Models and Experiments for Quality Handling in Digital Terrain Modelling

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Abstract

The work reported in this thesis considers the issue of *quality handling* in digital terrain modelling. Approaches to handling terrain information (TI) quality, to gain practical relevance, must embrace both a *TI producer* and a *TI user perspective*, and be equally applicable to both production and application domains. Of the five conceptions of quality briefly discussed, *fitness for use* is deemed best suited to provide an overall paradigm for the envisioned purpose, because:

- it provides a common baseline for TI quality handling across the heterogeneous communities using such information; and
- it apportions the responsibilities between producers and users of TI.

An understanding of both TI itself as well as the factors affecting its quality is prerequisite to the development of approaches to TI quality handling. Without such an understanding, the methods proposed will tend to remain restricted in their usefulness by the discipline-dependent assumptions implicit in their design. To gain such insight, the thesis develops a workflow framework for reliable application of TI in spatial modelling applications that traces the *life-cycle* of terrain data from its very conceptualisation to its actual storage. The workflow arranges the influence domains of TI producers and users on the terrain data life-cycle.

Investigation of this life-cycle reveals three major *factors affecting TI quality*, namely: *Uncertainty*, which arises from deficiencies in the individual steps of the digital terrain model (DTM) generation and the TI derivation process; *consistency*, which is a measure of the integrity of the actual digital terrain modelling workflow; and *validity*, which is a measure of the genericness of given TI. Besides these major aspects, a few additional factors affecting TI quality are identified, such as the *history* of the TI.

To organise knowledge about these factors affecting TI quality, three primary *metadata components* are suggested. These are:

- *Modifiers*, which make explicit the mappings involved in the digital terrain modelling workflow, and thus provide part of the *quality norm*.

- *Descriptors*, which provide *informative* data explaining the supplied TI by, for instance, recounting its life-cycle or describing details about its technical realisation or its availability.

- *Quality elements*, which measure the actual performance of the supplied TI.

The above components are substantiated by naming *metadata elements* necessary for comprehensive TI quality description. The components are accompanied by basic *principles* for TI quality reporting, including mechanisms for reporting quality on field-type data and for supporting quality assurance.

The *scale dependence* of landforms is recognised as an important yet rarely considered factor affecting TI quality. The issue, therefore, receives specific consideration here. Wavelet techniques are presented as useful tools for:

- *Local estimation* of the uncertainty introduced when representing terrain at different levels of scale. To achieve this, a *selective filtering* procedure is proposed *in the wavelet domain.*

- Enabling *'scale-aware'* modelling techniques, particularly through easy integration of methods for multiscale modelling.

Finally, the *Pluggable Terrain Module* (PTM) integrates the achievements of this thesis. On the one hand, the PTM realises the concepts discussed for *reliable digital terrain modelling* by:

- simulating a *continuous terrain surface* with the help of supporting data and terrain reconstruction methods driven by *application-specific rules*; and

- embodying *derivable TI* based on stored and thus persistent functions. By embedding the functionality needed for terrain analysis, the PTM provides an application with proper tools that prevent it from choosing inappropriate approaches to handle TI.

On the other hand, the PTM is basically a *conceptual model for storage of metadata* documenting TI quality using the metadata components and principles proposed in this thesis.

Zusammenfassung

Die vorliegende Dissertation beschäftigt sich mit Fragen und Aspekten der *Qualität in der digitalen Geländemodellierung.* Ansätze zur Behandlung der Qualität von Geländeinformation (GI) müsen, sollen sie je praktische Relevanz erlangen, sowohl die *Perspektive der Hersteller* als auch diejenige der *Benutzer* solcher Information berücksichtigen. Von den fünf kurz diskutierten Konzeptionen des Begriffs Qualität erscheint daher der *'fitness for use'* Ansatz als das für diese Arbeit am besten geeignete Paradigma; dies aus zwei Gründen:

- Der Ansatz verleiht den in verschiedenen Anwendungsgebieten unterschiedlichen Behandlungsweisen der Qualität von GI eine gemeinsame Basis;
- er regelt die Verantwortlichkeiten zwischen den Herstellern und den Benutzern von GI.

Die Entwicklung von Ansätzen zur Behandlung der Qualität von GI erfordert einerseits ein Verständnis der GI selbst. Andererseits setzt sie ein Verständnis der Faktoren voraus, die einen Einfluss ausüben auf die Qualität der GI. In Ermangelung eines solchen Verständnisses besteht die Gefahr, dass die vorgeschlagenen Methoden von den ihnen zugrunde liegenden anwendungsspezifischen Annahmen in ihrer allgemeinen Nützlichkeit beschränkt werden. Um ein derartiges Verständnis zu erlangen, wird ein Framework für die zuverlässige Anwendung von GI im Rahmen räumlicher Modellierungsaufgaben entwickelt. Das Framework erfasst den *Lebenszyklus* der Geländedaten, von ihrer Konzeptualisierung bis zu ihrer eigentlichen Speicherung und allfälligen Wiederverwendung. Es regelt auch die Einflussbereiche von Herstellern und Benutzern von GI im Lebenszyklus der Geländedaten.

Sorgfältige Analyse des gezeichneten Lebenszyklus endet in der Identifikation von drei Faktoren, die die Qualität von GI wesentlich beeinflussen (im Folgenden *Qualitätsfaktoren* genannt): *Unsicherheit* (uncertainty), deren Aufkommen in Unzulänglichkeiten der einzelnen Verfahrensschritte bei der Herstellung von digitalen Geländemodellen (DGM) und der Extraktion von GI begründet ist. *Konsistenz* (consistency), die ein Mass für

die Integrität des eigentlichen Modellierungsprozesses ist, und schliesslich *Gültigkeit* (validity), ein Mass für die Generizität der produzierten GI. In Ergänzung dieser drei Hauptaspekte benennt die Arbeit einige weitere Qualitätsfaktoren (so zum Beispiel die *Abstammung* der Geländedaten).

Strukturierung von Wissen und Erkenntnis über die genannten Qualitätsfaktoren geschieht anhand dreier *Metadaten Komponenten*, nämlich:

- So genannten *'modifiers'*, die der expliziten Erfassung aller im Rahmen des digitalen Geländemodellierungsprozesses notwendigen Abbildungen dienen. Die *modifiers* stellen daher einen Teil der *Qualitätsnorm.*
- *'Descriptors'*, die *informative* Angaben zur inhaltlichen Erklärung von GI bereitstellen.
- *'Quality elements'*, die die eigentliche 'Leistung' (performance) der Geländedaten bemessen.

Metadaten Elemente, deren Umfang eine umfassende Beschreibung der Qualität von GI sicherstellen soll, werden zur Konkretisierung der Metadaten Komponenten angeführt. Ebenso werden *Richtlinien* zur Beschreibung von GI Qualität dargelegt, insbesondere Mechanismen zur Beschreibung der Qualität von feldhaften Daten oder zur Umsetzung von Aspekten der Qualitätssicherung und -kontrolle.

Als wesentlicher, wenn auch dato kaum gebührend berücksichtigter Qualitätsfaktor entpuppt sich die *Massstabsabhängigkeit* digitaler Darstellungen von Geländeformen. Diesem Problem widmet sich daher der letzte Teil der Arbeit. Wavelet-basierte Methoden erweisen sich als nützliche Werkzeuge, um:

- Unsicherheiten, die bei der Darstellung einer Geländeoberfläche in einem bestimmten Massstabsbereich anfallen, *lokal abzuschätzen.* Dabei wird zum Zwecke der Darstellung eines Geländes in einem bestimmten Massstabsbereich ein *selektives Filterverfahren im Wavelet Raum* (wavelet domain) vorgestellt.
- Unter besonderer Berücksichtigung von Massstabsfragen zu modellieren. Dies gelingt dank der in den Wavelets einfach verfügbaren Methoden zur Multiskalen Modellierung.

Schliesslich werden die vorgeschlagenen Konzepte und Werkzeuge im *'Pluggable Terrain Module'* (PTM) integriert. Das PTM wurde einerseits zur Umsetzung der diskutierten Ansätze für eine *zuverlässige digitale Geländemodellierung* entworfen. Dazu wird:

- eine *stetige Geländeoberfläche* simuliert. Die Simulation erfolgt im Zusammenspiel von Stützdaten und Geländerekonstruktionsmethoden nach *anwendungsspezifischen* Regeln;

- *extrahierbare, 'virtuelle' GI* bereitgestellt. Das PTM 'speichert' die herleitbare GI in Form persistenter Methoden (stored functions). Dank der Einbettung von DGM Analysefunktionalität ins PTM werden dem Anwender geeignete Werkzeuge zur Verfügung gestellt, womit der Verwendung von unangebrachten Methoden zur Verarbeitung von GI ein Stück weit vorgebeugt wird.

Andererseits ist das PTM Design im Wesentlichen ein *konzeptuelles Datenmodell für die Speicherung von Metadaten* zur Qualitätsbeschreibung von GI unter Verwendung der in dieser Arbeit vorgestellten Metadaten Komponenten und Dokumentationsrichtlinien.

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Contents

Chapter 1

Introduction

1.1 Motivation

Uncertainty is an unavoidable property of geographic information (GI), and thus of terrain information (TI) alike. Major reasons for the intrinsic nature of uncertainty in GI include first the physical world's inherently indistinct and indeterminate nature. Real-world phenomena that may be identified or categorised are indisputably exceptions rather than the rule (Russel 1912, Lakoff and Johnson 1980, Burrough and Frank 1996). Second, even the most skilled and careful observer using the most advanced equipment will not be able to measure the physical world beyond some limits of precision (Vckovski 1998). Third, real-world phenomena show variation over different levels of scale (Burrough and Frank 1995). The resolution of the observing device and the actual digital representation of some reality thus introduce scale-based effects into GI. Fourth, all information is provided within the context of a theory (Raper and Livingston 1995). Although GI science aims at favouring those theories most faithful to the physical world, the effects of personal, cultural, and institutional impact on information production should never be discounted (Nyerges 1991). For the particular case of TI, two implications finally arise from terrain surface shape not obeying an a-priori known, mathematically or physically determined field function: digital representation of terrain requires discretisation when sampling terrain data; incomplete knowledge of the phenomenon then impedes its lossless reconstruction. Digital terrain representations are, therefore, in principle incomplete.

In recent years, digital terrain models (DTMs) became an integral component to a wide range of applications aiming at modelling, analysis, simulation, and visualisation of environmental processes and human activities in order to support planning, forecasting, and spatial decision making. However, the quality of the TI used in such applications is not always satisfactory. Causes for this nuisance are mentioned above; additionally, sourceing problems may arise. Many elevation data sets are derived from contour maps, where the original observations have been altered by effects of cartographic generalisation in the interests of (cartographic) simplicity and readability. Many sets of elevation data result from complex ‘black-box’ sampling techniques[1] and preprocessing methods[2], or even from subjective operator judgements[3], and thus may not be replicable between users.

Although an inescapable relationship between uncertainty and GI is widely acknowledged, to date commercial geographic information systems (GIS) have yielded little or no uncertainty handling capabilities (Duckham 1999). Alike, applications requiring TI have little or no facilities at their disposal to cope with the inevitable uncertainties. The subjects of TI quality and uncertainty have, thus, recently gained increasing interest, as DTM users have come to realise that the quality of the used products may be less than that required for their tasks; and that without knowledge of the reliability of the results produced, the integrity of past, present, and future decisions may be jeopardised (Hunter and Goodchild 1997). Not the least, because the omission of uncertainty handling capabilities raises questions about the legal liability for erroneous information and decisions based on GIS technology (Cho 1998, Epstein et al. 1998).

Investigation of quality issues has, therefore, in recent years been accorded a high priority in GI science leading to respective efforts of a number of institutions[4] and of a variety of researchers[5]. Increased research has been dedicated to quality issues in digital terrain modelling as well (see, e.g., Lee et al. (1992), Fisher (1993), Li (1993b,a), Wood and Fisher (1993), Brown and Bara (1994), Monckton (1994), Wood (1996b), Florinsky (1998), Hunter and Goodchild (1997), Wise (1998), Schneider (2000)). Common to these analyses is the investigation of isolated factors affecting DTM quality[6], mainly on a data level. Another look at the issue, however, reveals the

[1]Such as remote sensing technologies, including laser-scanning or air- or space-borne sensors such as InSAR or SAR.

[2]Such as georeferencing, geometric corrections, or denoising.

[3]Such as required for semi-automated stereo-plotting.

[4]Such as the National Center for Geographic Information and Analysis (NCGIA, Abler (1987)), the National Committee for Digital Cartographic Data Standards (NCDCDS, National Committee for Digital Cartographic Data Standards (1988)), the International Cartographic Association (ICA, Guptill and Morrison (1995)), the Open GIS Consortium (OGC, Bühler and McKee (1998)), and the International Standards Organisation (ISO, ISO/TC211 (1999), Godwin (1999)).

[5]Amongst many others Goodchild and Gopal (1989), Chrisman (1983, 1997), Goodchild and Jeansoulin (1998a), Heuvelink (1998), Duckham (1999).

[6]Such as the accuracy of the surveying process, uncertainty introduced through interpolation, uncertainty associated with institutional providers of terrain models, error propagation in DTMs, or implications of elevation uncertainties in derivative products and applications.

quality of digital terrain representations to be a complex system of given, generated and propagated uncertainties as well as related factors on both data and model levels. Yet, a holistic insight into this complex 'cycle' is lacking, and integrated strategies to cope with its implications are missing.

1.2 Error and Truth: Definitions and Concepts

Unfortunately, in the context of GI, several terms are common surrogates for quality, and the literature is full of varied meanings for the key terms in the discussion of GI quality.

Error is commonly defined as the discrepancy between reality and some observation or representation of reality. The definition of error implies the existence and knowledge of some 'truth'. Error is a term with a negative connotation, since it implies something that can be corrected, whereas not all sources of uncertainty in GI are inherently correctable (Goodchild 1995a). Likewise, deviations between a given set of GI and some perceived reality can not always be interpreted in the narrow sense of mistake[7]. Thus, in this thesis, error will only be used in situations requiring consistency with terminology common in statistics, where an error model is simply a model of the uncertainty present, without regard to source (Goodchild 1995a).

The ISO Standard 19113 (Geographic Information - Quality Principles) defines *accuracy* as difference of observations from true values or values accepted to be true (ISO/TC211 1999). Accuracy assessment thus requires comparison between the information to be assessed and some accepted truth. However, there are abundant difficulties in defining an accepted truth in connection with GI. And if accepted truth is replaced by the weaker *source of higher accuracy*, the definition takes on an awkward aspect of circularity (Goodchild and Jeansoulin 1998b, Goodchild 1995a). According to this definition, accuracy statements are always relative, since they depend upon the frame of reference chosen and the comparison method applied.

Since error always implies some knowledge of truth, and the notion of accuracy is both problematic and restrictive in connection with GI, the term *uncertainty* is preferred in this thesis. Following Hunter and Goodchild (1997), uncertainty denotes a lack of knowledge of the true value that would be found making a perfect observation using a perfectly accurate instrument. Used in this way, the term includes not only uncertainties due to imperfections in measuring instruments, but also uncertainties due to modelling effects such as abstraction or generalisation, as well as uncertainties

[7] As Duckham (1999) notes, for instance, the widespread use of two-dimensional Cartesian grid coordinates for the mapping of small spatial extents is not taken to imply that the world, like the Cartesian plane, is flat. It is simply that these representations are adequate as far as they adhere to an abstracted idea of reality.

due to inadequate or ambiguous definitions and the resulting variation in observation or interpretation.

For the purposes of discussion, *precision* is defined as the sharpness of definition; this notion includes the degree of detail used in reporting measurements or attributes[8]. Note that precision refers to the act of representation and not to the thing being observed and represented.

The aim of quality handling in digital terrain modelling is to address the problem arising from objective truth being often undesirable and always unobtainable through the provision of a *context* for TI that reflects the factors likely to affect its quality. The concept of *contextual information*, or information about information - referred to as *metainformation* - is used to generally refer to the structure imposed on the context required to appraise TI quality[9].

1.3 Quality in Digital Terrain Modelling

The various "information communities" (Bühler and McKee 1998) that use TI have traditionally developed different notions of *quality*. Ehrliholzer (1996) attempted to group the different conceptions of quality held in cartography. The grouping she proposed is applicable to the domain of digital terrain modelling in a straightforward manner. Five *concepts of quality* are discerned, which, however, can not always be strictly distinguished from one another:

- **Process-related** concept: In this approach, quality is understood as the ability to satisfy product specifications. To (theoretically) guarantee a product that meets its specification, quality control in each phase of the production process is called for. Integrating quality control into the production process, this conception finally aims at *quality assurance.*

- **Cost-benefit** concept: This approach puts quality in relation to time, and hence, indirectly, to price.

- **Fitness for use** approach: This notion defines quality from a user's perspective. In this sense, fitness for use means the ability of a product to satisfy the requirements of an application. It is unrealistic to expect that fitness for use can always be assessed entirely objectively. Rather than a simple 'yes' or 'no' answer, 'fit' or 'unfit', the question of fitness for use will almost always yield an answer qualified by a degree of subjectivity (Duckham 1999).

[8] 24.2461 is more precise than 24.2, though it may not be more accurate.

[9] Information on uncertainty undoubtedly is a major subset of such metainformation.

- **Product-related** concept: In this approach, quality is understood as a definable and exactly measurable quantity. In contrast to the notion of fitness for use, user needs are not considered relevant to quality assessment; quality is entirely defined by measurable product characteristics.

- **'Transcendent'** concept: A 'transcendent' notion understands quality as something not definable or even measurable; as something that can be assessed only from experience. Quality in this sense means a "universally recognisable" high standard (Ehrliholzer 1996)[10].

Of these notions of quality, *fitness for use* is considered most suitable to provide an overall paradigm for quality handling in digital terrain modelling, as it provides a common baseline for TI quality handling across the heterogeneous communities using such information. As mentioned above, the various information communities requiring TI have traditionally developed different approaches to the handling of TI quality. Fitness for use accommodates all of these different approaches without prejudicing one over another (Duckham 1999). However, the nature of digital terrain modelling is rather that of a 'process' than of a 'product'. Thus, when attempting to develop integrated approaches to cope with the uncertainties and other factors affecting TI quality given, generated, and propagated through this process, *process-related quality notions* also deserve consideration.

Fitness for use apportions and emphasises the responsibilities between providers and users of TI (Duckham 2000, Chrisman 1986). The *production domain* has a responsibility to provide explicit, appropriate information on uncertainty and other factors affecting the quality of data and information along with these data and information. Clearly, to accomplish this task, notions lent from a *product-related* understanding of quality have to be integrated within the fitness for use paradigm. The *user* of TI, on the other hand, has the responsibility to ensure the information is only applied to problems where such a use is appropriate.

An important implication of fitness for use is that the reduction of error is a peripheral consideration (where error is meant in the strict sense of perceived discrepancy between reality and observation or representation). The desire to reduce error is predicated upon the view that error is in some way bad. Conceiving deviations between perceived reality and its digital representation as uncertainty in the sense of section 1.2 rather than as error in a narrow sense, fitness for use aims at an understanding of uncertainties rather than at error reduction. An understanding of the uncertainties occurring is often correlated with an understanding of spatial phenomena,

[10] Much like, for instance, the conception of beauty, according to this view the conception of quality defies implicit definition (Walmüller 1990).

their sampling and analysis (Burrough and McDonnell 1998). Therefore, uncertainty is best conceptualised as a description of characteristics and limitations of data and information, rather than as faults to be eradicated from data (Chrisman 1982, Duckham 1999).

No definition of the term quality has been provided yet. Based on the above discussion, this thesis will comply with the ISO Standard 19113 (Geographic Information - Quality Principles), where *quality* is defined as the *"totality of characteristics of a product that bear on its ability to satisfy stated and implied needs"* (ISO/TC211 1999). Note that herewith quality is formalised without reference to truth (Buttenfield 1993), and it is not restricted to the notions of accuracy or precision. *Quality handling in digital terrain modelling*, then, basically means:

- Definition of a context that reflects the relevant DTM and TI characteristics[11],
- comparison of the TI characteristics with stated needs, and
- interpretation of the comparison results (within the frame of the TI context).

1.4 Problem Statement

Digital terrain modelling is the discipline of numerical modelling and representation of the terrain surface with digital computers in order to allow derivation of TI required in spatial modelling applications. Hence, its core task is to provide a mapping between the terrain surface and a suitable mathematical representation; thus enabling the use of mathematical inference rules to derive statements about the topographic surface being investigated. With this view, discussion of quality issues in digital terrain modelling means to raise fundamental questions, such as:

- What qualities characterise 'good' terrain models?
- How can 'topographic reality' be appropriately represented in a formal (mathematical and/or digital) system?
- What is the relationship between the 'real' terrain and its formal representation?
- How can different models of the same portion of the Earth's surface be compared?

[11] In other words, definition of apt metainformation.

- How can consistent encoding into a formal system be ensured in the sense that the rules applied in the formal system become reliable and valid statements about the topographic surface being studied when appropriately decoded back into ‘real-world’ relations?

Although no uniform and complete answers to these questions can be expected, a basis for considering these issues under conditions appropriate to a given setting may be provided.

1.4.1 Gaps in Understanding TI Quality

Despite issues of the above kind having gained growing interest, and increasing research having been directed towards identifying sources and implications of factors that influence TI quality, such as uncertainty, there are a number of, as yet, unresolved problems (cf. discussion in section 1.1):

- Most research efforts concentrate on isolated aspects of DTM quality. The quality of digital terrain representations is a complex system of given, generated, and propagated uncertainties on metric, attribute, and semantic levels. Although the components of this system are strongly interrelated, DTM quality is often reduced to the mere statement of the metric accuracy of given or computed elevations (Fisher 1998b, USGS - United States Geological Survey 1998). No ‘model of terrain data life cycle’ is at hand, on which to ground the quality discussion, and a holistic identification of the roles of uncertainty and related factors in such a model is still missing.

- Existing approaches to the documentation of spatial data extensively describe the actual data (as their names imply) but rarely describe models nor underlying model assumptions. However, there is more to the ‘systemness’ of DTMs than just the individual data items (as will be discussed in section 2.2 and chapter 4). Yet, the step from description of *data* quality to the discussion of *model* quality still has to be made.

- Although the *scale dependence* of terrain surface shape is widely recognised, it is rarely regarded as a factor affecting TI quality, and consequently it is not generally considered for DTM quality evaluation.

1.4.2 The Issue of Reliability

As mentioned in section 1.3, fitness for use does not aim at error reduction but at understanding the uncertainties occurring, that is, at understanding the degree of *reliability* of the information produced. Although the term ‘reliability’ has an intuitive significance, its exact meaning in digital terrain modelling has not yet been specified. According to the Merriam-Webster's Collegiate Dictionary (2000), the term 'reliable' denotes:

(a) suitable or fit to be relied on;

(b) giving the same results on successive trials.

Transferred to digital terrain modelling, this definition implies two fundamental requirements:

(a) Users of TI must know whether a DTM at hand is appropriate for their particular application. *Strategies for quality handling in digital terrain modelling thus must include capabilities for estimation of the quality of both the used and the produced TI* in order to enable the users to properly interpret their results and to draw sound conclusions.

(b) TI obtained from DTM analysis must be *replicable* and thus logically and internally *consistent*. Reliable handling of TI, therefore, should strive for producing results determined mainly by actual DTM characteristics and not by extrinsic factors, such as the GIS package or algorithms used, the DTM data format, etc. However, *strategies for quality handling in digital terrain modelling must make available to the users all the information necessary to ensure replicability of the produced TI.*

1.5 Research Aims

The overall aim of this thesis is to elaborate *methods to comprehensive quality handling in digital terrain modelling* consistent with the concept of fitness for use, paying special attention to the problems and issues highlighted in the previous section. The aim is to derive a *framework for comprehensive quality handling* from a conceptual and detailed investigation of the processes involved in digital terrain modelling, of the intrinsic modelling circumstances and constraints, and of the nature and endeavour of reliably representing the topographic surface in the computer.

An understanding of the features and behaviour of both the actual TI as well as the factors affecting its quality is prerequisite to the development of such a framework. Without such an understanding, the strategies proposed will tend to remain restricted in their usefulness by the discipline dependent assumptions implicit in their design. Central to quality handling in digital terrain modelling is the concept of *metainformation*. Both, fitness for use as well as the concept of reliability emphasise the importance of metainformation. Quality information is only useful if it can be made available to the users and thus be incorporated into the geographical or spatial decision-making process. The general objectives of this research are, therefore:

- Development of a comprehensive *quality model* for organising knowledge about TI quality, and
- elaboration of a *structure to describe TI quality*, which basically encompasses *guidelines for the description of TI quality* and *components for reporting DTM quality*.

