelets. For example the tensor product wavelet with favoured directions can barely reflect the full complexity of landforms.

The thresholding algorithm applied to detect significant structures has been based on the mean value of the coefficients at the respective scale. The threshold could also be set to specific values corresponding to an absolute amplitude or energy of features. This yields a result independent of the feature size distribution in the analysed terrain. However, for demonstration purposes, a mean threshold is more reasonable in order to achieve approximations of various intensities in the same terrain model.

The recognition of landscape features is a further field where wavelets can contribute to finding better solutions. Especially the issue of scale is approached by the very nature of wavelets. Dikau (1989) proposes a synthesis of landform components of different size and complexity. The suggested landform components are defined quantitatively as logical combinations of geometrical measures, such as gradient, curvature, distance to drainage divide or distance to channel divide. The underlying principle of wavelets is very similar: A signal is transformed to simple components and reconstructed by their re-combination. However, a wavelet transform cannot assign real-world landforms to one specific scale. The Heisenberg uncertainty principle suggests that any classification of local features referring to wavelength must be fuzzy. It might be an option to subsume various levels of the wavelet transform to 'super-classes' of different orders of magnitude. Dikau (1990) used a 'hierarchy of size orders' discriminating between classes spanning two orders of magnitude. Nevertheless, it is believed that assigning a range of scales to a landform or a fuzzy classification could address the problem more adequately. Further research could adapt the fuzzy landform classification of Wilson and Burrough (1999) or MacMillan and Pettapiece (2000) to accommodate also the issue of fuzzy scale membership of these landforms. Arrell et al. (2001) advocate a fuzzy landform classification corresponding to various resolutions. Such a combination of multiresolution and fuzzy logic techniques is considered to be promising for landform recognition.

The second derivative is believed to have a high relevance to assess the importance of many landscape features. However, for proper feature extraction a more sophisticated curvature assessment than the one suggested in section 4.4 is advisable. In addition to the east-west and north-south direction diagonal cross-sections from the original model could be transformed as one-dimensional profiles. Gradient values, which are necessary to obtain slope and contour direction, can be obtained by Haar wavelets equivalent to curvature detection (Beyer and Meier (2001)).
The actual feature extraction has been of secondary interest in this thesis. Nevertheless, the illustrations of Figures 4.4 and 5.8 strongly suggest making use of multiscale curvature information to extract significant structures.

Much research needs to be done in order to achieve approximations more closely related to generalisation based on semantic information rather than purely geometric and statistical measures. The wavelet concept, being somehow related to human vision (see section 3.2), could contribute significantly to such a task. A major problem is the detailed understanding of generalisation processes (Weibel (1992)). A formalisation of generalisation techniques is necessary in order to design algorithms for generalisation processes.
However, the desired type of generalisation is very much dependent on the application of the model. Generalisation is a problem that needs to be solved repeatedly, according to the type of simplification and the degree of abstraction required to be able to model a complex reality.

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Appendix A

Curvature from Wavelet Coefficients

In an informative paper, Beyer and Meier (2001) describe how to obtain derivatives by a stationary wavelet transform. Gradient, curvature and higher order derivatives can be obtained directly from stationary wavelet coefficients without prior inverse wavelet transform into the spatial domain. This is particularly useful due to the multiscale properties of wavelets. The derivative addressed by the wavelet is equivalent to the number of vanishing moments. All wavelets have zero mean (0. moment). A wavelet with an additionally vanishing first moment is proportional to the second order derivative. Say $v(j), j = 1, 2, \ldots, s$ is a discrete wavelet where s denotes its support interval. The first moment is defined as:

$$m_1 = \sum_{j=1}^{s} j v(j). \tag{A.1}$$

This is zero for a Daubechies wavelet with support interval 4. Hence, the 2. moment is:

$$m_2 = \sum_{j=1}^{s} j^2 v(j). \tag{A.2}$$

Let us define:

$$\tilde{m}_3 = \sum_{j=1}^{s} j^3 v(j). \tag{A.3}$$

In order to derive the curvature from a wavelet coefficient two types of adjustments need to be considered.

Scaling. Firstly, a scaling, also termed 'amplitude correction' has to be performed. For the second derivative, the wavelet coefficients have to be weighted by:

$$\alpha_2 = \frac{2}{m_2(\Delta x)^2}. \tag{A.4}$$

Δx denotes the sampling interval. It depends on the sampling interval of the original data set Δx_0 and on the scale k of the respective wavelet coefficient.

$$\Delta x = \Delta x_0 2^{k-1}, \qquad k = 1, 2, \ldots, K. \tag{A.5}$$

K is the coarsest scale regarded by the wavelet transform.

Phase shift. Secondly, a phase shift x_ϕ or spatial translation has to be taken into account. According to Beyer and Meier (2001), this is for the second derivative:

$$x_\phi = (n_c 2^{k-1} - \frac{\tilde{m}_3}{3m_2})\Delta x, \tag{A.6}$$

where n_c denotes the length of the mother wavelet. The actual significance in terms of curvature of the stationary wavelet coefficient $w(f; x_i)$ at location x_i is:

$$f''(x_i - x_\phi) = \alpha w(f; x_i). \tag{A.7}$$