More specifically, the research objectives include:

- *Definition* of what the concept of 'quality' may denote (departing from the fitness for use concept, as set out in section 1.3),
- development of a 'model of the terrain data life cycle', and identification of the *sources* and *roles* of factors affecting TI quality within it,
- identification of a set of *metadata components* suitable to TI quality description, and
- development of methods for *assessing* and *coping* with factors affecting TI quality.

To conclude, it is worth pointing out that digital terrain modelling exposes a fundamental dichotomy: *terrain model generation* on the one hand, and its *application in spatial modelling applications* on the other. These activities are commonly carried out by different persons: the 'DTM producers' and 'DTM users'[12]. Reliable application of digital terrain models in spatial modelling applications challenges both producers and users of DTMs. Therefore, to gain practical relevance, methods for quality handling in digital terrain modelling must embrace both a producer- and a user-oriented perspective, and be equally applicable to both the production and the application domains.

[12]Even if they should be carried out by the same person, the distinct roles of DTM generation and DTM application still remain distinguishable.

Chapter 2

Continuous Fields in Geographical Information Science

Two key terms epitomise this thesis: 'quality handling' and 'digital terrain modelling'. This chapter provides some insight to the background of digital terrain modelling, and focuses on clarification of the terms 'data', 'information', and 'model'. An introduction to research on uncertainty in TI and current approaches to describe and report on data quality is given in the subsequent chapter.

Understood as *function* $z(\cdot)$ assigning each location $\boldsymbol{x} := (x, y)$ of some planar 2-dimensional domain $\mathbb{D}$ a unique elevation value $z(\boldsymbol{x})$, the terrain surface typifies a *continuous field*. Strictly speaking, the interpretation of a terrain surface as a function is not universally valid; think for instance of overhangs, natural bridges, or caves. However, because such topographic features are relatively rare and small, the notion of terrain as a continuous field is considered adequate (Schneider 2001). The thesis will adhere to this understanding.

Section 2.1 briefly introduces the notion of continuous fields. *Digital terrain modelling* is the discipline of numerical modelling and representation of the terrain surface with digital computers. Respective *mathematical models* are a prerequisite to the translation of real-world phenomena (such as terrain surfaces) into computers. The discipline of mathematical modelling provides the theoretical basis for numerical modelling and digital representation of continuous fields. Section 2.2, therefore, provides some background on mathematical modelling in general. For the sake of clarity, the chapter concludes by distinguishing terms and concepts of *data*, *information*, and *models*. The chapter is not meant to be an exhaustive review of the literature, but more to provide an explanation of the scientific background and the respective terminology. More comprehensive reviews of the characteristics of fields as they are used in data processing systems include Vckovski (1995a,b, 1998), Bucher (1998). A profound introduction into mathematical modelling issues may be found in Casti (1989, 1992).

2.1 Continuous Fields

2.1.1 Definition

The concept of a field originates from physics and describes an entity that is distributed over a space $\mathbb{A}$ and whose properties are functions of space coordinates: A function $z(\cdot)$, more detailed:

$$(id, z(\cdot)) : \mathbb{D} \subset \mathbb{A} \longrightarrow \mathbb{D} \times \mathbb{V}, \quad \boldsymbol{s} \mapsto (\boldsymbol{s}, z(\boldsymbol{s})), \tag{2.1}$$

which assigns every location $\boldsymbol{s} \in \mathbb{D}$ a unique value $z(\boldsymbol{s}) \in \mathbb{V}$, is called a *field* on $\mathbb{D}$. $\mathbb{D} \subset \mathbb{A}$ is called the *domain* of $z(\cdot)$, $\mathbb{V}$ is its *range* (or value domain)[1, 2].

Assume the space $\mathbb{A}$ to be an infinite and dense set. A field is called *continuous*, if its domain $\mathbb{D} \subset \mathbb{A}$ is also infinite[3].

The notion of continuity of the domain $\mathbb{D}$ implies the existence of a *metric* $\|\cdot\|$ defined on $\mathbb{D}$, which allows to express 'distance' $d(\boldsymbol{x}, \boldsymbol{y})$ between two elements of the domain $\mathbb{D}$[4]:

$$d(\boldsymbol{x}, \boldsymbol{y}) := \|\boldsymbol{x} - \boldsymbol{y}\|, \quad \boldsymbol{x}, \boldsymbol{y} \in \mathbb{D}. \tag{2.2}$$

2.1.2 Sampling of continuous fields

Surveying a continuous field requires a measurement model for the physical quantities to be observed. Such a *measurement model* basically defines the

[1] In spatial data handling, $\mathbb{A}$ usually relates to the Earth's surface and therefore commonly is 2-, or sometimes 3-dimensional ($\mathbb{A}$ is sometimes defined as 4-dimensional space-time). $\mathbb{D}$ refers to the portion of the Earth's surface under investigation. The range $\mathbb{V}$ is usually a subset of $\mathbb{R}^n$. If $n = \dim(\mathbb{R}^n) = 1$, the field is called a *scalar field*, otherwise it is a *vector field*.

[2] The notion of $z(\cdot)$ as a function mapping $\boldsymbol{s} \in \mathbb{D}$ to exactly one value $z(\boldsymbol{s}) \in \mathbb{V}$ is not realistic for many natural phenomena, as it neglects any randomness or uncertainty. However, there are several ways to account for such uncertainties, such as probabilistic approaches, which model each field value as a random variable $Z(\cdot)$, interval methods, which map each $\boldsymbol{s} \in \mathbb{D}$ to an interval $[z(\boldsymbol{s})]$, fuzzy sets, etc. A more comprehensive discussion of the issue may be found in Vckovski (1998).

[3] As Vckovski (1998) points out, it is important to note the fundamental difference between the notion of *continuous functions* and *continuous fields*. While the first relates to continuity in the range $\mathbb{V}$, the latter refers to continuity in the domain $\mathbb{D}$.

[4] In spatial data handling, where $\mathbb{A}$ (or $\mathbb{D}$, respectively) most often refers to a Euclidean space, a common metric to compute distances is the Euclidean norm $\|\boldsymbol{x} - \boldsymbol{y}\| := (\sum_{i=1}^{n}(x_i - y_i)^2)^{1/2}$, where usually $n = \dim(\mathbb{A})$ (or $\dim(\mathbb{D})$, respectively) $\in \{2, 3\}$.

way to describe the properties of the field (e.g., terrain elevation) by values $z(\boldsymbol{s})$.

It is important to note the fundamental difference between the level of knowledge about *field behaviour* $z(\cdot)$ (e.g., identification of $z(\cdot)$ with a sine curve) and the measurement of the *value* $z(\boldsymbol{s})$ of the field $z(\cdot)$ at some specific location $\boldsymbol{s}$. It is perfectly possible to measure $z(\boldsymbol{s})$ at location $\boldsymbol{s}$, even if the actual field function $z(\cdot)$ is unknown[5]. This is exactly what the measurement model is about: providing actual field values $z(\boldsymbol{s})$ at specific locations $\boldsymbol{s} \in \mathbb{D}$. The measurement model, thus, defines a mapping from 'reality' to a mathematical object. Note that this mapping is always an 'idealisation' and never bijective (due to variability in the real-world phenomenon, measuring error, etc.).

For the moment, assume such a measurement model is defined, and it is known how to observe a property to get the associated field value $z(\boldsymbol{s}),\ \boldsymbol{s} \in \mathbb{D}$. Recall that the definition of the term continuous field identifies $\mathbb{D}$ as an infinite (even uncountable) set; that is, the function $z(\cdot)$ relates infinitely many elements $\boldsymbol{s}$ of $\mathbb{D}$ to values $z(\boldsymbol{s}) \in \mathbb{V}$. Thus, it is obvious that, in the general case, fields can never be surveyed (or recorded) completely[6]. Surveying a field, therefore, necessarily involves its *discretisation*, that is, a reduction of relation (2.1), which maps $\boldsymbol{s} \in \mathbb{D}$ to $z(\boldsymbol{s}) \in \mathbb{V}$, to a mapping between a finite set of selected locations $\boldsymbol{s}_i,\ i = 1, ..., n$ and corresponding observed field values $\tilde{z}_i := \tilde{z}(\boldsymbol{s}_i),\ i = 1, ..., n$ (where $\tilde{z}(\boldsymbol{s}_i)$ shall denote a *measured* field value, in contrast to the *actual* value $z(\boldsymbol{s}_i)$). However, no measuring device will manage to measure a field value *exactly* at location $\boldsymbol{s}_i$. Rather, the ascertained field value $\tilde{z}(\boldsymbol{s}_i)$ corresponds to a weighted average of the field $z(\boldsymbol{s}_i)$ 'in some neighbourhood' G of $\boldsymbol{s}_i$. The relation between the measured value $\tilde{z}_i$ and the actual field $z(\cdot)$ can be characterised as follows:

$$\tilde{z}(\boldsymbol{s}_i) = \int_{\mathbb{D}} \phi_i(\boldsymbol{s}) z(\boldsymbol{s}) d\boldsymbol{s}. \tag{2.3}$$

[5]Consider, for instance, measurement of air temperature. Using a thermometer, the air temperature at some specific spot $\boldsymbol{s}$ may be measured without any knowledge of its variation in the surroundings of $\boldsymbol{s}$. The respective measurement model is based on the notion of air temperature being an expression of the kinetic energy of the air. The temperature is measured by comparing the current extension of the thermometer's mercury column with the normed extension of a mercury column at zero and one hundred degrees, respectively.

[6]This does not hold true for the special case of physically determined fields, that is, for fields whose field function $z(\cdot)$ is *a priori* known. Consider, for instance, the electric field $\mathbf{E}(\cdot)$ generated by a point charge Q at location r_0 (in the assumed absence of any magnetic field):

$$\mathbf{E}(\boldsymbol{r}) = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{\boldsymbol{r} - \boldsymbol{r_0}}{|\boldsymbol{r} - \boldsymbol{r_0}|^3},$$

where ε_0 denotes the permittivity of free space. $\mathbf{E}(\cdot)$ is completely determined by ε_0, r_0 and Q; that is, with values recorded for ε_0, r_0 and Q, the field is completely definite. The above equation then allows exact modelling of field behaviour throughout its domain.

$\phi_i(\cdot)$ is called a *selection function*[7], and is usually normalised:

$$\|\phi_i\| := \int_{\mathbb{D}} \phi_i(\boldsymbol{s}) d\boldsymbol{s} \stackrel{!}{=} 1,\ \forall i = 1, ..., n.$$

To sum up, typical *sampling of a field* with unknown field function $z(\cdot)$ consists of a finite set of selected locations $\boldsymbol{s}_i$ and corresponding observed field values $\tilde{z}_i$ which together provide the view we have of the field $z(\cdot)$, the *'raw data'* so to speak. A *dataset* describing a field can therefore be written as a set of tuples[8]:

$$(\boldsymbol{s}_i; \tilde{z}_i), \quad i = 1, ..., n. \tag{2.4}$$

Extending (2.4) with a set M of additional data, which renders the dataset's appropriate context (*metadata*, see chapters 1.2 and ??), a formal notation of a generic dataset $\mathcal{D}$ describing continuous fields can be specified:

$$\mathcal{D} = \{M, \{\boldsymbol{s}_i; \tilde{z}_i\}_{i=1}^{n}\}, \quad \boldsymbol{s}_i \in \mathbb{D},\ z_i \in \mathbb{V}. \tag{2.5}$$

2.1.3 Reconstruction of continuous fields

Given a dataset $\mathcal{D}$ as a representation of a field $z(\cdot)$, it is often required to reconstruct the field from the given data values, that is, to estimate a field $\hat{z}(\cdot)$ using the information provided in $\mathcal{D}$. This task may be achieved by *spatial interpolation* or *approximation*[9], where the basic idea is to make use of the dataset $\mathcal{D}$ to provide parametrisation of a model that simulates the continuous field behaviour. In other words, the model provides *information* about the field behaviour 'in between' the sampled data. A broad discussion of interpolation and approximation techniques is beyond the scope of this chapter. Excellent reviews of respective methods may be found in Lam (1983), Myers (1994), Burrough and McDonnell (1998), Bucher (1998), or Schneider (1998). Nonetheless, note that interpolation or approximation models always imply some idealised assumptions about the behaviour of the field (with respect to its continuity properties), adding to the uncertainties inherent to the field representation provided by $\mathcal{D}$.

[7] If, for instance, $\boldsymbol{s}_i$ is a point location and the measurement retrieves the field value exactly at that point, then $\phi_i(\cdot)$ is the Dirac impulse function centred at $\boldsymbol{s}_i$: $\phi_i(\boldsymbol{s}) = \delta(\boldsymbol{s}_i - \boldsymbol{s})$.

[8] The set of sampled locations $\{\boldsymbol{s}_i\}$ is sometimes not given explicitly but implicitly, if there is some regularity. for instance, if the $\boldsymbol{s}_i$ are nodes of a regular lattice, it is sufficient to specify the lattice by definition of its spacing in every dimension, its orientation, starting point, and number of items in each dimension.

[9] *Interpolation* usually is required to reproduce the function values z_i 'at' $\boldsymbol{s}_i$, in contrast to *approximation* methods, which usually aim at finding a function $\bar{z}(\cdot)$ that 'best' fits the original function $z(\cdot)$.

2.2 Mathematical Modelling

This section outlines a general framework which provides a theoretical basis for the mathematical modelling of continuous fields. The discussion basically follows the discourse held more generally in Casti (1989, 1992), where the monumental task of developing a theory of models is attempted. Casti's work is commendable for the clarity it provides on a number of issues that are normally obscured by application specific aspects. According to Casti, knowledge of the properties of models, the techniques for encoding specific realities into formal systems, and the procedures for interpreting the properties of these formal systems in terms of the real-world, is the key to development of good models (Casti 1989).

States Consider a particular subset S of the observable world. For the sake of this discussion, let S be a field-type real-world phenomenon (e.g., terrain elevation, air temperature, or precipitation). Assume that the properties of S can exist in a set of distinct states $\Omega := \{\omega_1, \omega_2, ...\}$. The innumerable set of states Ω fully describes the total condition of the system S under study. Modelling fields, the properties of S will most often vary with their location $\boldsymbol{s}$ in space. Ω is called the *set of abstract states* of S[10]. Ω may be finite or infinite, and in general Ω need not be a set of numbers[11]. An observer investigating S may or may not be able to determine which of these states S is actually in (at any particular point in space or time). It all depends on the resolution of the measuring device at the observer's disposal. As well, what is considered an 'observable state' is not inherently an intrinsic property of the system S itself. Rather, it depends on the observer's intent and knowledge and the ways he or she has of probing S and distinguishing one state from another.

Observables Assume a system S is given together with a set of abstract states Ω. A rule f associating a mathematical object with each $\omega \in \Omega$ is called an *observable* of S. Thus, f is a map: $f: \ \Omega \longrightarrow \mathbb{V}$, where $\mathbb{V}$ denotes some mathematical space. An observable may be interpreted as generalisation of the everyday idea of a measuring device. In this sense, an observable generalises the notion of a measurement model as introduced in section 2.1.2. Consider, for instance, the electric field $\mathbf{E}(\cdot)$ generated by n

[10] If S, for instance, is a portion of the Earth's surface, then a reasonable set Ω of abstract states for S might comprise the range of elevations above sea level within S. Then, $\forall i: \ \Omega \ni \omega_i \subset \mathbb{R}$, with $h_{min} \leq \omega_i \leq h_{max}$, where $h_{min}, h_{max} \in \mathbb{R}$ denote the minimal and maximal elevations in S, respectively.

[11] Stick to the above example of S referring to a topographic surface. Then Ω could also relate to the characteristics of the locations $\boldsymbol{s}_i$ of S: $\Omega = \{\omega_{1,i} = \boldsymbol{s}_i \textit{ is a peak}, \ \omega_{2,i} = \boldsymbol{s}_i \textit{ is a pit}, \ \omega_{3,i} = \boldsymbol{s}_i \textit{ is a saddle}, \ \omega_{4,i} = \boldsymbol{s}_i \textit{ is none of these}\}$.

point charges Q_i situated at locations $\boldsymbol{r}_i,\ i = 1, ..., n$, acting on a specimen charge (of charge q and position $\boldsymbol{r}$). A reasonable set of observables to describe such a system would consist of the charges and positions of the $n + 1$ particles making up the system (n of them generating the field and one acted upon), and the force acting on the specimen charge (the point charge q at location $\boldsymbol{r}$). If S is referring to a topographic surface, a natural choice for a set of observables describing S could be the elevation at all locations of S, or the respective slopes or curvatures.

Generally, to completely describe S will require an infinite number of observables $f_i : \Omega \longrightarrow \mathbb{V}$, where i ranges over some possibly uncountable index set. However, for practical modelling purposes, such a large set of observables is inconvenient, or plainly not ascertainable. Hence, most of the observables f_i are boldly neglected, whilst a proper subset A of $F := \{f_i\}$ is selected and focused upon. A is called an *abstraction* of F, as it results from abstracting, i.e. throwing away, the information contained in $\{F \backslash A\}$. In the context of mathematical modelling of continuous fields, this process of 'abstraction' (or 'simplification') basically performs the discretisation step discussed in section 2.1.2. Strictly speaking, the finite set of observables A does not describe the original system S any more; rather it provides a partial view of S by describing a subsystem S' of S. According to this view, the process of abstraction is an expression of the belief that a particular facet of the system behaviour of S of specific interest to the observer remains invariant under the replacement of the original system S by a proper subsystem S'.

The properties of S' may be measured using different coordinate and reference systems. For instance, air temperature may be measured in degrees Celsius or on the Fahrenheit scale, respectively. Geometric constructs may be specified in Cartesian or polar coordinates and they may be referenced to varying origins. Different countries relate their levelling (terrain elevation surveying) to different bench marks. Therefore, the only aspects of the system S' that have any right to be termed *intrinsic system-theoretic properties* are the quantities that remain invariant under coordinate changes. In answering questions about the investigated system, ideally, only such *invariants* should be used (Casti 1989). Other observables, which are 'coordinate dependent', may be interpreted as artifacts of the observation method. This comment is particularly relevant to fields which are spatial models, as it implies that conclusions based solely on how the continuous world is discretised not truly describe the system, but rather describe aspects of our construction of it (Kemp 1993).

'Systemness' The finite set of observables $A = \{f_j\},\ j = 1, ..., n$, provides the partial view we have of S, the *'raw data'*. But when modelling field-type phenomena, there is more to the 'systemness' of the abstraction S'

of S than just the separate observables through which it is seen. The essential 'systemness' of S' is contained in the relationships linking the individual observables $\{f_j\}_{j=1}^n$, formally written as[12]:

$$\varphi_i(f_1, ..., f_n) = 0, \quad i = 1, ..., m, \tag{2.6}$$

where the $\varphi_i,\ i = 1, ..., m$ are the mathematical relationships expressing the dependency relations among the observables. In the sense of the discussion in section 2.1.3, the φ_i correspond to the attempt to model the field behaviour from the raw data available, that is, to estimate a field $\hat{z}(\cdot)$ from the finite set of observables $\{f_j\},\ j = 1, ..., n$[13]. Note that equation (2.6) establishes a *deterministic* relationship between the observables. No information whatsoever is contained about implications among the elements of the set $\{f_j\}$[14].

Types of Observables Using the equation of state, a number of different types of observables can be identified. The first ones are those whose values remain fixed for every state $\omega \in \Omega$; these observables are called *parameters*[15]. Suppose, there are r such parameters among the n observables $\{f_j\}_{j=1}^n$, with $r < n$. For convenience, assume these are the first r observables. Let $f_i(\omega) =: \alpha_i,\ \alpha_i \in \mathbb{V},\ i = 1, ..., r$. Then, the equation of state can be written:

$$\varphi_{\alpha_1,...,\alpha_r}(f_{r+1}, ..., f_n) = 0, \tag{2.7}$$

indicating the dependence of the description upon the values of $\alpha_1, ..., \alpha_r$ explicitly[16]. For each set of parameters $\{\alpha_i\}$, equation (2.7) describes a

[12] Such a set of relationships is usually called the *equation of state*.

[13] Let S again be a system of $n+1$ point charges, where the first n point charges are making up an electric field $\mathbf{E}(\cdot)$ acting on the $(n+1)$-th (the so called specimen charge). Again, take Ω to be the charges and positions of each of the $(n+1)$ particles, and the force acting upon the specimen charge. Define the $2(n+1)+1$ observables $Q_i(\omega) =$ *charge of the i-th particle when in state* ω, $R_i(\omega) =$ *position of the i-th particle when in state* ω, $F(\omega) =$ *force acting upon the specimen charge when in state* ω; where $i = 0, ..., n$. Then, a single equation of state may be put forward:

$$\varphi(Q_0, ..., Q_n, R_0, ..., R_n, F) = \sum_{i=1}^{n}\left(\frac{Q_i}{4\pi\varepsilon_0} \cdot \frac{R_0 - R_i}{|R_0 - R_i|^3}\right) - \frac{F}{Q_0} = 0,$$

where Q_0, R_0 are arranged to be the charge and position of the specimen charge; ε_0 denotes the permittivity of free space.

[14] A *causal direction* may be introduced into equation (2.6) by solving for some of the observables in terms of the others. The problem, however, is that there may be many ways to do so.

[15] The gravitational pull of the Earth, for instance, would be a typical example for such a parameter.

[16] In other words, an r-parametric family of descriptions results.

different system. The second ones are those observables which are functions of others. In other words, observables may be identified as *inputs* or *outputs* of the system (thus introducing a causal direction, cf. footnote 14).

Equivalence The family of descriptions (2.7) immediately leads to the question of when two descriptions of S' are equivalent. Assume, the observables $\{f_{r+1}, ..., f_n\}$ of equation (2.7) have further been separated into inputs and outputs. Consider then the two descriptions of S':

$$\varphi_\alpha : U \longrightarrow Y, \quad \varphi_{\hat{\alpha}} : U \longrightarrow Y,$$

where U and Y are the input and output spaces, respectively. These two descriptions are considered to be *equivalent*, if there exists a (usually nonlinear) coordinate change in U and/or Y such that the description φ_α is transformed into $\varphi_{\hat{\alpha}}$. Diagrammatically, bijections $g_{\alpha,\hat{\alpha}} : U \longrightarrow U$ and $h_{\alpha,\hat{\alpha}} : Y \longrightarrow Y$ are sought such that the following diagram commutes:

$$\begin{tikzcd} U \arrow[r, "\Phi_\alpha"] \arrow[d, "g_{\alpha,\hat{\alpha}}"'] & Y \arrow[d, "h_{\alpha,\hat{\alpha}}"] \\ U \arrow[r, "\Phi_{\hat{\alpha}}"'] & Y \end{tikzcd}$$

Modelling Relations The core epistemic principle in mathematical modelling is to map a real-world phenomenon into a *formal mathematical system* $\mathcal{F}$[17] in order to use the inference rules of this formal system to derive new relations interpretable as statements about the phenomenon being studied when appropriately decoded back into real-world behaviour. When modelling fields, the goal is to find relations among the available raw data interpretable as field behaviour. The essential step in mathematical modelling, therefore, is the translation of a system of interest S into a particular formal system $\mathcal{F}$, that is, the specification of an encoding map $\mathcal{E} : \{\Omega; f_1, ..., f_n\} \longrightarrow \mathcal{F}$. Equally essential is the ability to decode the statements in $\mathcal{F}$, if they shall be interpreted in terms of behaviour in the real-world system S. So, *reliable* mathematical modelling essentially means:

- to ensure consistent encoding in the sense that relations in $\mathcal{F}$ become valid statements about S when appropriately decoded back,

[17] A formal mathematical system $\mathcal{F}$ is a collection of abstract symbols together with a set of rules expressing how strings of these symbols may be combined to create new symbol strings. Additionally, a set of symbol strings that are taken to be axioms (i.e. which are assumed without proof) is usually part of the definition of $\mathcal{F}$. The definition is completed by a set of rules of logical inference enabling the generation of new and correct strings from earlier ones (Casti 1992).

- to make adroit choices of the encoding and decoding operations, so that as little information as possible is lost in the transition from S to $\mathcal{F}$ and back again.

2.3 Data, Information, and Models: Digital Modelling of Fields

A major challenge in processing data describing continuous fields arises from the discretisation inevitable to the survey of field-type phenomena (as explained in sections 2.1.2 and 2.2). Extent and nature of this discretisation depend on both the level of knowledge about the behaviour of the field $z(\cdot)$ and the investigation's aim. However, a major implication of discretisation is that the sampled dataset $\mathcal{D}$ provides explicit representation only of a clipping of reality, while information about reality 'in between' the sampled locations is contained *at best* implicitly.

For spatial modelling applications dealing with field-type phenomena, however, rarely are the sampled data $\mathcal{D}$ themselves of interest, rather characteristics of the real-world phenomenon being modelled (such as its state or its behaviour $z(\cdot)$) are the primary foci. Thus, implicit information derivable from the sampled data needs to be made available. That is, the actual field $z(\cdot)$ needs to be reconstructed from the observed data in order to make deducible whatever information the dataset $\mathcal{D}$ may contain. Section 2.2, outlining a theoretical basis for the mathematical modelling of continuous fields, proposed the notion of the equation of state as a mathematical formalisation of the task. This section intends to structure the work flow of modelling continuous fields in view of a respective implementation in an information system (IS) context.

Digital modelling of continuous fields aims at providing information on the state or behaviour of continuous fields under specific circumstances or at points not sampled in either space or time. In principle, three components are involved in this process[18]: data, information, and models. *Data* constitute the 'raw material', which may be sampled or provided (cf. sections 2.1.2 and 2.2). Data themselves are not very informative. As pointed out above, they usually refer to a small, discretised piece of reality, whereas the behaviour of a phenomenon 'in between' the sampled data, or the process causing such behaviour, remain unknown. *Information* is the product of analysis. Information *is* the statement about the real-world phenomenon strived for in the modelling attempt. Thus, information may be understood as data 'turned to good use' by its *interpretation* in the modelling process. Throughout this work, the terms *data* and *information* are used in the sense explained above, rather than synonymously as is sometimes done

[18]Which is sometimes called (continuous field) *analysis* (Bucher 1998)

in literature.

The role of *models* is more complex. In principle, digital representation of continuous fields is based on the nexus of data and models (Bucher 1998). Data represent the state of real-world phenomena at selected points sampled in either space or time. Models are simplified images of reality. Models typify the level of knowledge as well as the current ideas about the behaviour of the phenomenon under study. For the term model various definitions may be found. Accordingly, the contexts in which models may be used are diverse. Bucher (1998), based on Kemp (1993), notes that in spatial data handling the concept of models is used in two fairly different connections: connoting models of physical phenomena or processes[19] and spatial data models.

2.3.1 Models of Physical Phenomena or Processes

Models of physical phenomena or processes are representations of physical structures, processes, and functional relationships which shape, or at least influence, the behaviour of real-world phenomena. Thus, they have a *functional* connotation. Though the comments to follow are of general validity, for the sake of this discussion assume that the phenomena being modelled are of field-type. Notions and understanding of the modelled phenomena may vary considerably. Accordingly, Burrough (1997) distinguishes four types of models of physical phenomena or processes, three of which may be promisingly used to model continuous fields, depending on the notions and level of knowledge about the respective field behaviour $z(\cdot)$[20]:

(1) *Empirical (regression) models* (or *black-box models*, often also called *response function models* or *transfer models*; Bouma and Bregt (1989)), which are data driven and may be based on (multivariate) regression or similar statistical methods. Empirical models do not guarantee causality (e.g., many spurious correlations may be observed), but through long and widespread use they may achieve a broad acceptance;

(2) *deterministic (physical) models*, which attempt to explain the phenomena being modelled in terms of basic physical and chemical laws or

[19]Discussion in this section is confined to the characteristics of mathematical models of physical phenomena or processes although much of what is said can be equally applied to models of human activities.

[20]For the sake of completeness, the fourth type of models of physical phenomena or processes identified by Burrough (1997) are *rule-based models* (or *logical models*), which are founded on the basic axioms of logic and on straightforward set theory operations on discrete binary, ternary, or similar data. While rule-based models aim at explaining or predicting a specific state (e.g., nitrate pollution hazard) at a specific point in space (and time), they do not attempt to deal with spatial continuity in the sense of simulating continuous field behaviour.

determined mathematical relations; and

(3) *stochastic (physical) models*, which, as well, characterise phenomena in terms of physical or chemical driving forces or mathematical relations, but at least one of the model inputs is described by a probability distribution rather than an exact value.

Levels of Abstraction for Modelling physical phenomena or processes It is useful to consider the building of models of physical phenomena or processes as a sequence of steps at distinct stages of abstraction. The following paragraph is based on material merged from several sources including Bishr (1997), Bucher (1998), Bekey (1985) and Kemp (1997). From the real world to an actual model of physical phenomena or processes three levels of increasing abstraction may be identified (see figure 2.2, left branch):

(1) The *universe of discourse* (UoD), constituted by the set of real-world features of interest.

(2) The *reference model*[21], which is a purely conceptual representation of the UoD. The reference model may be understood as an intentional definition of the UoD, including three types of abstractions:

    (i) *Definition of categories*, where categories are collections of real-world features with similar characteristics.

    (ii) *Geometric description*, which is the process of assigning geometric types to the categories distinguished.

    (iii) *Establishment of relationships* between these categories. Relationships may be identified on a logical, functional, and/or topological level.

(3) The *mathematical model*, which provides formal expression of the phenomenon-characteristic variables and formulation of the relationships between them in mathematical terms. Mathematical formulation of the conceptual reference model is a prerequisite for its digital representation. Drawing up a mathematical model includes three steps:

    (i) *Definition of variables (classes* or also *properties)*, that is, definition of a finite set of variables (classes or properties) to describe the categories defined in the reference model. Involved in variables definition are determination of the *states* (cf. section 2.2)

[21]Sometimes also denoted as *abstract view of the universe* (David et al. 1996), *context world view* (Bishr 1997), or *terrain nominal* or "nominal ground" (Nyerges 1991).

the features represented by the variables may be in, and definition of respective *observables*[22].

(ii) *Geometry definitions*, whose end is to provide formalised geometric descriptions of the variables.

(iii) *Formalisation of relationships*, that is, mapping of the dependency relations established between the variables to mathematical expressions.

The mathematical model may be understood as an unparameterised (mathematical) description of the phenomenon being modelled. It is used to survey the UoD (figure 2.1). However, computer representation of models of physical phenomena or processes still requires encoding of the mathematical model in a formal computer language, and provision of model parametrisation by means of respective data. These last two steps, which will be shown to happen in close interaction with spatial data models, are discussed in section 2.3.3.

2.3.2 Spatial Data Models

Spatial concepts structure the human perception of geographical spaces[23]. However, the concepts used to understand geographical space may be based on notions which can not be implemented directly, either for lack of formal definition or for lack of discretisation (Frank 1992). Digital representation of space, therefore, requires their abstraction and formalisation, a premise provided by spatial data models. *Spatial data models* formalise the semantics of spatial concepts and, thus, supply a formalised representation of space and spatial properties. Hence, their connotation is clearly *descriptive*.

In the reminder of this section, just a short comment on data models for the representation of fields is made. A broad review of spatial data models is beyond the scope of this discussion. A general introduction to the topic may be found in Burrough and McDonnell (1998), for a review of data models for the representation of fields please refer to Goodchild (1992), Kemp (1993, 1997), Vckovski (1998). Goodchild (1992) distinguishes two types of data models for representation of fields: piecewise models and sampled models.

Piecewise Models *Piecewise models* conceptualise continuous space as dissected into contiguous regions. A field value is explicitly assigned every-

[22]That is, development of a measurement model, or, outlining of rules by which real-world features are identified and measured or associated with variables; see also chapter 2.2.

[23]According to Frank (1992), spatial concepts, or specifically *spatial concepts and geometry* denote "ideas, notions, and relations between them which are used by humans to organise and structure their perception of reality". They differ depending on the task at hand, the circumstances, and the experience of the persons.

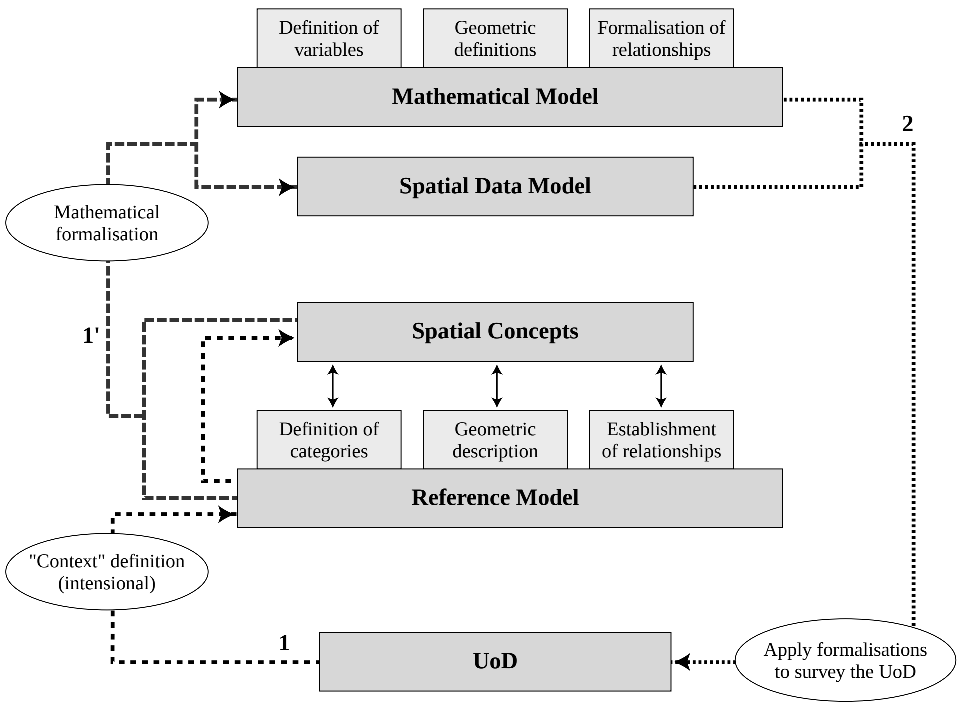


Figure 2.1: Levels of abstraction for modelling physical phenomena or processes. 1: Abstraction of real-world features (definition of categories, geometric description, establishment of relationships). 1': Formalisation of this abstraction. 2: Application of these formalised rules to survey the real world (modified after Bishr (1997)).

where in space by drawing up a simple function piecewise defined on each dissected region to estimate the character of the spatial variation. Usually, constant or linearly-varying functions are used. The crucial assumption in all piecewise models is that the value or function assigned to each region is representative of the average value or general trend of the field rather than an exact measurement for any specific location.

Sampled Models *Sampled models*, on the other hand, represent continuous space by sampling the field at selected locations while providing no information about the values in between these locations; values in between the sampled locations must be interpolated.

To sum up, piecewise models presume knowledge about the behaviour of the field's spatial variation and then seek an according domain partitioning, while sampled models attempt to specify a field function fitting the data sampled according to chosen discretisation criteria. However, note that both approaches to digital representation of continuous fields provide a specification of mathematical function(s) to simulate continuity as an integral component of the formalism.

2.3.3 Interaction between Models of Physical Phenomena or Processes and Spatial Data Models

Roughly speaking, the process of designing a digital representation of continuous fields starts with identifying the *UoD*, and *ideally* proceeds to outlining a respective *reference model*, as illustrated in figure 2.2. Essential to this end are *problem specification* and statement of the modelling *objectives*; both tasks entailing an intentional definition of the UoD and formulation of requirements upon the model and data[24]. Involved in this is a step for definition of categories (cf. section 2.3.1), which implies a specific concept of space, and leads to what was denoted as *spatial concept* in the last section. Formalisation of the conceptual reference model results in the *mathematical model*. Involved in this step are, roughly speaking, expression of phenomenon-characteristic variables and formulation of the established dependency relations in mathematical terms, both with respect to the objectives stated. These two operations basically imply how to deal with continuity, that is, how to discretise and subsequently simulate by mathematical means the continuity of the field being modelled. They thus confine formalisation of the spatial concept(s) implied by the reference model; and hence the resulting *spatial data model*. Aiming at digital representation of the phenomenon under study, the *functional* (mathematical model) and

[24]The requirements upon model and data establish the so called *quality norm* (Brassel et al. 1995), which finally allows valuation of the data quality (an issue discussed in more detail in chapters 6 and 7).

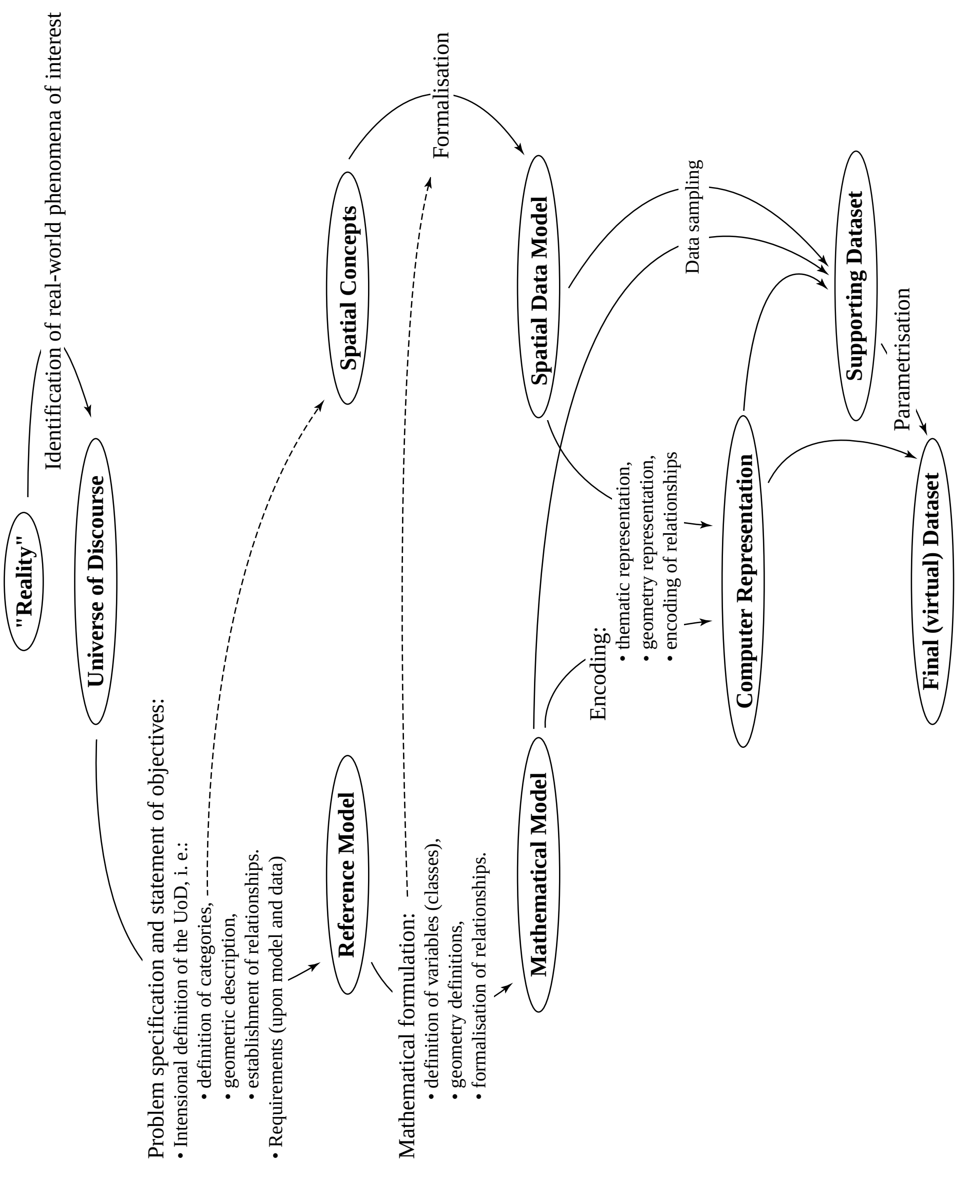


Figure 2.2: An ideal modelling sequence. The dashed lines illustrate modelling steps confining other steps in the modelling sequence.

descriptive (spatial data model) models developed so far yet have to be merged to form a *computer representation*; this calls for *encoding* of the mathematical model into a formal computer language with respect to the logical structures provided by the spatial data model. Mapped to a digital representation, the *variables* (and their geometry and relationships) defined in the mathematical - and spatial data model, will be referred to as *property types*. The computer representation, in this sense, may be understood as an unparameterised digital representation of the phenomenon being modelled. *Parametrisation* then is provided by respective data, ideally sampled subject to the rules implied by the mathematical model (see also figure 2.3).

The above 'ideal modelling sequence' may not always be *feasible*, however. Especially definition of a spatial data model and data sourceing (purely) as a function of the mathematical model for the phenomenon under study may turn out to be wishful thinking. Digital field representation then turns out to be a trade-off between modelling objectives and the data and resources available, as reflected in figure 2.4. In this case, the model of physical phenomena or processes developed is usually kept simpler, at the cost of expressiveness.

However defined, the parameterised digital representation of the field being investigated forms the basis for computation of *information* requested by the user. Models of physical phenomena or processes essentially contribute to this task by creating a nexus between the continuous behaviour of real-world fields and their discrete and finite computer representation. By exploiting mathematical relationships to simulate continuity, they put the discrete data in the continuous context required for reliable modelling of real-world structures and processes. Thus, they contribute to bridging the gap between continuous real-world field behaviour and its discrete and finite computer simulation at the expense of presuming the phenomenon investigated is suitable to algorithmic compression (cf. chapter 2.4).

Finally, note that explicit specification of both model of physical phenomena or processes and spatial data model at all levels of abstraction is a prerequisite to quality control and quality assurance throughout the modelling task, as will be discussed in more detail in chapters 5 and 6.

2.4 Algorithmic Compressibility

In science, pattern recognition and model building are formalised activities for discovering and describing patterns or processes that repeat in a way that is to some degree predictable (Burrough and Frank 1995). The patterns recognised and models built are exploited to condense vast arrays of observational data into compact formulae (or, at least, to code their information content more concisely). Any sequence of sampled observations

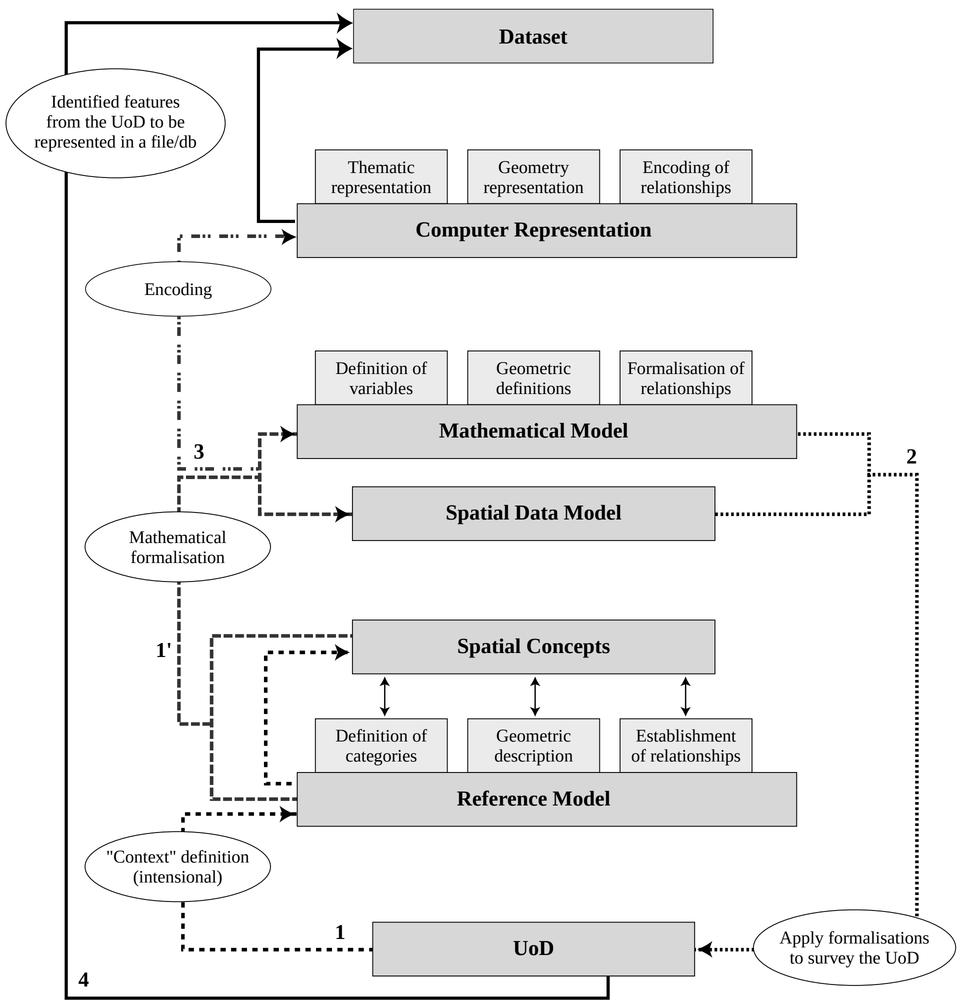


Figure 2.3: Abstraction and representation of field-type phenomena. 1: Abstraction of real-world features (definition of categories, geometric description, establishment of relationships). 1': Formalisation of this abstraction. 2: Application of these formalised rules to survey the real world. 3: Encoding. 4: Representation in a file/db according to the schema specified by the computer representation (modified after Bishr (1997)).

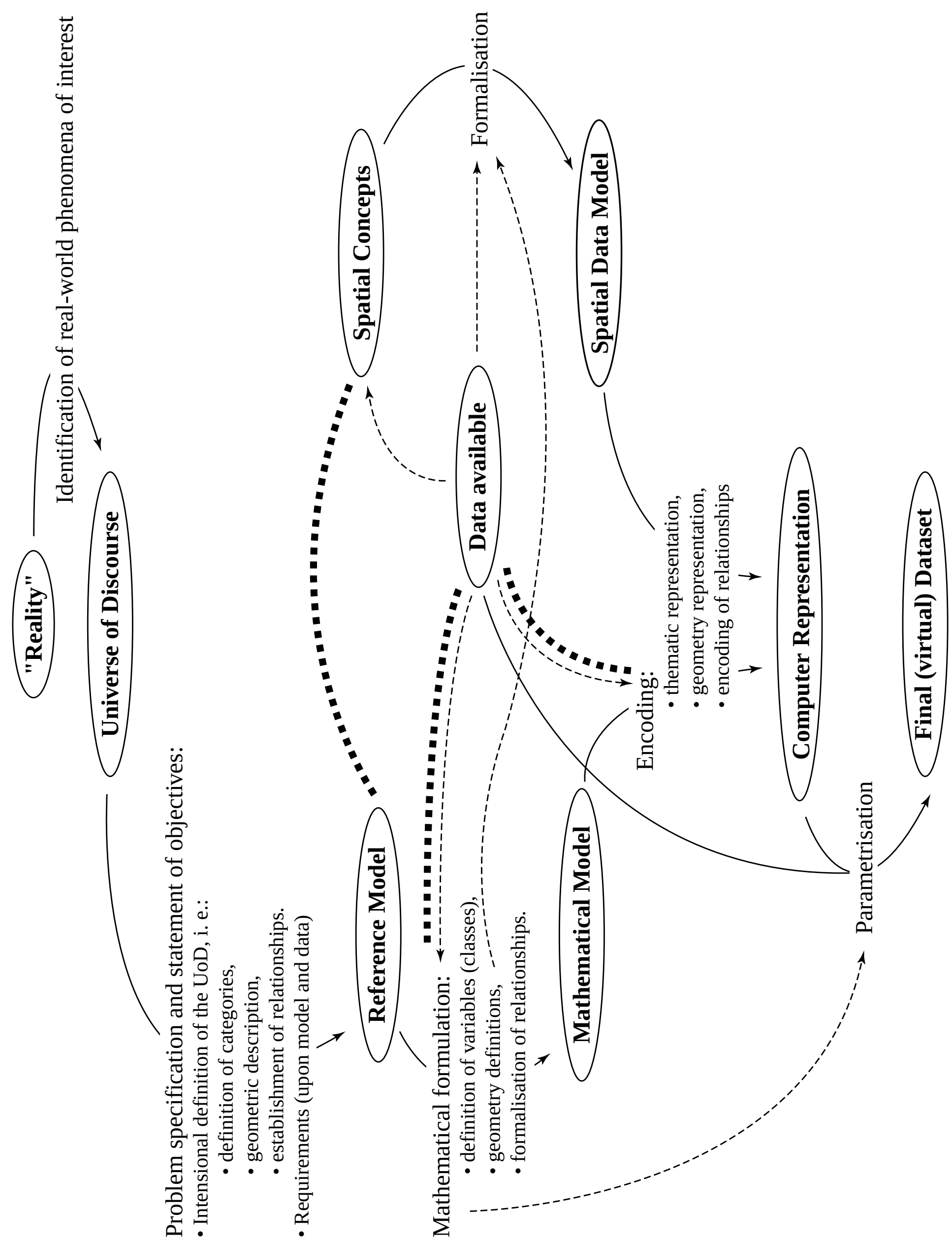


Figure 2.4: A possibly more realistic modelling sequence. The dashed lines illustrate modelling steps confining other steps in the modelling sequence. The bold dotted lines stand for potential conflicts between the available data and the modelling objectives.

that can be given an abbreviated representation is called *algorithmically compressible* (Barrow 1991).

In this sense, science is the search for algorithmic compressions (Barrow 1991). “Without the development of algorithmic compressions of data all science would be replaced by mindless stamp collection - the indiscriminate accumulation of every available fact” (Barrow 1991).

According to this view, digital terrain modelling is the search for algorithmic compression of topographic data; the search for an abbreviated representation of the logic behind terrain surface properties that can be written down in finite form. Raising quality issues in digital terrain modelling, then, includes an investigation of the suitability of topographic surfaces for algorithmic compression; an investigation of whether the topographic surface can be replaced by shorthand formulae which possess the same information content. If apparently no such lossless compression exists, investigated must be what may can be said about the information loss of the abbreviated representation achieved, and how this information loss may be qualified in view of potential applications of the algorithmically compressed terrain model.

2.5 Review

Digital representation of fields necessarily involves *discretisation* when sampling the field, and *reconstruction* of the field by interpolation or approximation. If the field behaviour is not known a-priori, the field can never be exhaustively surveyed, and its lossless reconstruction is impeded by the incompleteness of the available knowledge. Furthermore, there exist many ways in which a field may be discretised, and, given a set of sampled data, there remains an abundant number of continuous functions honouring the information provided. The task of digital field representation is, therefore, somehow underdetermined. Thus, it is necessary to incorporate additional information into the modelling process. Appropriate digital representations of continuous fields are based on the nexus of *data* and *models*; the models stipulate the rules of the modelling attempt, and the data provide parametrisation of the models.

Chapter 3

Quality Management in GI Science

While the last chapter dealt with the background of digital terrain modelling, this chapter focuses on the complex subject of quality handling. In order to manage quality in digital terrain modelling, it is first necessary to develop a framework for discourse about the factors affecting DTM quality. The term 'metainformation' serves to generally refer to the structure imposed on the context in which DTM quality can be appraised. The use of *metainformation* is implied by the definition of fitness for use. TI providers can only document the characteristics of their TI, and TI users can only determine the suitability of TI for a particular application, if there is some common arena of discourse between the two.

There is a variety of structures which have been proposed to describe the quality of GI in general and TI in particular. The aim of this chapter is to explore these various approaches from different perspectives. The following section investigates the role of standards organisations and the use of so-called quality standards. Uncertainty is likely to be a major factor influencing TI quality. Approaches to quality handling in digital terrain modelling, therefore, require a formal model of uncertainty. An understanding of different conceptions of uncertainty is the foundation of such a model. Section 3.2 thus explores the representation of TI uncertainty within the research literature. Finally, section 3.3 looks at an approach to modelling common in GI science which may prove to be useful in examining the factors affecting TI quality.

3.1 Data Quality Standards

Over recent years, a variety of government and private organisations have worked on the definition of data quality standards, mostly as part of more extensive spatial data transfer standards. On a supranational level, the moving power in Europe is the Comité Européen de Normalisation (CEN), whose Draft Standard for Geographic Information includes a data quality standard (CEN/TC287 1996). Resting upon the CEN work, a working group of the International Standards Organisation (ISO) is currently developing an international standard for spatial data quality (ISO/TC211 1999, Godwin 1999). Further remarkable efforts include the Digital Geographic Information Exchange Standard (DIGEST, Digital Geographic Information Working Group (1997)), which was designed by the Digital Geographic Information Working Group (DGIWG, in fact a NATO spin-off). The International Cartographic Association (ICA), as well, has been involved in attempts to standardise spatial data quality (Guptill and Morrison 1995). Data quality and metadata are, of course, topics of the OpenGIS specifications (Open GIS Consortium 1999a,b, Bühler and McKee 1998), where these issues are handled in close collaboration with ISO/TC211. On a national level, 22 separate national spatial data quality standards are identified by an authoritative study (Moellering and Hogan 1997), all of which mention the importance of spatial data quality, and almost all of which make significant attempts to standardise data quality. However, as Duckham (1999) remarks, the majority of these national data quality standards is closely related to the United States' Spatial Data Transfer Standard (SDTS, National Committee for Digital Cartographic Data Standards (1988)). The SDTS defines five elements of spatial data quality: *lineage, positional accuracy, attribute accuracy, logical consistency*, and *completeness*. These five spatial data quality elements have gained widespread acceptance not only amongst standards organisations, but also amongst GI researchers and professionals around the world (Morrison 1995). They have dominated the horizons of spatial data quality for a decade, earning them the affectionate appellation "the famous five" (Duckham 1999).

3.1.1 The SDTS "Famous Five"

At the core of the SDTS design lies the concept of *truth in labelling*. Like fitness for use, truth in labelling rejects the formulation of prescriptive (and arbitrary) quality thresholds in favour of the provision of detailed quality information (Duckham 1999). The five elements of spatial data quality defined to serve this purpose are:

- **Lineage** (Clarke and Clark 1995), which refers to the history of a dataset and describes the source material from which the data were derived, the methods of derivation, and the operations and transformations performed upon them. The SDTS, however, leaves the detailed structure of lineage largely unspecified.

- **Positional accuracy** (Drummond 58), which is a measure of the closeness of positional data to the 'true' or ideal value.

- **Attribute accuracy** (Goodchild 1995a), which is quality information on the thematic component of spatial data. The SDTS quality element, attribute accuracy, consists of two distinct components: *Categorical attribute accuracy*, which describes the accuracy of categorical data, and *continuous attribute accuracy*, which is associated with quantitative data.

- **Logical consistency** (Kainz 1995), which analyses whether the data are contradictory or inconsistent. This, generally, involves reporting the results of logical tests of validity run upon the dataset.

- **Completeness** (Brassel et al. 1995), which identifies the exhaustiveness of a dataset in comparison with its abstraction of reality, i.e. it checks whether all real-world phenomena that have a conceptualisation are represented.

When discussing data quality standards, it is difficult to understate the importance of the five SDTS elements of spatial data quality. Their extensive use in one or another form within many national and international standards organisations is an indication of their value. However, despite the success of the "famous five", it is arguable that neither SDTS nor any other standard forms a suitable basis for a GI science approach to GI quality. A rationale for this view is given in the following sections.

3.1.2 Exhaustiveness and Expressiveness

Even a cursory review of the literature suggests that the *exhaustiveness* of the five SDTS elements of spatial data quality is disputed. The ICA Commission on Spatial Data Quality, for instance, accepts the SDTS data quality elements, yet feels the need to augment these with *semantic accuracy* and *temporal accuracy* (Guptill and Morrison 1995). Similarly, several of the national data transfer standards described in Moellering and Hogan (1997), whilst usually supporting at least some of the core SDTS quality elements in one form or another, still propose a number of additional quality descriptors. For example, Duckham (1999) mentions "map information level", which is included as an essential element of spatial data quality in the Japanese Standard Procedure and Data Format for Digital Mapping (SPDFDM). "Reliability" and "cartographic identifiability" are considered indispensable for comprehensive quality documentation in the Netherland's standard (NEN1878). The use of "textual fidelity" as an additional descriptor of spatial data quality suggested by CEN/TC287 (1996).

Further, Duckham (1999) refers to Drummond (1996) who notes the lack of a firm agreement on the definitions of a number of elements of spatial data quality in standards and more generally in the literature. Duckham (1999) proceeds to argue that standards are by their nature both prescriptive and proscriptive, leading to an inflexibility in standards limiting their *expressive* range. Therefore, he suggests that no single data quality standard is likely to be *exhaustive* or *expressive* enough to suit every user's needs. Hence, no standard classification of uncertainty and/or other factors affecting GI quality will ever be universally accepted (Hunter 1996).

3.1.3 Responsibility

Fitness for use emphasises the *responsibilities* of users and providers of GI. As set out in section 1.3, the production domain has a responsibility to provide detailed and appropriate information on quality along with data and information. Given that no spatial data quality standard can be exhaustive or expressive enough for every eventuality, it is arguable that compliance with such a standard represents the most appropriate quality information. Ultimately, this comment reflects the persuasion that GI providers are more likely to understand the peculiarities of their data than standards organisations. Enabling the production domain to express this understanding more freely is considered a deterministic factor for the success of GI quality handling (Duckham 1999).

3.1.4 'Systemness'

Data quality standards, as their name implies, attempt to standardise reporting of the quality of spatial *data* only. However, as pointed out in section 2.2, digital representation of continuous fields comprise more than the discrete field representation supplied by the sampled data; mathematical functions to simulate continuity are an integral component of the formalism. Likewise, reporting of the quality of digital field representations must provide *descriptors for both* the *data discretising* the phenomenon being modelled and the *functions reconstructing* it. It is worth placing this point in the context of algorithmic compressibility (cf. section 2.4). Clearly, the compression algorithm is an integral part of the abbreviated, finite form representation of the logic behind the things sought for in science. Therefore, investigation of the quality of the compressed representation necessarily must include critical appraisal of the compression algorithm[1]. However, today's quality standards enable documentation of the discretising data only,

[1]Transferred to digital terrain modelling, in Li's (1993a) words this means that factors affecting DTM quality comprise both factors affecting the quality of the raw data and factors affecting DTM quality introduced through the processes of sampling and reconstruction (in which case Li speaks of "accuracy loss").

while the factors affecting TI quality introduced through sampling and reconstruction remain unconsidered.

3.2 Error Models [2]

There is a considerable volume of research concerned with DTM uncertainty. A striking characteristic of this body of work is its diversity and the lack of integration between the different research threads. Due to the complex nature of uncertainty in TI, however, the current disaggregation is not surprising. Nevertheless, uncertainty handling in digital terrain modelling depends upon the development of a coherent conceptual framework for uncertainty. This development may substantially benefit from an understanding of the existing error models. This section, therefore, aims at reviewing research concerned with uncertainty in TI by highlighting the different representations of uncertainty used.

3.2.1 Uncertainty in the Sampled Data

Confidence Level One of the simplest error models for description of elevation accuracy specifies an uncertainty value, such that the true or theoretical value falls within ± that uncertainty value a certain percentage of the time. This is equivalent to reporting vertical accuracy at a certain *confidence level*. The Vertical Map Accuracy Standard (VMAS), which is part of the United States National Map Accuracy Standard (NMAS, U. S. Bureau of the Budget (1947)), for instance, requires vertical (map) accuracy to be reported at a 90% confidence level, where the reported accuracy value shall not be greater than one half the contour interval (CI), at all contour intervals. Similarly, the National Standard for Spatial Data Accuracy (NSSDA, Federal Geographic Data Committee (1998b)) stipulates vertical accuracy to be reported in ground distances at a 95% confidence level; the reported accuracy value is proposed to be computed depending on the dataset's RMSE (root mean squared error; cf. discussion in the next paragraph). Confidence levels are simple and efficient models of error. However, they fall short of a comprehensive and sophisticated error model for a number of reasons.

Measures of accuracy including confidence levels, always raise the vexed question of what the true value is (cf. section 1.2). Adding to the fundamental comment that no observation or measurement can be made to an

[2] As argued in section 1.2, the term 'uncertainty' is preferred to the term 'error'. However, to be consistent with usage in statistics, 'error' is used in this section, in the context of 'error model'. Yet, 'error' may, here, be understood in the broader sense of uncertainty rather than in the narrow sense of mistake.

arbitrary level of precision[3], in the case of TI there is the complication that TI cannot be defined independent of *scale*. The terrain surface displays many characteristics of a fractal surface. Although not a true fractal in the mathematical sense, the terrain surface does possess the property of always manifesting detail at a scale larger[4] than that sampled. This suggests that DTMs implicitly model at a certain scale (implied by the resolution of the sampled data). Consequently, first, what is really being tested when reporting DTM accuracy at a certain confidence level is how well DTM and derived TI manage to represent the reference data used to reflect the true ground positions, and not how well the true but unknown terrain is represented. Second, there is no indication of the scale dependence of the reported accuracy.

Empirical evidence suggests that DTM errors can be positively spatially autocorrelated (Guth 1992, Monckton 1994). Confidence levels, however, provide a single global description of accuracy, completely ignoring any spatial structure of the error.

As pointed out in section 3.1.4, TI quality reports must provide descriptors for both the data discretising the terrain surface and the methods involved in its reconstruction. Clearly, confidence levels fall short to meet this demand as they usually provide description of only the discretising data. The approach lacks a surface-based extension of the point-based concept of confidence levels (such as, for the 1-d case, the Perkal- or ε-bands; Perkal (1966), Blakemore (1984)).

Root Mean Squared Error (RMSE) Generally, more sophisticated error models are based on conventional descriptions of frequency distributions that include measures of central tendency and dispersion. The *standard deviation* (SD) and the *root mean squared error* (RMSE) can be used to communicate the precision and accuracy of a point location respectively (Drummond 58). In the context of reporting DTM accuracy, probably the most widely used measure is the RMSE (see, for instance, USGS - United States Geological Survey (1998), Ordnance Survey (1993)), whose computation is as follows:

$$\mathrm{RMSE} := \sqrt{\frac{\sum_{i=1}^{n}(z_{DTM}(\boldsymbol{x}_i) - z_{Ground}(\boldsymbol{x}_i))^2}{n}},$$

where $z_{DTM}(\boldsymbol{x}_i)$ are the DTM elevations at sampled locations $\boldsymbol{x}_i$, $z_{Ground}(\boldsymbol{x}_i)$ are the vertical coordinates of the corresponding points in the reference dataset, and n is the number of points being checked.

[3] It is not important for this discussion whether the limits of precision are inherent and therefore never can be overcome, or if these limits are of a more practical nature.

[4] Note, that in this thesis the term ‘scale’ is used in a cartographic sense, that is, large scales referring to more detailed rendering.

Although such a model is highly robust and allows the production of derived measures of uncertainty[5], a number of problems remain with the measure and the way in which it is often derived.

The RMSE does not involve any description of the mean deviation between the DTM elevations and the elevation values assumed to be true[6]. The common interpretation of the RMSE entails an assumption of zero mean deviation, which is presumptive and not always valid (Li 1993b,a, Monckton 1994)[7]. A zero mean also implies *stationarity*, which is neither an always desirable nor an always justified property of a general error model (Heuvelink 1998, Cressie 1993).

Most of the reservations voiced against confidence levels are raised with equal validity against the RMSE. Again, the pattern of deviation between two datasets is described, one of which usually is the dataset being tested and the other the one assumed to represent ground truth - and not the pattern of deviation between the data set being tested and the real terrain. Also, there is no indication of the dependence of the RMSE value on the spatial scale of sampling.

Finally, the RMSE is a vertex-based measure; it accounts for the data discretising the terrain surface only. The approach lacks a clear theoretical relation between variability in elevation at point locations and elevation variability over entire surface patches.

3.2.2 Uncertainty Propagation

Taylor Series Based Error Propagation *Analytical error propagation* methods such as *Taylor series based variance propagation* were proposed to calculate the uncertainty associated with derived TI (Schneider 2000, Florinsky 1998). Taylor series based approaches can be applied if the derived TI is a differentiable function of the given terrain data (Heuvelink 1998, Vckovski 1998), if the errors of the sampled data can be expressed with mean

[5]The NSSDA (Federal Geographic Data Committee 1998b), for instance, stipulates reporting vertical accuracy at a 95% confidence level computed based on Greenwalt and Schultz (1968), who suggest derivation of the reported accuracy value according to:

$$\text{Accuracy}_z := 1.9600 \cdot \text{RMSE}.$$

Assumed herewith are zero mean deviations and that the vertical error is normally distributed.

[6]The most common use of the RMSE is to provide a single global measure of deviation, involving no indication of spatial variation over the DTM. Inevitably, a single measure of dispersion does not describe the form of the deviation's frequency distribution. Since the variance of deviations in elevation is likely to be attributable to a set of processes leading to variation in the mean, a single measure may hide important information (Wood 1996a).

[7]An important impact of the mean not being zero is that the SD's and the RMSE no longer coincide.

and SD, and if the mean of all errors is zero. Besides offering the opportunity to avoid comparison with some value assumed to be true, a second important strength of the method is its ability to cope with (auto)correlated errors, provided the errors of the given data are known[8]. If the correlation values ρ_{ij} between each pair of errors σ_i and σ_j are known, the estimator of the propagated uncertainty can be expressed as follows (Heuvelink 1998):

$$\sigma := \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}(\rho_{ij}\,\frac{\partial f}{\partial \boldsymbol{x}_i}\sigma_i\,\frac{\partial f}{\partial \boldsymbol{x}_j}\sigma_j)}, \tag{3.1}$$

where σ is the SD of the propagated error, f is the function $f(\boldsymbol{x}_1, ..., \boldsymbol{x}_n)$ deriving the desired TI, and $\boldsymbol{x}_1, ..., \boldsymbol{x}_n$ are the data affected by errors expressed as SD's $\sigma_1, ..., \sigma_n$[9]. A further benefit of the approach is the quite detailed, locally adapted description yielded by the predicted error, specifying its distribution by means of the SD.

However, the Taylor series based variance propagation has limitations. Major restrictions are the mentioned assumption of zero mean error in the data (which is not always the case as stressed in the last section), and the method's applicability limited to derived TI which can be expressed as a differentiable function of the given terrain data.

It should be stressed that Taylor series based approaches, unlike the error models discussed in the previous section, are not concerned with evaluation of the uncertainty occurring in the given data; rather, they model the propagation of these uncertainties through TI derivation. Examined are the fundamental errors accrued from the algorithms applied rather than the uncertainty associated with how well these methods model the real topography. Florinsky (1998) and Schneider (2000) both used first order Taylor methods to analyse different algorithms for the delineation of simple TI (such as slope angle, slope aspect, or curvature)[10]. The formulae for the SD of the propagated error in the derived TI thus produced, without exception, suggest the counter-intuitive conclusion that the resulting SD is inverse proportional to the resolution of sampled terrain data.

In cases where the application of Taylor series based methods for uncertainty propagation is impeded by their restrictive assumptions and premises,

[8] Known, for instance, from estimation during measurement.

[9] If the errors $\sigma_1, ..., \sigma_n$ are spatially independent (that is, if $\rho_{ij} = 0, \ \forall i, j$), formula (3.1) simplifies to:

$$\sigma = \sqrt{\sum_{i=1}^{n}((\frac{\partial f}{\partial \boldsymbol{x}_i})^2\,\sigma_i^2)},$$

an expression known as Gaussian error propagation (Vckovski 1998).

[10] TI derivation from both discrete, raster-based DTMs (by finite differences methods) and continuously specified surfaces is investigated.

interval arithmetic (Schneider 2000, Hugentobler 2000) or stochastic simulation methods (Ehlschlager and Shortridge 1996, Hunter and Goodchild 1997, Fisher 1998a) were proposed as viable alternatives. While *interval mathematics* are simple and runtime efficient, the expressiveness of their error estimation is rather scanty. *Stochastic simulation*, on the other hand, can yield the entire distribution of the probable outcome of TI derivation at an arbitrary level of accuracy, though at the expense of heavy computing load and considerable storage requirements.

Interval Arithmetic The basic idea behind *interval mathematics* (Moore 1966, Bauch et al. 1987, Mayer 1989) is to represent real-valued data as closed connected subsets of the real line, for instance, as *intervals* in $\mathbb{R}$ or $\mathbb{R}^n$. These intervals represent all possible values of a quantity. Owing to the well established calculus of intervals, interval arithmetic can be used for uncertainty propagation by replacing all quantities within computation of any TI with intervals yielding intervals as results, and, therefore, some estimate of the resulting uncertainty (Vckovski 1998, Schneider 2000, Hugentobler 2000). The main advantage of interval mathematics is its simplicity. Interval arithmetic is a suitable and intuitive tool for predicting maximum error in all common DTM analysis tasks, as it does not make demands on the data, such as requesting the errors to be normally distributed or to have zero mean. As well, the functions propagating the uncertainties do not have to be continuously differentiable with respect to the variables introducing uncertainty, and the method is also applicable if the extraction of TI is based on geometric constructions rather than on functional expressions (Schneider 2000). However, there is no measure associated with the intervals, so only an approximation to the maximum error can be calculated. Details about distribution and probability are not given. Furthermore, the approach ignores any structure of spatial dependence between the errors.

Stochastic Simulation The basic idea of *stochastic simulation*[11] is to think of the uncertain variables as random variables (or random functions). The information to be computed is then calculated using a set of realisations $\{x_i^{(1)}, ..., x_i^{(m)}\}$ of each random variable X_i. For m large enough, the empirical distribution of the information computed will approach its true distribution.

In the case of digital terrain modelling, the sampled data are assumed to be the sum of a true value and an error term. The error term is conceived as a random variable of a known probability distribution function (p.d.f.). In the most simple approach, a random error field is generated from the

[11] The best known stochastic simulation approach is likely to be the Monte Carlo method (Hammersley and Handscomb 1983, Johnson 1987, Lewis and Orav 1989).

p.d.f. and added to the elevations. Naturally, any number of versions of the error field can be generated. Hence, multiple realisations of the sampled data are provided, each incorporating a slightly different version of the error field but the same mean and SD. However, as already mentioned briefly, empirical evidence suggests that the error term is positively spatially autocorrelated (Guth 1992, Monckton 1994). Therefore, still based on the assumed p.d.f., the error term is usually modelled as a *spatially correlated random field* (Ehlschlager and Shortridge 1996, Hunter and Goodchild 1997, Fisher 1998a). Finally, specific TI is derived from each sample realisation. By studying the distribution of the results, conclusions can be drawn about how the uncertainty in the sampled data affects the result of the derivation of this particular TI from this particular data set[12]. In short, stochastic simulation allows locally focused estimation of the impact of the uncertainty occurring in the sampled data, accounting for the structure of spatial dependence of the errors.

Yet, a number of problems persist:

- The p.d.f. of the occurring error must be known, and, if spatial dependencies of the error term are to be considered, information about them must be available. If such knowledge is lacking, as is often the case, makeshift assumptions are usually made about the error's p.d.f. and spatial autocorrelation structure. The error is often assumed to be normally distributed with zero mean, and the (auto)correlation function is often modelled as realisation of an autoregressive process (see, for instance, Hunter and Goodchild (1997)). Another solution is to estimate p.d.f. and spatial dependence structure from comparison with reference data (Ehlschlager and Shortridge 1996). This approach may yield a more realistic error model, however at the expense of the ground truth problem.

- Both, mean and SD of the error term are modelled as spatially stationary[13], which is an unrealistic assumption. Fisher (1998a) and Kyriakidis et al. (1999) therefore suggest the acquisition of improved error estimates by using more accurate reference data - again however, at the expense of the vexed ground truth problem - to derive statistical summaries, and to make use of *geostatistics* to examine the spatial distribution of the errors (e.g., by computing a *variogram* of the errors). Also, reference data are sometimes used to condition the stochastic simulation process[14] (Fisher 1998a, Kyriakidis et al. 1999).

[12] This approach is known as *unconditional stochastic simulation* (Ehlschlager and Shortridge 1996).

[13] Furthermore, often there is the unjustified assumption that the mean error is zero, or unbiased (for further discussion of this topic please refer to the RMSE discussion).

[14] In which case one speaks of *conditional stochastic simulation* (Deutsch and Journel

- Finally, the results of stochastic simulations do not come in a convenient analytical form. The resulting equations of Taylor series based error propagation methods, for instance, may be analysed to see how a reduction of the input error will impact the output. With stochastic simulation methods, the only solution is to run the entire simulation again (Heuvelink 1998).

3.2.3 Uncertainty in the Reconstructed Surface

Models Based on (Auto)covariance and the Variogram Another family of approaches to TI uncertainty modelling leans from *geostatistics*. Kubik and Botman (1976) proposed calculation of the variances of *interpolation errors* based on the *(auto)covariance* values of sampled data heights[15]. Based on the assumption that the terrain surface is homogeneous and isotropic (i.e., that the covariance function is independent of the absolute positions and orientation of the lag vector) they derived simple mathematical expressions for mean and maximal variances and SD's of interpolation errors as functions of sampling density and correlation distance. Expressions were derived for various interpolation techniques for both regularly spaced error-free and error-prone data. The general form yielded for a *covariance based accuracy model* is:

$$\sigma^2_{DTM}(\boldsymbol{x}) = \sigma^2_{int}(\boldsymbol{x}) + K(\boldsymbol{x}) \cdot \sigma^2_{raw}, \tag{3.2}$$

where $\sigma^2_{DTM}(\boldsymbol{x})$ is the variance of the resulting DTM elevation at location $\boldsymbol{x}$, $\sigma^2_{int}(\boldsymbol{x})$ denotes the proportion of this variance due to sampling and interpolation, $K(\boldsymbol{x})$ is a scalar-valued function and σ^2_{raw} is the proportion of the variance due to measuring error.

The approach is very valuable for providing simple mathematical expressions[16] which allow *separation* and *quantification* of the proportions of the *different error sources* (sampling, measurement, and reconstruction of a continuous surface). Estimates are given for DTM error variances considering the technique used for reconstruction and the spatial structure of the errors occurring in the sampled data[17].

1992, Journel 1996, Goovaerts 1997).

[15]Actually, the covariance values were approximated by either the exponential or Gaussian covariance models.

[16]Although the simplicity of the expressions should not be overestimated as it mainly results from the 'simplifying' premise of the sampled data being regularly and equally spaced in all dimensions.

[17]However, the dependence of $\sigma^2_{DTM}(\cdot)$ on the location $\boldsymbol{x}$ suggested by relation (3.2) has to be qualified insofar as the resulting variance value depends only on the location of $\boldsymbol{x}$ within the surface mesh. The pattern of local error variance variability is therefore reproduced over the entire model.

The major limitation of the approach lies in its demanding premise of terrain homogeneity (i.e. wide sense stationarity[18] of the terrain surface) and isotropy. It is questionable whether this assumption is indeed verifiable for real terrains. In any case, this assumption finally means that one has to average statistics over the whole model and, therefore, to define any spatial dependence of errors independently of their location $\boldsymbol{x} \in \mathbb{D}$ as a function of length of the separation vector $\boldsymbol{h}$ only.

In a manner very similar to the one just described, Frederiksen et al. (1986) used the *variogram* as a terrain descriptor. Variogram-based formulation of the accuracy model (3.2) is in principle preferable to the model's formulation in terms of covariances, because the variogram has a larger degree of generality than the covariance function. In particular, the variogram makes less limiting demands on the data, requiring just *intrinsic stationarity*[19] rather than the stronger *wide sense* or *second order stationarity* premised by the covariance function. Yet, the variogram, like the covariance, averages statistics and thus inherently renders a (globally) homogeneous pattern of spatial dependence of the errors occurring. With the variogram-based formulation of equation (3.2), therefore, no substantial progress is made beyond relaxing the demands upon the data.

Variograms are sometimes also used for *kriging*-based interpolation techniques (Šiška et al. 1997). Kriging-based predictions of the error variances for computed TI are more sophisticated than the results obtained with relation (3.2), or with its variogram based pendant, because kriging yields variance estimates which are truly local, that is, which are truly dependent on the locations $\boldsymbol{x}$ for which they are computed. However, the reservations voiced against the concepts of variogram and covariance, particularly, against the underlying stationarity assumptions remain valid. Finally, the question whether terrain is a phenomenon meeting the premises of a *probabilistic interpolation* tool such as kriging, is disputed.

[18] A random function $Z(\boldsymbol{x})$ is called *wide sense* or *second order stationary*, if its first and second order moments exist and are invariant under translation:

$$\begin{aligned} \mathrm{E}[Z(\boldsymbol{x})] &= m, \ \forall \boldsymbol{x} \\ \mathrm{Var}[Z(\boldsymbol{x})] &= \sigma^2, \ \forall \boldsymbol{x} \\ \mathrm{Cov}[Z(\boldsymbol{x}), Z(\boldsymbol{x}+\boldsymbol{h})] &= C(\boldsymbol{h}), \ \forall \boldsymbol{x}, \boldsymbol{h}. \end{aligned}$$

[19] $Z(\boldsymbol{x})$ is called an *intrinsic random function* (IRF) if, for every vector $\boldsymbol{h}$, the increment $Y_{\boldsymbol{h}}(\boldsymbol{x}) = Z(\boldsymbol{x}+\boldsymbol{h}) - Z(\boldsymbol{x})$ is wide sense stationary in $\boldsymbol{x}$. An IRF is characterised by the following relationships:

$$\begin{aligned} \mathrm{E}[Z(\boldsymbol{x}+\boldsymbol{h}) - Z(\boldsymbol{x})] &= m(\boldsymbol{h}), \ \forall \boldsymbol{x}, \boldsymbol{h} \\ \mathrm{Var}[Z(\boldsymbol{x}+\boldsymbol{h}) - Z(\boldsymbol{x})] &= 2\gamma(\boldsymbol{h}), \ \forall \boldsymbol{x}, \boldsymbol{h} \end{aligned}$$

where $m(\boldsymbol{h})$ is the linear drift of the IRF and $2\gamma(\boldsymbol{h})$ is its variogram function.

Models Based on Fourier Analysis Many methods for DTM reconstruction from sampled data, such as polynomial interpolation approaches including spline interpolation or finite element methods, constitute linear systems (Tempfli 1982). The accuracy of such linear systems can be estimated by *spectral analysis*[20]. Makarovič (1972) established a quantitative relationship between the sampling density of a regularly spaced point grid and the fidelity of the surface reconstructed from the sampled data. He defined *fidelity* as the ratio between the amplitude spectrum of the reconstructed terrain function to the amplitude spectrum of the input data for each spatial frequency. Thus, fidelity can be represented for the whole spectral band of interest by the system's *transfer function*. Makarovič (1974) then tried to convert the fidelity figures into SD values. In a very similar way, Tempfli (1980) related the variance σ^2_{DTM} of the resulting DTM error to the amplitude spectrum of the sampled and measured data and the system's transfer function. He facilitated derivation of an estimator for the error variance σ^2_{DTM} by a conceived separation into *sampling* as an impulse modulation of the real terrain $z(\cdot)$, and subsequent measuring:

$$z(\cdot) \overset{sampling}{\rightarrow} z_{sam}(\cdot) \overset{measuring}{\rightarrow} \tilde{z}(\cdot) \overset{reconstruction}{\rightarrow} \tilde{z}_{rec}(\cdot).$$

Sampling can be formulated as:

$$z_{sam}(x_1, x_2) = \sum_{l=0}^{n_1} \sum_{m=0}^{n_2} z(l\Delta x_1, m\Delta x_2)\, \delta(x_1 - l\Delta x_1)\, \delta(x_2 - \Delta x_2), \qquad (3.3)$$

where $\delta(\cdot)$ is the Dirac impulse function, Δx_1 and Δx_2 are the sampling intervals along the dimensions x_1 and x_2, and $(n_1\Delta x_1 \times n_2\Delta x_2)$ is the extent of the terrain to be modelled (that is, $(n_1\Delta x_1 \times n_2\Delta x_2) =: \mathbb{D}$, where $\mathbb{D}$ denotes the modelling domain). Subject to reconstruction, i.e., to interpolation or approximation, then are the discrete terrain data: $\tilde{z}(\boldsymbol{x}) = z_{sam}(\boldsymbol{x}) + m_{sam}(\boldsymbol{x})$, where $m_{sam}(\boldsymbol{x})$ is the measuring error at the sampled locations $\boldsymbol{x}$.

Based on the assumption that the measuring errors were purely random, Tempfli defined $m_{sam}(\cdot)$ as a sequence of uncorrelated values, which are normally distributed with zero mean and variance σ^2_M[21]. In the case of sampling at a rate *higher than the Nyquist rate*[22], σ^2_{DTM} then results as:

$$\sigma^2_{DTM} = \sigma^2_{rec} + \sigma^2_m, \qquad (3.4)$$

[20] Provided the terrain surface is understood as function $z(\cdot)$ assigning each location $\boldsymbol{x} := (x_1, x_2)$ of some planar 2-dimensional domain $\mathbb{D}$ a unique elevation value $z(\boldsymbol{x})$.

[21] As repeatedly discussed, both the assumptions of uncorrelated errors and zero mean deviations are in contradiction to empirical evidence.

[22] That is, if sampling intervals $\Delta x_1, \Delta x_2$ along x_1 and x_2 are chosen such that:

$$\hat{z}(\xi_1, \xi_2) = 0,\ \forall\, |\xi_1| \geq \xi_{1_n},\ \forall\, |\xi_2| \geq \xi_{2_n},$$

where $\xi_{1_n} = \frac{1}{2\Delta x_1}$, $\xi_{2_n} = \frac{1}{2\Delta x_2}$ and $\hat{z}(\xi_1, \xi_2)$ is the Fourier transform of $z(\cdot, \cdot)$.

where σ^2_{rec} is the variance of reconstructing the terrain from the sampled data by interpolation or approximation and σ^2_m denotes the influence of the measuring error. The mathematical expressions for σ^2_{rec} and σ^2_m result to be:

$$\begin{aligned}
\sigma^2_{rec} &= 2\int_0^{\xi_{1n}}\int_0^{\xi_{2n}} (\Delta x_1 \Delta x_2 - \hat{a}(\xi_1,\xi_2))^2 \, |\hat{z}_{sam}(\xi_1,\xi_2)|^2 \, d\xi_1 \, d\xi_2 \\
&\quad + 2\int_{\xi_{1n}}^{\infty}\int_{\xi_{2n}}^{\infty} \hat{a}^2(\xi_1,\xi_2) \, |\hat{z}_{sam}(\xi_1,\xi_2)|^2 \, d\xi_1 \, d\xi_2, \qquad (3.5) \\
\sigma^2_m &= 2\int_0^{\infty}\int_0^{\infty} \hat{a}^2(\xi_1,\xi_2) \, |\hat{m}_{sam}(\xi_1,\xi_2)|^2 \, d\xi_1 \, d\xi_2, \qquad (3.6)
\end{aligned}$$

where $\hat{a}(\cdot)$ is the system's transfer function, $\hat{z}_{sam}(\cdot)$ is the Fourier transform of $z_{sam}(\cdot)$, and $\hat{m}_{sam}(\cdot)$ is the Fourier transform of the measuring error $m_{sam}(\cdot)$.

Remarkable about relations (3.5) and (3.6) is that by treating wavelength as equivalent to scale, they can be interpreted as explicitly modelling the *scale dependence* of the resulting error variances[23]. Theoretical separation into sampling as an impulse modulation of the real terrain $z(\cdot)$ and its subsequent distortion by measuring, allows for an estimator of the error variance σ^2_{DTM} itself consisting of separate terms which account for the *different error sources*. Practical evaluation of equations (3.5) and (3.6) (that is, practical quantification of the variance proportions), however, requires knowledge about either ground truth or measuring error. Relations (3.5) and (3.6) are, therefore, of mainly theoretical relevance.

The major limitation of the approach, however, is the premise of sampling at a rate higher than the Nyquist rate. Since functions can never be both band- and space limited, and practically the portions of terrain sampled are always of finite extent, sampling a real terrain $z(\cdot)$ theoretically always causes a deformation of the spectrum of $z(\cdot)$, an effect known as 'aliasing'. Unfortunately, there is no research known to the author investigating the effects of *aliasing* due to sampling of terrain data. However, in the presence of aliasing, σ^2_{DTM} depends not only on the amplitude spectrum of $z(\cdot)$, but also on its phase spectrum. Tempfli (1980) only states that the effect of aliasing on σ^2_{DTM} depends on the relationship between the phase angles of the components with frequencies $(\boldsymbol{\xi}_n + \Delta\boldsymbol{\xi})$ and on those with frequencies $(\boldsymbol{\xi}_n - \Delta\boldsymbol{\xi})$, without suggesting convenient mathematical expressions.

Frederiksen (1980) approached the problem by suggesting an error model on the basis of the summation of Fourier spectra computed from the recon-

[23] Though, as inherent to Fourier methods, the good localisation of the error variances in frequency domain goes at the expense of their localisation in space domain. Relations (3.5) and (3.6) thus render a global measure of the amounting error variance, but provide no indication of possible spatial variation over the entire DTM surface.

structed terrain model for wavelengths $\boldsymbol{\xi} := (\xi_1, \xi_2)$, where $\xi_1 > \frac{1}{2\Delta x_1}$ and $\xi_2 > \frac{1}{2\Delta x_2}$ and $\Delta x_1,\ \Delta x_2$ are the sampling intervals along the dimensions x_1 and x_2. This model, however, ignores the fact that the magnitude of the spectra is not only a function of the accuracy of the interpolated data but also of the spacing between the test data points used to compute the spectra. Therefore, this model may well produce too optimistic DTM accuracy predictions (Li 1993b).

General Accuracy Model Based on Slope as Terrain Descriptor Based on Ackermann (1980), Li (1993b) suggested that a general model of DTM accuracy should take the following form:

$$\sigma_{DTM}^2 = k_1 \sigma_{raw}^2 + k_2[\boldsymbol{c} \cdot (\Delta x_1, \Delta x_2)^T]^2, \tag{3.7}$$

where σ_{DTM}^2 is the error variance of the final DTM, σ_{raw}^2 denotes the error variance of the raw data, $\boldsymbol{c}$ is a parameter depending on the characteristics of the terrain surface, $\Delta x_1, \Delta x_2$ are the sampling intervals along the dimensions x_1 and x_2, and k_1, k_2 are constants depending on the interpolation method used.

The problem remains of what kind of function should be used to represent $\boldsymbol{c}$ in equation (3.7). From experimental tests (Ackermann 1980), Li (1993b) deduced that in general $\boldsymbol{c}$ is a function of the mean *slope* value, the mean *wavelength* and the *sampling interval*. He then concluded that a combination of slope and wavelength may be recommended as the main terrain descriptors for DTM purposes, admitting at the same time that the adequacy of such descriptors depends primarily on the availability of accurate slope values for acceptable terrain description. Li himself proposed a parametrisation of his model based on empirical tests, but the general validity of his results has never been investigated or proven.

3.2.4 Taxonomic Approaches

In contrast to the formal error models discussed above, a number of *taxonomic approaches* to GI quality have been proposed. Burrough (1986) suggested an error classification based on the source of the error[24]. Group I errors then comprise "obvious error sources" such as (map) scale and temporal factors. Group II errors result from the natural variation within the phenomena being measured, and Group III errors occur as a result of data processing. Burrough's approach has been influential in a number of later studies. Veregin (1989) presented a hierarchy of needs for treatment of error whose three basic levels reflect Burrough's taxonomy. Clearly related

[24]Following Burrough's original terminology, the term 'error' is used here to denote any kind of factor affecting GI quality.

to Burrough (1986) as well, Maffini et al. (1989) identified three sources of error: the inherent properties of nature, the nature of measurement, and the data models used. In a subtly different manner, Goodchild (1989) distinguished between "source errors" and "processing errors", the former relating to error resulting from deviations between ideal and observations, the latter relating to errors introduced through processing of the data.

While the same themes often recur, there is seemingly no limit to the number of equally plausible taxonomies of factors affecting GI quality. Burrough himself dropped the three group taxonomy in favour of a more capacious seven point description of factors affecting GI quality comprised of currency, completeness, consistency, accessibility, accuracy and precision, sources of error in data and sources of error in derived analyses (Burrough and McDonnell 1998). Goodchild (1995b) identifies factors affecting GI quality resulting from measurement and definition, lack of documentation, interpretation, processing errors and physical distortions. While many of the different error taxonomies may even not claim to be comprehensive, systematic, fundamental, or generic, they are all to some degree useful representations of factors affecting GI quality.

3.2.5 The Role of Error Models

This section explored the large quantity of contrasting models of uncertainty used in GI science and digital terrain modelling. An overview on the discussed models is provided in table 3.1. They range from using a single measure of uncertainty that applies to an entire DTM, to locally focused measures that apply to individual locations, rendering spatial variation of the occurring uncertainties. Some approaches use clearly defined statistical measures (such as the RMSE), whilst others suggest relatively vague, unstructured indices (such as Burrough's three error groups). Certain models make rather limiting demands on the data (such as requiring the errors to be normally distributed with zero mean), many require ground truth knowledge, and most do not contain any indication of the dependency of their error measures on the spatial scale of sampling. Many approaches are clearly discrete in nature, meaning they consider the sampled terrain data only (such as confidence levels); only a few (such as models based on Fourier analysis) provide estimates for the error variances at arbitrary locations by separating variance proportions due to sampling and reconstruction from influences due to measuring errors. However, no single model stands out as the most appropriate choice to represent the entire spectrum of uncertainty. The decision of which error model is the most appropriate for a given DTM involves weighing up in terms of a specific application's requirements the relative merits of simple approaches with more sophisticated ones, of generic approaches with specific ones, of clearly defined approaches with vague ones.

	Prerequisites	*Required a-priori information*	*Ground truthing*	*Consideration of scale dependencies*	*Global or local measure*	*Consideration of spatial error structure*	*Evaluation of sampled data*	*Evaluation of reconstructed data*	*Separation of error sources*
Confidence level	-	-	+	-	Global measure	-	+	-	-
RMSE	Zero mean deviation	-	+	-	Global measure	-	+	-	-
Taylor series	Differentiability; error normally distributed	Mean and SD of error	- / (+)	Depends on data resolution	Local mean and SD estimates	+	(input)	+	-
Interval arithmetic	-	Data intervals	- / (+)	?	Local estimate of max. error	-	(input)	+	-
Stochastic simulation	-	p.d.f. of error term	-/ +	?	Local outcome distribution	+	(input)	+	-
Covariance and variogram models	Homogeneity and isotropy of the terrain surface; stationarity	Variances of sampled data	- / (+)	Depends on sampling resolution	Local estimates of mean and maximal variance	Within a surface mesh	(input)	+	+
Fourier analysis	Sampling at a rate higher than the Nyquist rate	Measuring error	- / +	Depends on data resolution	Local variance estimates	potentially	(input)	+	+
Slope as terrain descriptor	-	Variances of sampled data; slope	- / (+)	Considers wavelength	Local variance estimates	?	(input)	+	- / +

Table 3.1: An overview of the discussed error models.

3.3 A Conceptual Model of Information Systems

In the previous sections several approaches to dealing with GI quality were reviewed, mainly under two broad headings: data quality standards (commonly embedded in spatial data transfer standards), and error models. However, since neither one can claim to be fundamental or generic (Duckham 1999), both data quality standards and error models have been rejected as suitable frameworks for development of approaches to quality handling in GI science in general, and in digital terrain modelling in particular. Consequently, a more general approach is needed, able to support any error model without being tied to a particular data quality standard or research thread.

Such an approach may be found in what is denoted the "*conceptual model of Information Systems*"(Duckham 1999), illustrated in figure 3.1, which is fundamentally a generalisation of the work flow for modelling fields presented in section 2.3. The basic assumption is that the real world is infinitely complex and to some extent never knowable. As discussed in section 2.3, handling the real world by digital means, roughly speaking, comprises two operations: *abstraction* of the perceived reality to a *functional* model of the phenomenon being investigated; and its digital *representation* achieved by mapping the functional model to formal computer schemata and language[25]. Information on a real-world phenomenon, being contained either explicitly or implicitly in the actual dataset, thus always results from a representation of an abstraction of reality, not from a representation of reality itself.

This information flow works well when discussing digital modelling of physical geospatial phenomena, but needs extension when dealing with GI quality. In contrast to geospatial features, no meaningful concept of GI quality exists in the real world. Hence, a key concept is that GI quality is not usefully modelled by the process of abstraction and representation, rather it is but a *side effect of deficiencies in these processes* that quality issues arise at all (David et al. 1996, Duckham 1999, 2000).

The modelling process outlined in section 2.3 bridges the gap between reality and information with realising that information is compiled in order to fulfil the requirements for a useful abstraction of reality. This understanding relates directly to the conceptual model of Information Systems (IS) illustrated in figure 3.1. Hence, it is argued here that the conceptual model of IS, which models physical geospatial phenomena with the processes of abstraction and representation, and which ascribes GI quality to deficiencies

[25]Note that such a conceptual model of Information Systems is implicitly realist in epistemology, as it necessarily assumes the existence of a reality made out of phenomena being both perceivable and observable (Duckham 1999). Questions like whether reality exists without perception are not dealt with, as it is argued that it is always a perception of reality that is handled, and that therefore it is not necessary to differentiate.

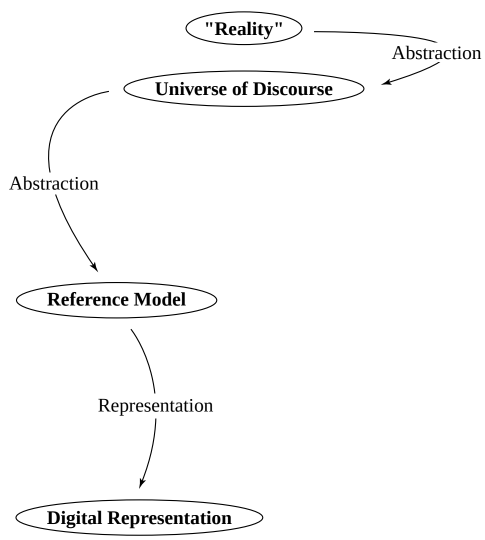


Figure 3.1: Conceptual model of Information Systems.

in these processes, provides a fundamental framework for discourse about the factors affecting GI quality (Duckham 1999). Particularly the model explicitly accounts for such factors being a product of both deficiencies in *representing* an abstract view of reality and deficiencies in *producing* such a useful abstraction.

3.4 Review

The challenge to quality handling in digital terrain modelling is clear. There is a rich diversity of approaches to describe GI and TI quality, embedded within both standards organisations and current research. While standards and error models provide an important resource base for TI providers and TI users, there is evidence that methods for quality handling in digital terrain modelling should be flexible enough to allow users to adapt or define quality measures for their own purposes.

The framework for approaching quality handling in digital terrain modelling is also clear. None of the quality standards and error models discussed has the expressive power or flexibility to represent all the others. Therefore, any approach to quality handling tied to a particular quality standard or error model is unlikely to be able to respond to the full range of user's requirements. The conceptual model of IS presented in section 3.3 is the only approach that may claim to be fundamental to metainformation generally, as it arguably encompasses the more specific data quality standards and the too narrowly focused error models. Consequently, it is a suitable candidate for use as a basis for quality handling in digital terrain modelling.

Chapter 4

Reliable Digital Terrain Modelling

The term *Digital Terrain Model* (DTM) is used generically to denote a surface model for digital terrain representation which provides an approximation of a topographic surface based on a finite number of samples and which features specific model properties. Terrain data are acquired either through sampling (on-site measurements or remote sensing technologies) or through digitisation of existing contour maps (for a review of terrain data acquisition methods, see Weibel and Heller (1991), Schneider (1998), Baltsavias (1999), Hutchinson and Gallant (2000), Meier et al. (2000)). Raw data come in the form of elevations at a set of either regularly or irregularly distributed points in the surface domain. Sequences of points may form lines (representing either linear features such as break lines, ridges or ravines, or contours) or simple surfaces (representing areal features such as water bodies).

As highlighted in section 1.5, *digital terrain modelling* exposes a fundamental dichotomy: terrain model generation on the one hand, and its application in spatial modelling applications on the other. For spatial modelling applications, rarely are the terrain elevations themselves of interest, rather derived TI such as slope, aspect, or topographic structures, are the primary foci. Hence, in the context of spatial modelling, the TI of interest may, to a great extent, not be directly accessible, rather it needs to be made available through specific extraction methods. Therefore, to embrace both producer- and user-oriented perspectives, the term digital terrain modelling is used in this thesis to denote not only terrain reconstruction in the sense of terrain model generation, but includes also supplication of any TI of interest (derived by respective extraction methods) and its use in spatial modelling applications.

In this chapter, the concept of a DTM is elaborated and formalised.

In section 4.1 some preliminary *notations* and *definitions* are introduced. Section 4.2 discusses the major *impediments* complicating practical reliable digital terrain reconstruction. Based on the considerations set forth, a *DTM* is defined in section 4.3 as digital terrain representation driven by *rules* directly determining its properties. As already pointed out, reliable digital terrain modelling challenges both providers and users of TI. Also, fitness for use, which was considered best suited to provide an overall paradigm to TI quality management (section 1.3), establishes the responsibilities of the two. In section 4.3, therefore, the implications of the proposed approach to *digital terrain modelling* for producers and users of TI are highlighted by interpreting digital terrain modelling as a *workflow process* of DTM generation and DTM application. The chapter ends with consideration of questions such as how different representations of the same terrain may be compared and when two representations of the same terrain may be said to be *equivalent*.

4.1 Digital Terrain Models

4.1.1 Notations and Definitions

According to Vckovski (1998), the notion of a field function $z(\cdot)$, together with its domain $\mathbb{D}$ and its range $\mathbb{V}$, provides an essential model[1] for digital representation of fields (section 2.1). With respect to digital terrain modelling[2], a *topographic surface* (or terrain) may be described by the triple:

$$T = (z(\cdot), \mathbb{D}, \mathbb{V}),$$

where $z(\cdot)$ is a bivariate function (with suitable continuity properties) modelling the topographic surface. $z(\cdot)$ is defined over a compact and connected domain $\mathbb{D}$ in the Euclidean space, and its range $\mathbb{V}$ most often is a subset of $\mathbb{R}$. T is called a *mathematical terrain model*[3] (De Floriani et al. 1994, De Floriani and Puppo 1992).

Because the shape of the terrain surface is a priori unknown, in practical applications the actual elevation is usually sampled for a finite set S, $S := \{\boldsymbol{s}_1, ..., \boldsymbol{s}_m\} \subset \mathbb{D}$, of points in the surface domain. S is called the *set of representative points* (De Floriani et al. 1996). Additionally, further terrain properties characterising the topographic surface other than the mere

[1] According to the object-oriented approach by Cook and Daniels (1994), the term "essential model" denotes model design at the highest level of abstraction.

[2] Consideration of the topographic surface as a continuous field is supported by the arguments set forth on page 11.

[3] The mathematical description of the topographic surface implied by the mathematical terrain model is (cf. equation (2.1)):

$$(id, z(\cdot)) : \mathbb{D} \longrightarrow \mathbb{D} \times \mathbb{V}, \quad \boldsymbol{s} \mapsto (\boldsymbol{s}, z(\boldsymbol{s})).$$

elevations may be sampled. The 'domain' of this additional information, then, is either S (in the case of point features, such as peaks or pits) or K, where $K := \{k_0, ..., k_n\} \subset \mathbb{D}$ is a finite set of simple or closed lines[4] having their endpoints in S and which, for convenience, shall be pieced of non-crossing line segments. The elements of K represent either *linear features* (e.g., break lines, ridges, ravines, contours, etc.; cf. figure 4.3, top right) or *areal features* (e.g., water bodies). Through *sampling*, the surface domain $\mathbb{D}$ is thus reduced to the pair of *finite sets* (S, K). Discussion of DTM quality issues is facilitated by separation of *sampling* into domain *discretisation* and subsequent *surveying* of the data. A *discretised terrain model* T_D then is a tuple:

$$T_D := (z(\cdot)|_S, TI_{(S,K)}, (S, K), \mathbb{V}, \mathbb{V}_{TI}),$$

where $TI_{(S,K)}$ denotes the selected properties characterising the terrain surface other than the mere elevations $z(\cdot)|_S$ of the representative points, and $\mathbb{V}_{TI}$ stands for the 'range' of these properties[5]. $TI_{(S,K)}$ is introduced to compensate (as well as possible) for the information omitted by reducing $\mathbb{D}$ to (S, K). T_D may be understood as an essential model for terrain survey. Measuring of $z(\cdot)|_S$ and apposite survey of $TI_{(S,K)}$, then, supplies what is usually understood as the *terrain dataset* $\mathcal{D}_T$:

$$\mathcal{D}_T := \{\{\boldsymbol{s}_i; \tilde{z}(\boldsymbol{s}_i)\}_{i=1}^{m},\ \{t_j; \tilde{\chi}(t_j)\}_{j=1}^{n' \leq (m+n)}\}, \quad \boldsymbol{s}_i \in \tilde{S},\ t_j \in \tilde{S} \cup \tilde{K},$$

where $\tilde{z}(\boldsymbol{s}_i)$ denotes the measured elevation value at location $\boldsymbol{s}_i$, $\tilde{S}$ is the set of points at which measurements are made, $\tilde{\chi}(t_j)$ denotes the piece of TI assigned to $t_j \in \tilde{S} \cup \tilde{K}$, and $\tilde{K}$ is the actual realisation of K[6]. $\mathcal{D}_T$ will sometimes also be denoted as the *'supporting data(set)'* (cf. figure 4.3, top row, where three possible supporting datasets $\mathcal{D}_T$ for a given site are illustrated).

4.1.2 Reliable Terrain Modelling

Reliable DTM application means that the TI derived must be *replicable* and *consistent* (section 1.4.2). If no topographic surface is explicitly specified from $\mathcal{D}_T$, methods to derive different types of TI may be based on heterogeneous interpretations of the supporting data, that is, on different implicit representations of the same terrain. For instance, methods for the

[4]Lines in the sense of curves.

[5]In the top right picture of figure 4.3, for instance, $TI_{(S,K)}$ consists of flow lines and ridges.

[6]The subtle distinction between S and its realisation $\tilde{S}$ is emphasised to explicitly indicate that measuring uncertainty encompasses not only measurement uncertainty, but also uncertainty about the exact measurement location. The same discussion holds, of course, for K and $\tilde{K}$.

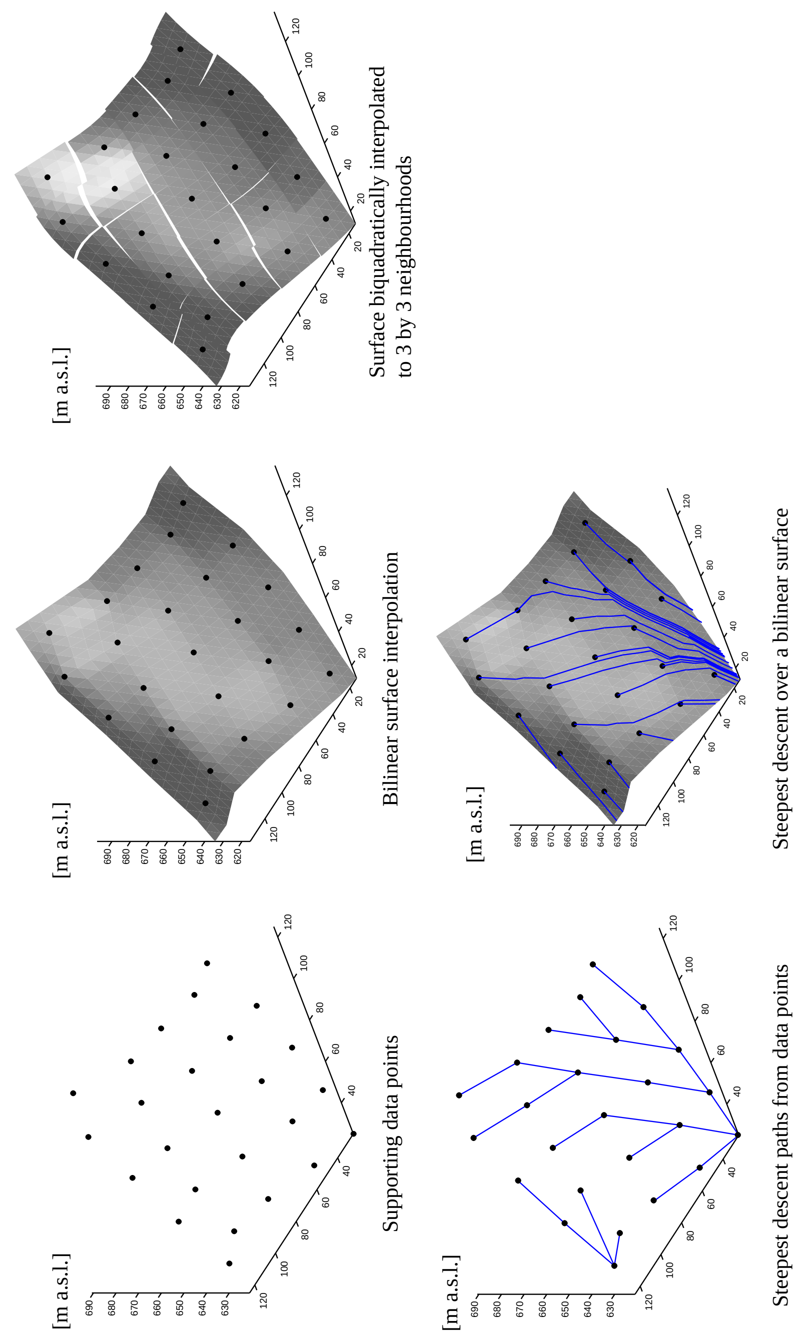


Figure 4.1: Different interpretations of the supporting data. The steepest descents are computed by choosing the direction of steepest descent from the eight connections to the neighbouring points in a 3 by 3 neighbourhood (left picture); and derived following the negative gradient direction of the bilinearly (right picture) interpolated surface (DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

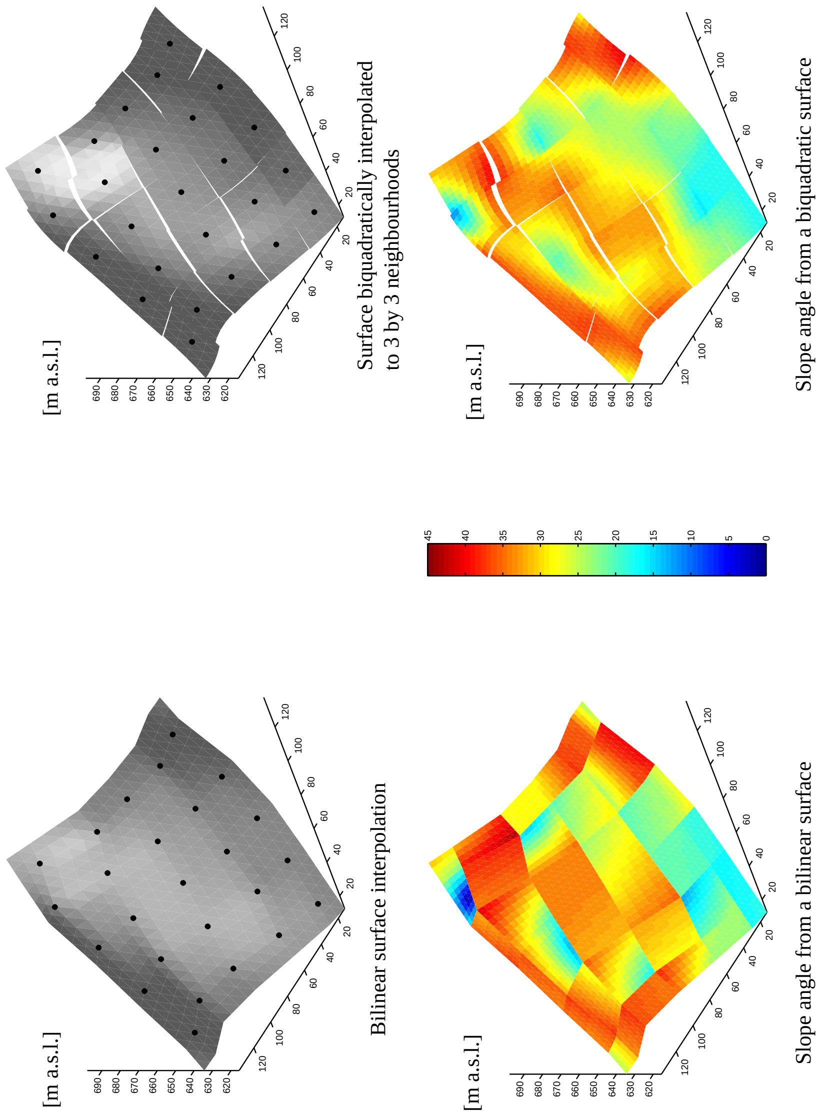


Figure 4.2: Different interpretations of the supporting data (slope angles expressed in degrees; DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

derivation of elevation values often interpret the data as pixels or as bilinearly interpolated surfaces. In contrast, other TI like slope, aspect or curvature is often derived with the help of local polynomial surfaces fitted to a 3 by 3 raster point neighbourhood. Paths of steepest descent as well as many hydrological parameters, finally are most often extracted from the data points only, without specification of a surface. (Figures 4.1 and 4.2 illustrate this problem: The terrain displayed is supported by 5 by 5 regularly spaced data points, whose elevation ranges from 621.00 to 690.70 m a. s. l. Exemplified is TI extracted from the supporting data only, without specification of a surface; TI derived from a bilinearly interpolated surface; and TI derived over a surface biquadratically interpolated to 3 by 3 neighbourhoods.) Therefore, consistency between different types of TI can not be guaranteed, unless the TI is derived from an explicitly pre-specified terrain reconstruction. Consequently, it presents a challenge to *reliable digital terrain modelling* to provide an *explicit* and *suitable reconstruction of the topographic surface*, that is, to supply a well-defined and replicable estimation $\hat{z}(\cdot)$ of the actual terrain behaviour based on the information provided by $\mathcal{D}_T$ (Wood (1998b), although not using the term 'reliable', stated that "the transformation of discrete gridded values to some form of continuous model is a necessary prerequisite to many GIS processing and visualisation procedures").

4.1.3 Digital Surface Reconstruction

A digital terrain reconstruction is defined as a piecewise approximation to the mathematical terrain model, obtained through a surface reconstruction $\hat{z}(\cdot)$ defined over a subdivision Σ of $\mathbb{D}$ in a piecewise fashion (De Floriani and Puppo 1992). Hence, a *digital terrain reconstruction* T_R is a triple:

$$T_R := (\hat{z}(\cdot), \Sigma, \mathbb{V}), \tag{4.1}$$

where $\Sigma := \mathcal{S}(\mathcal{D}_T)$ is a planar *subdivision* that partitions the domain $\mathbb{D}$ into regions $\{R_1, ..., R_p\}$. $\hat{z}(\cdot)$ is a *piecewise approximation* to the mathematical terrain description $z(\cdot)$ set up by a family of *continuous functions* $\hat{z}_i(\cdot)$, each defined on a region R_i of Σ, and $\mathbb{V}$ is the *range* of the $\hat{z}_i(\cdot)$[7]. A planar subdivision $\Sigma := \mathcal{S}(\mathcal{D}_T)$ of $\mathbb{D}$, is a planar line graph (V, E) constraint by

[7]That is, $\hat{z} := (\hat{z}_1, ..., \hat{z}_p)$, $\hat{z}_i : R_i \longrightarrow \mathbb{V}$, $(i = 1, ..., p)$, where $\hat{z}_i \in \mathcal{C}^k(R_i; \mathbb{V})$. $\mathcal{C}^k(R_i; \mathbb{V})$ is the set of all functions mapping from R_i to $\mathbb{V}$, which are continuous in the interior of R_i and whose partial derivatives up to order k exist and are continuous in the interior of R_i as well (where $0 \leq k \leq \infty$). The mathematical approximation to the topographic surface implied by T_R, then, is (cf. footnote [3]):

$$(id, \hat{z}(\cdot)) : \mathbb{D} \longrightarrow \mathbb{D} \times \mathbb{V}, \quad \boldsymbol{s} \mapsto (\boldsymbol{s}, \hat{z}(\boldsymbol{s})),$$

$\mathcal{D}_T$[8], having V as a set of vertices and whose edges (in E) are non-crossing line segments with endpoints in V (Preparata and Shamos 1985). $R := \{R_1, ..., R_p\}$ is the set of *polygonal regions* (or faces) induced by the graph defined by (V, E). It follows from this understanding of Σ that $\bigcup_{i=1}^{p} R_i = \mathbb{D}$, and that any intersection between two or more regions R_i is a null set in $\mathbb{D}$[9] (meaning that, if $\mathbb{D}$ relates to a 2-dimensional plane, any two or more regions R_i may meet, if at all, in a common vertex and/or edge, respectively; see figure 4.3, middle and bottom rows). Note that the sets V and E of subdivision vertices and edges, respectively, may or may not correspond to $(\tilde{S}, \tilde{K})$[10] (figure 4.3). It is worth stressing that definition (4.1) does not impose any conditions on the individual $\hat{z}_i(\cdot)$ - making up the reconstruction family $\hat{z}(\cdot)$ - except to be locally continuous on R_i. Particularly, no global continuity properties are imposed on $\hat{z}(\cdot)$.

The definition for a digital terrain reconstruction given in (4.1) is general enough to accommodate both local and global surface reconstructions. Whilst *local models* are piecewise defined on a partition of the domain into patches, *global terrain representations* are defined through a single function honouring all data. The latter case may be thought of as partition of the domain into one patch. Subdivision of the domain according to definition (4.1), then, corresponds to establishing the boundary of $\mathbb{D}$. Furthermore, the definition is general enough to comprise both *interpolating* and *approximating models*. As already mentioned, no global continuity properties are imposed whatsoever, thus allowing for piecewise continuous terrain representations (including stepped models; see figure 4.4, second row)[11].

---

where:

$$\hat{z}(\boldsymbol{s}) = \begin{cases} \hat{z}_1(\boldsymbol{s}), & \boldsymbol{s} \in R_1 \\ ... & \\ \hat{z}_p(\boldsymbol{s}), & \boldsymbol{s} \in R_p. \end{cases}$$

[8]That is, induced by the elements of $(\tilde{S}, \tilde{K})$, and possibly also depending on the data values $\tilde{z}(\boldsymbol{s}_i)$, $(i = 1, ..., n)$, and $\tilde{\chi}(t_j)$, $(j = 1, ..., n' \leq (m + n))$.

[9]A set $A \in \mathbb{R}^n$ is called a *(Lesbegue) null set*, if $\forall \varepsilon > 0$ $\exists$ countable set of open cubes $I \in \mathbb{R}^n$ such that

$$A \in \bigcup_k I_k, \quad \sum_k \mu(I_k) < \varepsilon,$$

where μ is the Lesbegue outer measure on $\mathbb{R}^n$.

[10]Let $\tilde{S}$, for instance, be a set of scattered points. If Σ corresponds to a triangulation of $\tilde{S}$ (that is, if $\Sigma = \mathcal{S}(\tilde{S})$ and the domain partition depends on $\tilde{S}$ alone), the resulting set of vertices exactly matches $\tilde{S}$ (figure 4.3, second row middle picture). If, on the other hand, a Voronoi diagram for the points of $\tilde{S}$ is computed, a new set of vertices V is generated for which there is no corresponding counterpart in $\tilde{S}$ (figure 4.3, bottom). Additionally, a set $\tilde{K}$ of line segments representing, for instance, break lines, may be given. Let Σ be a constrained triangulation, honouring the line segments recorded in $\tilde{K}$ (that is, $\Sigma = \mathcal{S}(\tilde{S}, \tilde{K})$). Then, the set E of edges obtained is a proper superset of $\tilde{K}$ (figure 4.3, second row left picture).

[11]Extension of $\hat{z}$ to a family of distributions would even allow for a 'continuous form' representation of discrete surfaces by $\hat{z}(\boldsymbol{s}) = \sum_{i=1}^{n} \tilde{z}(\boldsymbol{s}_i)\delta(s_1 - s_{i_1})\delta(s_2 - s_{i_2})$, where

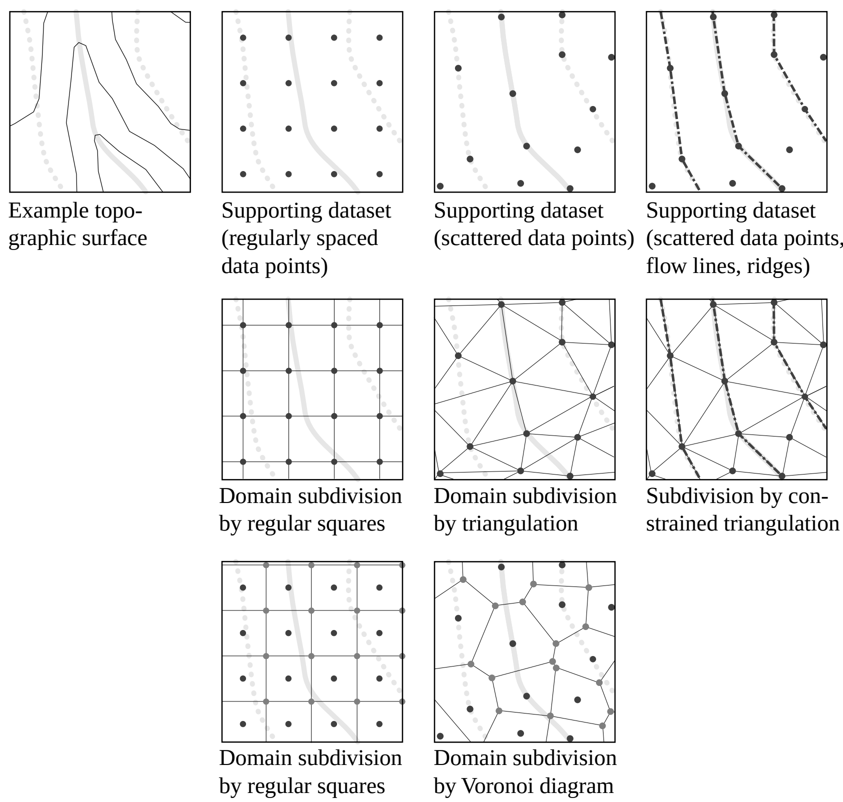


Figure 4.3: Subdivision of the domain $\mathbb{D}$. The top left picture depicts a topographic surface with a flow line (light gray line) and ridges (light gray dotted lines). Black dots are sampled data points (i.e., elements of $\tilde{S}$), black dashed lines denote further TI sampled (i.e., elements of $\tilde{K}$). Gray dots stand for subdivision vertices (i.e., elements of V) and the solid lines represent the edges of the subdivision. Top: Supporting datasets $\mathcal{D}_T$; middle: V exactly matches $\tilde{S}$. Additionally, in the middle right picture, E is a superset of $\tilde{K}$; bottom: V does not correspond to $\tilde{S}$.

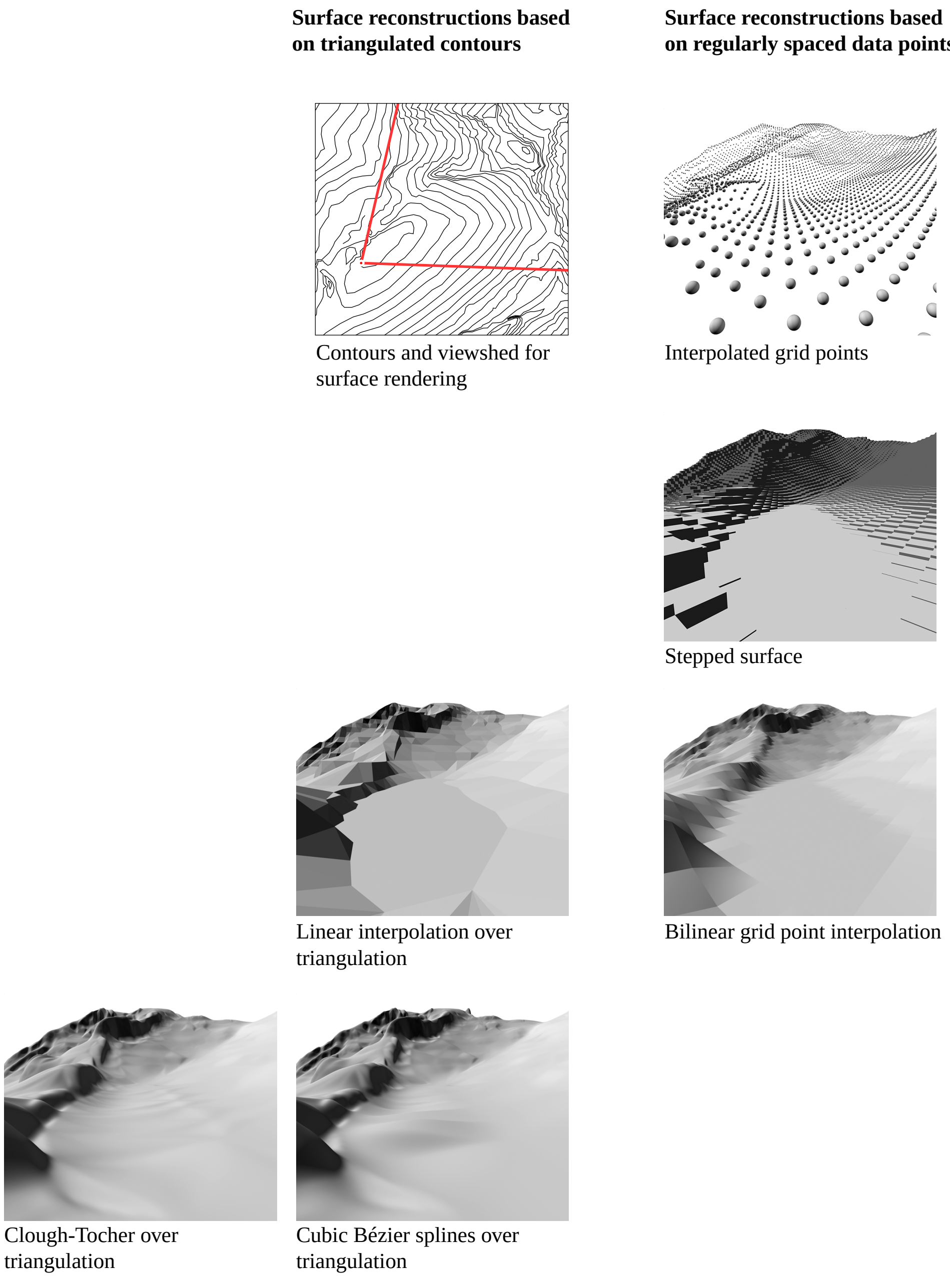


Figure 4.4: Various terrain reconstructions (interpolation and rendering by B. Schneider; DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

Two kinds of terrain representations common to GIS satisfy definition (4.1): grid elevation models and terrain representations based on polyhedral tesselations. *Grid elevation models* are defined by a domain partition into regular polygons. The most commonly used grid model is the *Regular Square Grid* (RSG), in which all regions are squares. For the functions $\hat{z}_i(\cdot)$ usually low order polynomials such as bilinear or bicubic polynomials are chosen (see figure 4.4, right hand side)[12]. With RSGs, smooth surface patches may be computed for each region of the subdivision, in order to achieve C^1- (or higher order) continuity of the resulting surface (see, for instance, Blaszcynski (1997), Mitasova et al. (1995)).

Terrain models built from a polyhedral tessellation are specified on the basis of a domain partition into polygonal regions induced by $\mathcal{D}_T$. More precisely, the subdivision is induced by the set $\tilde{S}$ of points at which the terrain is sampled, and possibly constrained by $\tilde{K}$ (in that segments of $\tilde{K}$ may appear as borders imposed on the subdivision patches, as illustrated in figure 4.3 in the middle right picture)[13]. The most common representatives of this kind of terrain model are *Triangulated Irregular Networks* (TINs), in which all regions are triangles. For the functions $\hat{z}_i(\cdot)$, again, polynomials or splines of a low order are usually chosen (see figure 4.4, left hand side). Also, smooth surface patches may be computed for each region of the subdivision, in order to achieve C^1- (or higher order) continuity of the resulting surface (see, for instance, Watson (1992), Sambridge et al. (1995), Schneider (1998))[14].

A third kind of terrain data common in the context of GIS are (digital) contours. Given a value $q \in \mathbb{V}$, the set $C_T(q) := \{\boldsymbol{s} \in \mathbb{D} \mid z(\boldsymbol{s}) = q\}$ is the set of *contours* of T at height q. Given a sequence $Q := \{q_0, ..., q_h \mid q_i \in \mathbb{V},\ i = 0, ..., h\}$, a set of *digital contours* of T is an approximation of the collection of contours $C_{T,Q} = \{C_T(q_i),\ i = 0, ..., h\}$. Digital contours are often represented as point sequences. A line interpolating the points of a contour may be obtained in different ways, ranging from the simple case of linear interpolation to spline curves of various orders. In the context

$\boldsymbol{s}_i = (s_{i_1}, s_{i_2}) \in \tilde{S}$ denote the sampled data.

[12] In some cases, a RSG is treated as a raster, and a constant function is used over each region. Clearly, such a stepped model is non-continuous across the edges of the grid (see figure 4.4, second row, left picture).

[13] Additionally, the subdivision may also depend on the data values.

[14] Considerable attention has also been directed towards methods for constructing a triangulation. The Delaunay triangulation is the probably most popular method and several efficient algorithms have been devised, for instance, in Guibas and Stolfi (1985), Heller (1990), de Berg et al. (1997). Triangulation methods have been seen as attractive because they can be adapted to various terrain structures. However, these methods are sensitive to the positions of the data and the triangulation needs to be constrained to produce useful results (Schneider 1998, Pries 1995, Weibel and Brändli 1995).

of this discussion, digital contours provide a discrete terrain dataset $\mathcal{D}_T$[15]. Given a set of digital contours, the specification of a family of continuous functions constituting $\hat{z}(\cdot)$ such that $\hat{z}|_{C_{T,Q}} \in Q$ is a delicate task. Therefore, digital terrain representations satisfying definition (4.1) are usually built from digital contours through the intermediate step of first inducing a grid or polyhedral domain partition. There exists a vast amount of literature on terrain interpolation from contours, including Schneider (1998), Aumann (1994), Huber (1992), Pilouk and Tempfli (1992), Brändli (1991), Hutchinson (1988) and Inaba et al. (1988).

4.2 Problem Statement

Definition (4.1) suggests an understanding of digital terrain representations as functions of three 'input quantities': the discrete terrain representation supplied by the dataset $\mathcal{D}_T$, the topological structure imposed on the domain $\mathbb{D}$ by the subdivision Σ, and the family of functions $\hat{z}(\cdot)$ providing the mathematical reconstruction of the topographic surface $z(\cdot)$. These three input quantities stand for the most basic operations involved in digitally representing terrain, namely:

- *discretisation*,
- *reconstruction*, involving:
  - establishing topological relations (*subdivision*),
  - simulation of continuity (mathematical *reconstruction*).

These operations are henceforth referred to as the *'basic operations'* of digital terrain reconstruction.

Discretisation is essential in that it reduces terrain description from a mapping $z(\cdot)$ that relates infinitely (even uncountably) many $\boldsymbol{s} \in \mathbb{D}$ to values $z(\boldsymbol{s}) \in \mathbb{V}$ to mappings between finite domain sets (S, K) and measured elevation values $\tilde{z}(\boldsymbol{s}_i)$, $(\boldsymbol{s}_i \in \tilde{S} \subset \mathbb{D})$, and surveyed TI $\tilde{\chi}(t_j)$, $(t_j \in (\tilde{S} \cup \tilde{K}) \subset \mathbb{D})$, respectively. Discretisation hence may be interpreted as *compression* of the terrain representation. The information loss due to compression is a function of the redundancy of the information omitted. Successful discretisation, therefore, becomes a task of identifying a finite set of TI (to be sampled), such that the information omitted by reducing the domain from $\mathbb{D}$ to $(\tilde{S}, \tilde{K})$ is redundant.

While discretisation decides upon information contents, *subdivision* influences the final mathematical terrain surface simulation $\hat{z}(\cdot)$. It does this

[15]Rather than a terrain reconstruction T_R. Lacking to fulfil the requirements of T_R is a family of functions $\hat{z}_i(\cdot)$ suitable to simulate the terrain surface between the contours.

by setting up the topological structure of the domain partition and hence the manner in which the individual $\hat{z}_i$ are assembled to form the piecewise terrain reconstruction. However, domain subdivision is but a rough diagram enabling elegant mathematical solutions for the task of terrain representation based on a finite number of samples. Subdivision hence is an auxiliary procedure which usually does not have a meaningful real-world interpretation (an exception is, for instance, the contour-flowline network of Hutchinson and Gallant (1999)). *Reconstruction*, finally, means simulation of the topographic surface continuity by means of mathematical functions specified from the information provided by $\mathcal{D}_T$[16]. These functions may be understood as a mathematical formulation of the redundancies exploited for compression of the terrain description in the discretisation step. Reconstruction, thus, prescribes how the information omitted through discretisation shall be recovered from the discrete terrain representation supplied by $\mathcal{D}_T$ (for several different reconstruction schemes based on the same supporting data, see figure 4.4, right and left hand sides, respectively). It follows that the *information content* of a digital terrain reconstruction is governed by discretisation. *Reliable reconstruction* must preserve the available information, in the sense that information shall neither be lost nor shall (new) information, which is not somehow deducible from $\mathcal{D}_T$, be 'invented'. For a more detailed discussion of this topic, please be referred to sections 4.3.1 and 4.3.2.

The *general problem* in digital terrain modelling is, however, that neither the 'true' elevation values nor the qualities of the actual topographic surface behaviour itself are known, because:

- Sampling of topographic surfaces is in principle *inexact* (due to imperfections in the measuring devices, due to scale effects, to implications of sampling resolution and data generalisation, to simplifications for the sake of reduction of problem complexity, etc.). Hence, terrain representations are always tainted with, more or less significant, un-

[16]Formally, a terrain reconstruction $\hat{z} = (\hat{z}_1, ..., \hat{z}_p)$ (where $\hat{z}_i \in \mathcal{C}^k(R_i, \mathbb{V})$, and $i = 1, ..., p$), is specified subject to the conditions:

$$\begin{aligned} \forall \boldsymbol{s} \in \tilde{S} \;\; \exists i \in \{1, ..., p\} : \quad \hat{z}(\boldsymbol{s}) \;&=\; \hat{z}_i(\boldsymbol{s}) \stackrel{!}{=} \varepsilon(\tilde{z}(\boldsymbol{s})), \\ \mathcal{T}(\hat{z}(t)) \;&\stackrel{!}{=}\; \mathcal{E}(\tilde{\chi}(t)), \quad t \in \tilde{S} \cup \tilde{K}, \end{aligned}$$

where $\varepsilon(\tilde{z}(\boldsymbol{s}))$ gives the value to be taken by the reconstruction at location $\boldsymbol{s}$ as a function of the measured value $\tilde{z}(\boldsymbol{s})$, depending on the actual uncertainty representation and the allowed accuracy tolerance. $\mathcal{E}(\cdot)$ stands for the translation of a selected property further characterising the terrain to a mathematical condition, and $\mathcal{T}(\cdot)$ denotes the operation deriving this TI from $\hat{z}(\cdot)$. For instance, if gradient $\nabla(\boldsymbol{u})$ is given for a selected location $\boldsymbol{u} = (u_1, u_2)$, then $\tilde{\chi}(\boldsymbol{u}) = \nabla(\boldsymbol{u})$, and $\mathcal{E}(\tilde{\chi}(\boldsymbol{u}) = (\frac{\partial \hat{z}}{\partial u_1}, \frac{\partial \hat{z}}{\partial u_2})|\boldsymbol{u} \stackrel{!}{=} \nabla(\boldsymbol{u})$, $\mathcal{T}(\hat{z}(\boldsymbol{u})) = (\frac{\partial \hat{z}}{\partial u_1}, \frac{\partial \hat{z}}{\partial u_2})|\boldsymbol{u}$.

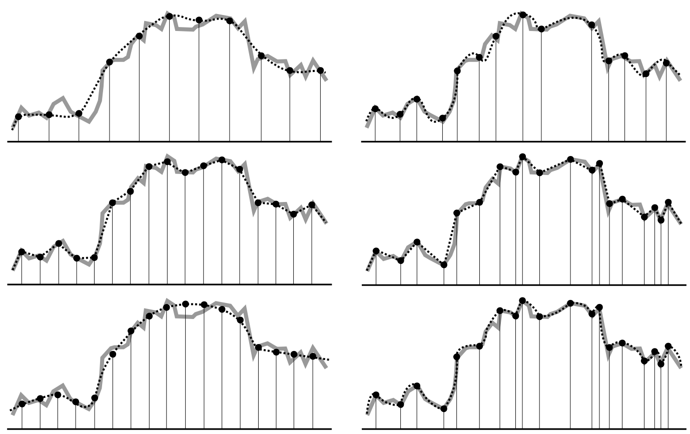


Figure 4.5: Different ways in which to plausibly idealise the same terrain profile. Left: Regular sampling schemes of varying sampling resolution, combined with varying reconstruction polynomials. Right: Irregular sampling schemes combined with varying reconstruction functions. The dashed lines denote the reconstructed profile, the gray line depicts the 'true' terrain (modified after Schneider (2001)).

certainties. Reliable processing of terrain data must thus explicitly consider these uncertainties.

- The terrain surface shape does not obey an *a priori* known, determined mathematical function (such as a sine curve). Descriptions of the topographic surface are, thus, in principle *incomplete* due to incomplete knowledge of the phenomenon terrain impeding its lossless algorithmic compression. Therefore, digital terrain representation necessarily involves some *idealisations*, including, in particular, simplification and abstraction. *Simplification* in the sense that, for the sake of complexity reduction, the topographic surface to be modelled must be portrayed in simplified terms. *Abstraction*, in that properties considered important to the actual terrain shape must be stressed, while those considered immaterial are neglected. Yet, the problem remains that there may be many ways in which to plausibly idealise the same terrain (figure 4.5). Therefore, without the specification of the desired model properties, the task of finding a finite terrain surface approximation is ill-defined, and an unambiguous solution to the problem does not exist.

4.3 The Rules

Equivalent to the conclusion that, in general, a unique finite terrain surface approximation does not exist, is to say that the problem is underdetermined. Therefore, it is necessary to incorporate *additional knowledge* and *assumptions* into the terrain modelling process. A *set of rules* is required to compensate for the incomplete understanding of terrain by supplying enough information about the desired model properties to allow reliable DTM specification. Rules are required that drive the representation of terrain by directly determining its properties[17]. Rules, in this context, portray the *semantics* of a DTM, where semantics are understood as the relationship between a real terrain and its actual digital representation[18]. In view of terrain surface *discretisation*, such rules must facilitate identification of a finite set of TI sufficient to portray a topographic surface bearing desired properties. With respect to the *reconstruction* step, sufficient knowledge and assumptions must be provided to allow well-defined surface specification from the supporting data. Based on these considerations, a DTM is defined as a *virtual dataset* (VDS[19], Stephan et al. (1993)):

$$DTM := \{\mathcal{M}; (T_R; \mathcal{D}_T)\}. \tag{4.2}$$

$\mathcal{M}$ denotes a *set of metadata* which records, amongst others, the rules (i.e., $\{\text{rules}\} \in \mathcal{M}$; for further discussion of $\mathcal{M}$, see chapters 6 and 7)[20]. T_R is a *terrain reconstruction* specified subject to these rules and parameterised by $\mathcal{D}_T$. A DTM, then, extends the *supporting* (and persistent) *dataset* $\mathcal{D}_T$

[17] Desired model properties could be, for instance, that the specified terrain surface shall be $\mathcal{C}^2$-continuous, that it shall display fractal characteristics, that the terrain shall be approximated at a very local scale, or that drainage continuity shall be enforced (for a more detailed discussion, see sections 4.3.1 and 4.3.2).

[18] It is essential to distinguish between semantics, as defined here, and computer model semantics, as defined in computer science (Bishr 1997). In the latter, semantics is the mathematical interpretation of the formal language expressions of a database (van Leeuwen 1990).

[19] The basic idea of a VDS is to extend sample data with methods to provide any derivable or predictable information. Instead of transforming original data to a standard format and storing them, the original data are enhanced with persistent (i.e., explicit) methods that only will be executed upon request (Stephan et al. 1993). An application that uses a VDS does not 'read' the data from a physical file (or query database), but it will call a set of corresponding methods defined in the VDS which return the requested data (Vckovski 1998). As the name suggests, a virtual data set contains virtual data. Virtual data is information which is not physically present, that is, which is not persistent. This data is computed upon request at run time (Stephan et al. 1993).

[20] The meta*information* derivable from these metadata, then, outlines the structure imposed on the context in which DTM quality can be appraised (section 1.2). With the set of rules being an element of the metadata set, the involvement of the rules in providing a *quality norm* (Brassel et al. 1995), which finally allows DTM quality assessment, is indicated. A more detailed discussion of this topic may be found in chapter 6.

with a terrain reconstruction T_R that provides a *(parameterised) family* $\hat{z} = (\hat{z}_1, ..., \hat{z}_p)$ *of stored functions* (in the sense of Open GIS Consortium (1999c)) which simulate a continuous topographic surface by providing virtual data for the locations not sampled.

4.3.1 Implications of Discretisation

As mentioned above, the rules are introduced to prescribe, amongst others, the desired DTM properties, including the required modelling scale. So, the challenge to terrain discretisation is to identify a supporting dataset whose resolution (or density) accords with the prescribed DTM scale and which provides an amount of TI sufficient to support mathematical specification of a surface featuring the desired properties (with the required accuracy). This task touches upon questions of information content and scale.

Information Content It is important that the rules address whether it is sufficient that the supporting data carry mere elevation values, or whether they must also support information like the presence of a peak, a pass, a break line, a water body boundary, etc. In terms of section 4.1, addressed must be whether or not the set S of representative points shall be completed by a set K, whose elements represent terrain properties further characterising the topographic surface[21]. The resulting information content of the supporting dataset $\mathcal{D}_T$ has a direct impact on the *reliability* of the final DTM.

At this point in the discussion, it is essential to carefully distinguish between the notion of *information content* and the notion of *storage volume*. Take, for instance, the example shown in figure 4.6 (top row): Despite the amount of data points being larger in the top right picture than in the top left one, the information content with respect to the modelled surface is the same in both cases; there are just more redundancies in the right picture. In other words, the case shown on the top left is *optimal with respect to redundancy*. In figure 4.6 (bottom row), on the other hand, the difference lies not in the number of data points, but in the spatial data model used (regular vs. scattered points). Again, the information content with respect to the modelled surface is the same, although the storage needs in the bottom left case may be kept lower than the ones in the bottom right example by exploiting the regularity of the supporting data. That is, the example in the bottom left picture is *optimal with respect to storage needs*[22].

[21] This decision has obvious consequences for the data model, as the explicit sampling of peaks, pits, passes, break lines, etc., excludes raster-based data models.

[22] If information content and storage volume were just two names for the same thing, terms like ‘lossless compression’, where ‘compression’ is meant in the sense of reduction of storage needs, would lose their meaning.

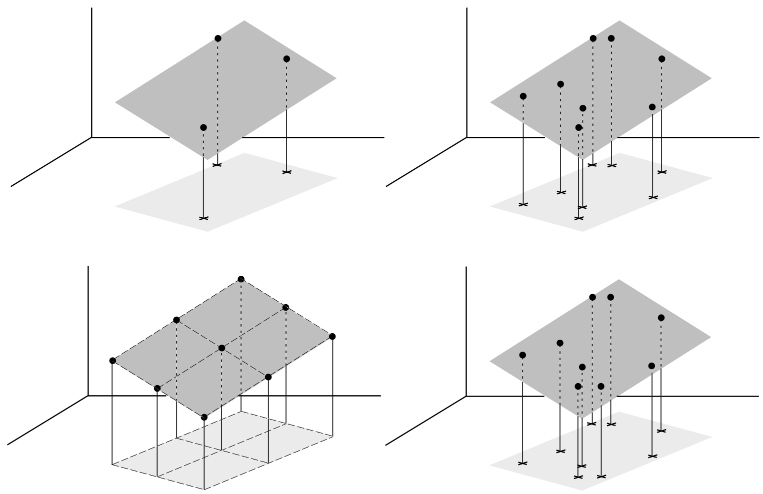


Figure 4.6: Information content vs. storage needs. Top: Same information content, but more redundancy in the right picture. Bottom: Same information content, but possibly less storage volume in the left picture.

For the current discussion, it is difficult to find an appropriate definition for information content. The classical information theory approach defines the *information content* I_i of an event E_i as a function of the probability p_i for the event E_i to happen:

$$I_i := -\log \frac{1}{p_i}, \quad 0 \leq p_i \leq 1.$$

Application of this definition to topographic surfaces would result in propositions of the form: “Because terrains shaped by processes of fluvial erosion feature relatively few sinks, the probability for a randomly chosen supporting point to be a pit is lower than its probability to be part of a hill slope. Thus, pits have a higher information content than points making up hill slopes.” Although such a proposition may somehow be in accordance with experience, useful application of this approach to information content in digital terrain modelling requires extension of the concept for two major reasons: First, it is not clear how to assign probabilities to terrain data elements. Second, the above definition provides a measure of the information content of an individual data element. Even if this measure could be quantified, because of the redundancies may be caused by the ‘systemness’ of the phenomenon terrain (cf. section 2.2), the information content of the entire supporting dataset would in general not amount to the sum of the individual elements’ information contents (cf. figure 4.6, top).

The sampling of more data in the sense of densification of the supporting data may enhances the dataset's information content with respect to the modelled terrain; however, always at the expense of altering the DTM's scale (on this topic, see also the next paragraph and figure 4.10). The prescribed modelling scale may impede the gathering of the amount of input data needed for surface specification by densification of the supporting data. Sampling density (or sampling resolution) is therefore not considered a convenient measure of information content. In cases where the prescribed modelling scale does not allow further densification of the supporting data, to get the required amount of input information, more information elements per sampling location may be provided instead of data samples for more locations.

Based on these considerations, identified as essential (and complementary) *measures of information content* are:

- The *degree of 'randomness'* of the information element provided with respect to the terrain being modelled (probability measure; cf. the pit example above);

- the *number of conditions contributed* to terrain reconstruction by a data element either by itself or in combination with other data ('systemness' measure);

- the *number of digital terrain modelling operations* (survey, subdivision, mathematical reconstruction, etc.) *influenced*.

Figure 4.8 may illustrate these different aspects of information content. The top left picture depicts a topographic surface with a drainage channel (gray line), breaklines (gray dotted lines), and a peak (triangle symbol). The contours sketch the general shape of the terrain. In the top right example, domain discretisation consists of a set $\tilde{S}$ of 15 by 15 regularly spaced supporting points, while the set $\tilde{K}$ is empty (that is, no additional terrain properties are recorded). Each data element may contribute one condition (in the sense of one piece of information) to mathematical surface specification (namely, the elevations at sampling locations). The degree of randomness of the informations (i.e. the elevations) provided is maximal, as the supporting points are not sampled depending on terrain-specific properties. The bottom pictures show a domain discretisation consisting of 60 irregularly spaced supporting points (32 lying in the interior and 28 on the boundary of the area covered). On the left, $\tilde{K}$, again, is empty, while in the case on the right it consists of 22 elements representing the course of the drainage channel, the breaklines and the peak. In both examples, the degree of randomness of the information provided is, in part, lower than in

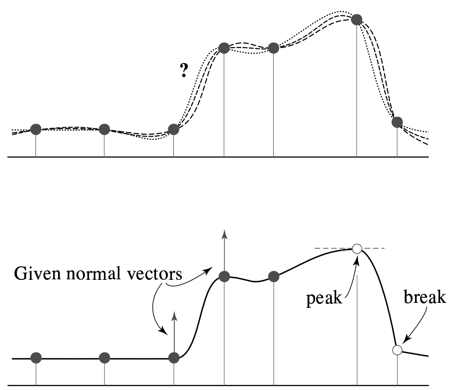


Figure 4.7: Information content of the supporting dataset $\mathcal{D}_T$: sampling more TI without altering sampling resolution. Top: the supporting data carry only elevation information. Bottom: the data carry also attribute information further characterising the terrain surface (modified after Schneider (2001)).

the top right picture, as part of the data is selected to capture critical topographic structures. The domain discretisation set up, therefore, impacts on the data survey, to some degree, by making prescriptions about sampling locations. In the right hand example, the supporting data may also impact on the subdivision step by forcing the resulting domain partition to reflect the topographic structures explicitly sampled. Conditions specified for surface reconstruction, then, may include the prescription that the peak be a local maximum, or that monotonous descent be enforced along the drainage channel.

To sum up, information content is more a measure of the amount of knowledge deducible from the supporting data that enables consistent and unambiguous digital terrain reconstruction than a measure of resultant storage needs. The richer the description of the simulated terrain provided by the supporting data - accounting for the envisioned modelling scale - the more reliable the terrain behaviour modelled and thus the derivable TI will be. Only TI that is explicitly or implicitly contained by the supporting data can be consistently derived with the help of reconstruction and extraction methods. Figure 4.9 visualises the relation between information content and reliability[23]. The bottom right picture depicts a terrain sur-

[23]The examples in figures 4.8 and 4.9 are chosen such that the storage needs for the supporting data amount to approximately the same volume for regular and scattered data ($15 \cdot 15 = 225$ elevation values to be stored in the case of regularly distributed supporting data; $60 \cdot 3 = 180$ coordinate values to be stored in the case of scattered data, and,

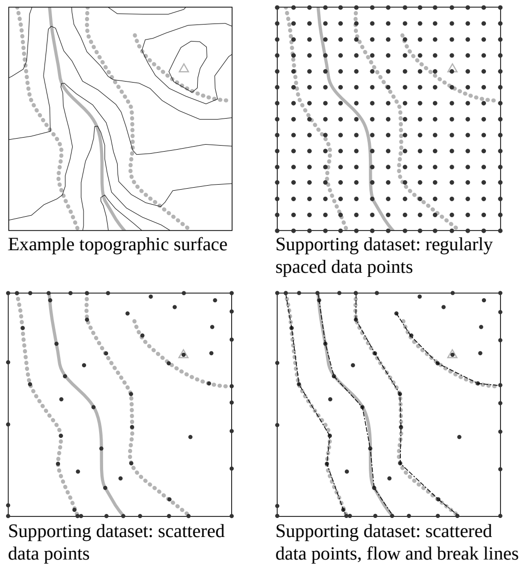


Figure 4.8: Information content of various supporting datasets $\mathcal{D}_T$.

face reconstructed using cubic Bézier splines specified over a constrained triangulation that honours the data shown in figure 4.8, bottom right. The drainage channel and the breaklines explicitly sampled are retained through the entire reconstruction process, resulting in a terrain representation displaying these topographic structures in a clearly pronounced way. The surface depicted in the bottom left case was also obtained using cubic Bézier splines, in this case specified over a Delaunay triangulation of the same representative points yet not further constrained by additionally reported terrain properties (cf. supporting dataset in figure 4.8, bottom left picture). The resulting surface clearly fails to properly capture the overall shape of the terrain. Particularly, the leftmost breakline and the drainage channel are portrayed in a rather distorted way, causing obvious artifacts such as an apparently interrupted drainage channel resulting in the formation of a pond in the bottom left quadrant of the picture. The top right example, finally, shows a surface obtained by bilinear interpolation of the supporting data pictured in figure 4.8, top right. While the overall terrain shape is

assuming edges are stored in the form of starting- and endpoints, $180 + 22 \cdot 2 = 222$ data values for the scattered data enhanced with further TI). This apparent equality is somewhat misleading, however, as the storage needs, the mathematical demands, and the computational complexity of the domain subdivision and surface reconstruction are not considered.

rendered rather well, the topographic structures are portrayed in a clearly less defined way than in the bottom right case. This lack of focus is caused by the limited localisation properties of the regularly spaced data.

Scale Scale decides the degree of detail displayed and the degree of detail neglected and therefore implicates both:

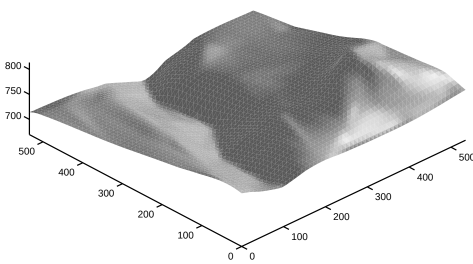
- the *magnitude* of the details building the modelled terrain, and
- the *information content* of the terrain representation.

Strongly related to the notion of scale are the notions of *resolution* (or *data density*) and of *frequency*. The relation between scale and data density is given by the resolution required to capture the details building the modelled terrain. Vice versa, a given resolution determines the magnitude of the details representable and hence indicates scale. However, caution should be exercised when treating resolution as scale, because resolution can be increased without altering scale by the addition of redundant data.

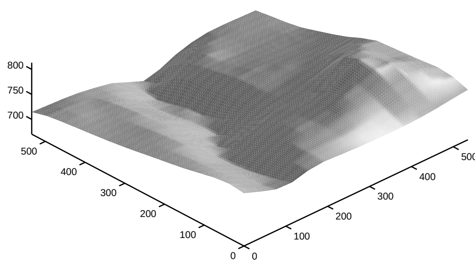
The notion of frequency is only of peripheral interest to the current discussion. The relation between frequency and scale is outlined in appendix D.3.2. A relation between frequency and resolution is provided by various sampling theorems, probably best known of which is Shannon's sampling theorem (Shannon 1949).

In the context of digital terrain modelling, the scale prescribed by a specific application determines the required resolution (or density) of the supporting dataset $\mathcal{D}_T$. The supporting data, in turn, determine the *scale range* of the digital terrain representation, and ultimately its information content, as information not indicated by the supporting data can not be reconstructed. This scale range is therefore implicitly assigned to all TI derived from the DTM (Wood (1996b); see also figure 4.10). If TI for a smaller scale is required, the scale of the DTM may theoretically be decreased with the help of surface generalisation. The maximum applicable scale, however, is given by the available data.

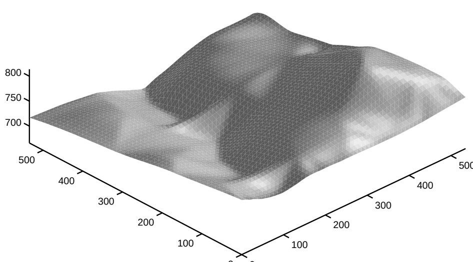
The challenge to terrain discretisation thus is to find a supporting dataset $\mathcal{D}_T$ that provides sufficient information to support mathematical specification of a surface displaying the desired properties and whose resolution (or density) complies with the prescribed modelling scale. Disaccording scales in the sense of too much or little detail displayed may lead to disappointing DTM analysis results because TI may well vary with scale (Wood 1998a, Gallant and Hutchinson 1996, Hutchinson and Gallant 1999), as illustrated in figure 4.10. The discussion of scale-dependence of DTMs is resumed in chapter 8, where a specific approach for dealing with scale issues in digital terrain modelling is discussed.



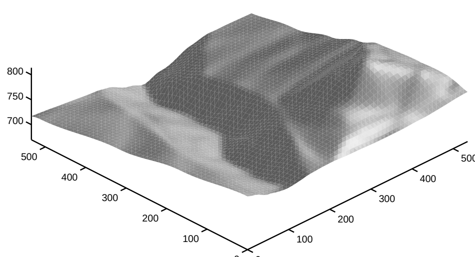
Cubic Bézier splines over constrained triangulation



Bilinear interpolation of regularly spaced data

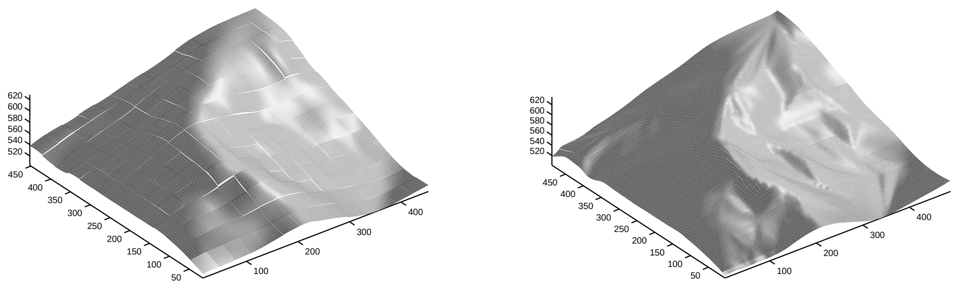


Cubic Bézier splines over Delaunay triangulation



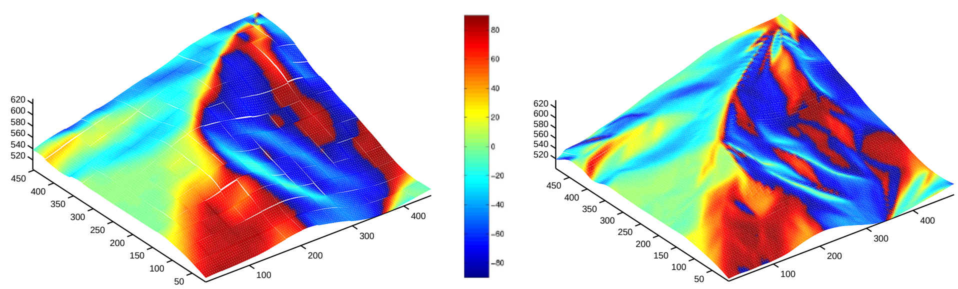
Cubic Bézier splines over constrained triangulation

Figure 4.9: Information content and reliability. Top left: Surface reconstruction based on the supporting data displayed in figure 4.8, top left picture (constrained triangulation honouring the contours and the TI, then interpolation with cubic Bézier splines). Top right: Surface obtained by bilinear interpolation of the supporting data of figure 4.8, top right picture. Bottom left: Interpolation with cubic Bézier splines based on a Delaunay triangulation of the data of figure 4.8, bottom left picture. Bottom right: Interpolation with cubic Bézier splines based on a constrained triangulation honouring the data of figure 4.8, bottom right picture.



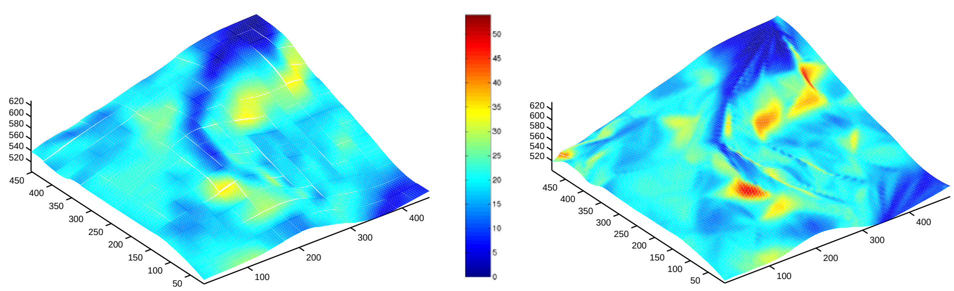
Piecewise biquadratic interpolation of 31.250 m resolution grid

Piecewise biquadratic interpolation of 7.825 m resolution grid



Aspect derived from piecewise biquadratic interpolation of 31.250 m resolution grid

Aspect derived from piecewise biquadratic interpolation of 7.825 m resolution grid



Slope angle derived from piecewise biquadratic interpolation of 31.250 m resolution grid

Slope angle derived from piecewise biquadratic interpolation of 7.825 m resolution grid

Figure 4.10: The impact of DTM scale on aspect and slope angle (aspect and slope angle both expressed in degrees; DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

4.3.2 Implications of Surface Reconstruction

Because the terrain surface shape is not a known, well-determined mathematical function, sufficient knowledge and assumptions describing the characteristics to be exhibited by the surface simulation must be supplied by a set of rules, as described above. Included in the task of applying the rules are the formulation of the desired *continuity properties*, and decisions about the range of the reconstruction[24]. Reasonable assumptions could be, for instance, that the surface must be C^1-continuous except where break lines or points occur; that the surface must not contain sinks where not explicitly indicated; or that it must display fractal characteristics. However, it is important to note that, in the general case, the assumptions and desired properties can not be interpreted as properties intrinsic to the topographic surface. Rather, they usually depend on a user's objectives. Assumptions perfectly reasonable may contradict and, therefore, exclude each other. If, for instance, terrain is modelled as a fractal surface, the terrain reconstruction, first, does not show any continuity properties of order higher than zero, and second, contains a multitude of sinks. While a user interested in visualisation applications may use a fractal terrain to achieve (photo)realistic effects, a hydrologist is likely to prefer a terrain representation not exhibiting spurious sinks.

4.3.3 Implications for Producers and Users of TI

Derivation of apt rules and mapping of such rules to mathematical conditions for terrain surface specification are demanding tasks which challenge both producers and users of TI. To fully appreciate the implications of the rules for producers and users of TI, digital terrain modelling must be placed in the context of DTM generation and application. A venture was undertaken by Martinoni and Bernhard (1998), where a workflow framework was proposed aiming at reliable application of TI in spatial modelling applications[25].

Application Model Generation Every spatial modelling or simulation application is motivated by the wish or need to model, simulate, or analyse a specific *real-world phenomenon*[26]. According to the framework of Martinoni

[24] This is mainly an issue of uncertainty representation and is not discussed further here. For an excellent review of the subject, please refer to Vckovski (1998).

[25] The framework extends the workflow process of modelling a single field (such as terrain, precipitation, or temperature) presented in section 2.3.3 by integrating modelling activities of individual fields into the workflow of an entire spatial modelling or simulation application (such as modelling potential erosion, or harvest predictions).

[26] In appendix A, a snow melt model for high alpine permafrost regions is outlined as an example to detail the individual steps of the workflow framework that is discussed here

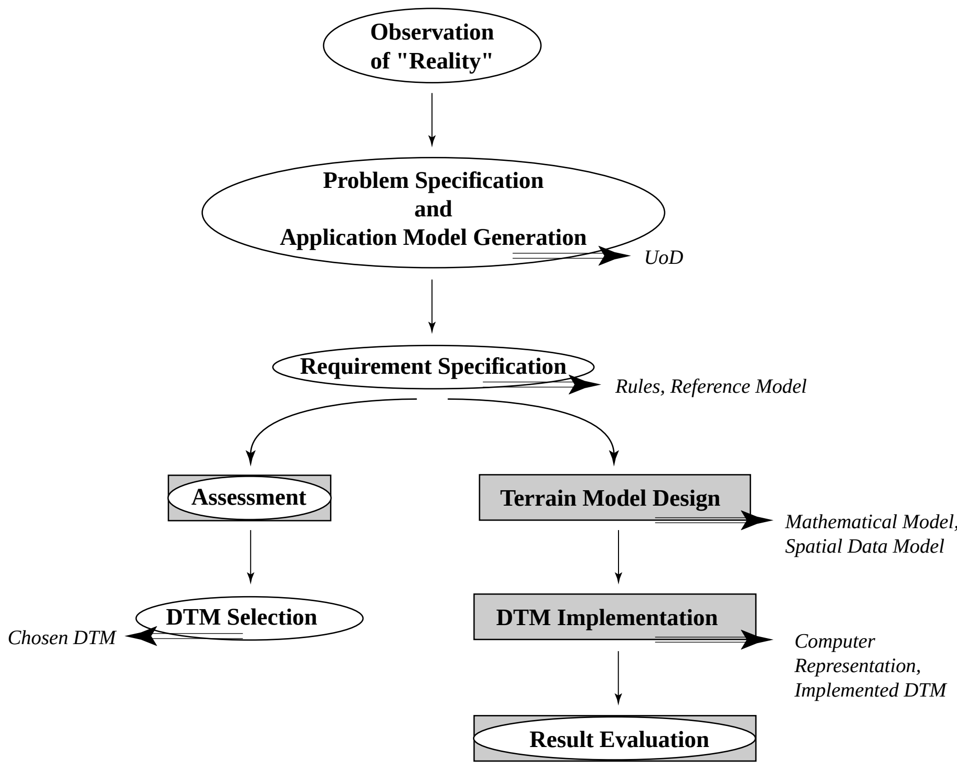


Figure 4.11: Workflow framework for reliable application of TI in spatial modelling applications (after Martinoni and Bernhard (1998)). Ellipses indicate duties of the TI users, while right angled, grey boxes denote tasks of the TI producers. Steps that may be carried out by either users or producers of TI are symbolised with both types of boxes. Italics: workflow steps induced for modelling an individual field as described in section 2.3.3.

and Bernhard (1998), a *spatial modeller*, in a first step, has to identify the real-world features to be modelled and to find suitable ways for their abstraction (figure 4.11). In this step, a conceptual model of the real-world phenomenon under study is generated. This is likely to be composed of several submodels with varying interaction schemes (one of which may be a terrain model). The resulting *application model* defines each submodel's *UoD* (as introduced in section 2.3.1; see also appendix A.1).

Requirement Specification If the problem under study is influenced by topographic factors, the application model generated necessarily includes an abstraction of the phenomenon terrain. Investigation of the application model subject to the *specified problem* has to provide a prescription of this abstraction by naming all respective properties and constraints (i.e., *requirements*) relevant to the application (*requirement specification* step). Issues such as level of abstraction, modelling scale (cf. figure 4.10 for the effect of scale on derived TI), desired semantic properties, accuracy demands, or required TI must be clarified. As already mentioned, for spatial modelling applications, often derived TI rather than the terrain elevations themselves is of interest. As only TI which is explicitly or implicitly represented in the data and retained in the final terrain representation can be reliably extracted, the required TI impacts on both supporting data and terrain reconstruction (think, for instance of the example depicted in figure 4.9). Hence, reliable DTM application crucially depends on explicit specification of the TI needed (for a requirement specification example, see appendix A.2). Subject to the requirements specified, then, a *reference model* for the input factor topography can be generated (cf. section 2.3.1; see also appendix A.2.1). The requirement specification and the reference model, together, form the pool from which *rules* driving DTM generation can be obtained. It follows from the above comment that the rules, amongst others (cf. sections 4.3.1 and 4.3.2), must name the TI required by the application, as this TI needs to be explicitly or implicitly represented in the resulting terrain representation. In other words, it needs to be consistently derivable from the final DTM (see also appendix A.2.2).

Based on the reference model, either a *new DTM is generated*, or existing DTMs are *compared* and *assessed* and the most suitable one selected.

Terrain Model Design In this case, a mathematical as well as a spatial data model must be outlined to support consistent and replicable derivation and representation of the TI required by the application. This *terrain model design* step, carried out by the *producer* charged with model construction, basically corresponds to the tasks of mathematical model formulation and

only theoretically.

data model formalisation introduced in sections 2.3.1 and 2.3.2[27]. As can be seen from figures 2.2 and 2.4, it is important that the reference model supplied by the spatial modeller enables the producer to understand the application's spatial concepts (for more on the terrain model design step, see also appendix A.3).

DTM Implementation and Evaluation Once the (terrain model) design is complete, its *implementation* is left to the producer, assuming the spatial modeller is satisfied with the proposal[28]. Model implementation embodies specification of what was termed *'computer representation'* in section 2.3.3, and the subsequent *model parameterisation* by means of the respective data (see also appendix A.4). Finally, in the sense of an *acceptance trial* the generated DTM must be *evaluated* with respect to the requirement specification and the rules given to drive the DTM construction.

Comparison and Assessment If there is no need to generate a new DTM, that is, if an existing DTM can be re-used, requirement specification is followed by a *comparison* and *assessment* step. In this step, *available DTM's* are compared and evaluated with respect to the application's requirements. Assessed must be, for instance, whether they meet the needed modelling scale or accuracy demands, or whether they contain, either explicitly or implicitly, the TI required. Hence, assessment is driven by the same rules as the generation of a new DTM. Clearly, in-depth assessment depends on the availability of metainformation sufficiently documenting the DTM to be evaluated. Depending on the comparison results, the *spatial modeller* may then decide on one of the DTM's. However, not existing DTM's (in the sense of definition (4.2)) but rather supporting datasets are usually available (for instance, a 7.5-minute USGS DEM without an explicitly specified function for terrain surface reconstruction). Strictly speaking, in such a case it would still be left to the producer to specify a terrain reconstruction subject to both the rules and the available data (amounting to DTM generation in a way much like the one sketched in figure 2.4).

In summary, by identifying the *spatial modeller* with the *TI user*, it remains a duty of the user to formulate and specify the application's requirements to the digital terrain representation and thus the *rules* driving

[27] To ensure consistent TI extraction, the DTM may be extended with specific methods to provide the required TI. In this sense, TI extraction methods may be considered as integral components of an extended DTM. Definition of a DTM as a virtual dataset (section 4.3) proves to be very useful in this context. Development of this idea is continued in chapter 7.

[28] The rules prescribed by the application may limit terrain model design considerably. In the worst case, they may even be impossible to be complied with (e.g., because they contradict). In this trade-off situation, a suitable solution must be found, that is always driven by the application's priorities.

DTM generation or selection, respectively. Generally, the terrain *abstraction* to be worked with, as well as the admissible simplifications and idealisations have to be elaborated by the user. The *producer*, on the other side, is the keeper of the *encoding* and supplier of the *decoding operations*. He or she creates and maintains the dictionaries by which translation is provided from the real world to the mathematical (and digital) world and back. Of course, the producer has to make these dictionaries available in a well documented manner to the user, as it is usually the latter who actually performs decoding, i.e. interpretation of the results. Finally, the production domain has a responsibility to provide *explicit* and *appropriate explanation, specification*, and *quality documentation* of the TI supplied (cf. chapter 7).

4.3.4 Embedding the Spatial Modelling Workflow into the Spatial IS Architecture Proposed by Falcidieno et al.

The workflow framework for reliable DTM application presented above corresponds to a workflow interpretation of the concepts on which the spatial IS architecture proposed by Falcidieno et al. (1992) is founded, namely the notions of "descriptive" and "prescriptive" modelling (figure 4.12). *Prescriptive modelling* is a top-down approach to provide a conceptual model of the real-world phenomenon being modelled, naming all properties and constraints that are relevant from a particular application's point of view. Prescriptive modelling thus may be identified with the user's part in the above spatial modelling workflow. *Descriptive modelling*, on the other hand, corresponds to a bottom-up approach to terrain data handling. The terrain under study is surveyed and measured, yielding raw data. Upon these data the digital terrain reconstruction ('mathematical terrain reconstruction') is built via structuring the data and specifying a suitable reconstruction scheme ('model generation'). The aim of the descriptive modelling approach is to provide a digital reconstruction of the topographic surface from which powerful tools reliably extract the TI needed for a spatial modelling application. Descriptive modelling, which basically corresponds to what was outlined in section 4.1, thus, denotes the generation-specific approach.

Both, prescriptive and descriptive modelling deal with the same real-world phenomenon. Consequently, their results have to be in accordance with each other (as implied by figure 4.12). This is a crucial requirement for *reliable* DTM application. However, as set forth in section 4.2, in the absence of more precise specifications, the descriptive modelling branch is underdetermined. That is there exist a number of equally plausible mathematical terrain reconstructions featuring very different properties. Therefore, only descriptive modelling driven by rules supplied by the prescriptive approach may ensure a terrain reconstruction consistent with the respective reference model.

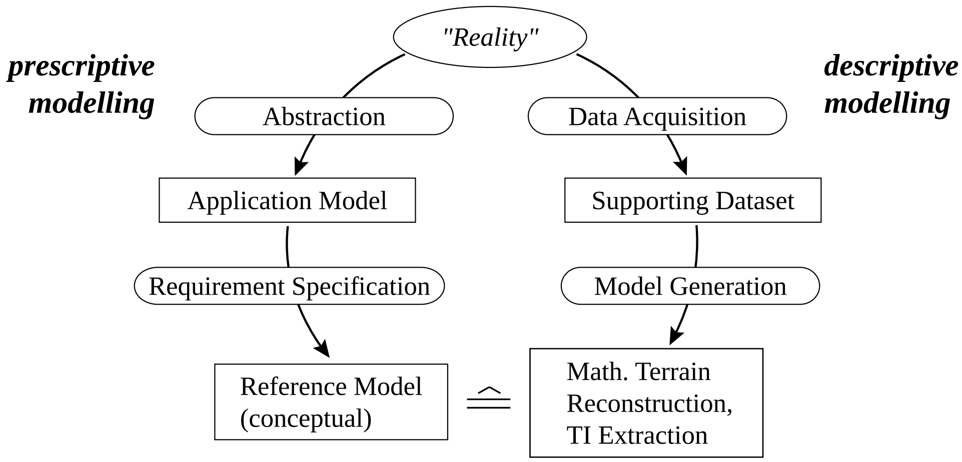


Figure 4.12: Conceptual scheme, on which the spatial IS architecture of Falcidieno et al. (1992) is founded, re-interpreted as two-sided approach to digital terrain modelling.

4.4 Comparison of Different Terrain Models

In the last section it was argued that specification of a well-behaved, finite digital terrain reconstruction necessarily requires extension of (purely) geometric supporting data with additional knowledge and assumptions derived from application-specific rules. Also, it was argued that enriching the (geometric) supporting data with additional information capturing characteristic terrain properties was likely to improve on DTM reliability. At the *data level*, this introduction of an additional body of knowledge through modelling assumptions and/or enriched data samples may be seen to provide the data an *enhanced 'information base'*[29, 30]. Clearly, lossless transformation of one terrain reconstruction T_R (terrain reconstruction in the sense of definition (4.1)) to another representation $T_R^{'}$ must preserve information contents and model properties. $T_R^{'}$, thus, must adhere to the same rules as T_R. Two digital terrain reconstructions T_R and $T_R^{'}$ are therefore considered to be *equivalent*, if

(i) they agree on the information base and displayed properties,

[29]Discretisation provides a projection of the terrain being modelled into a finite dataset $\mathcal{D}_T$ located in some inhomogeneous space $\mathbb{R}^3 \times \mathbb{I}$, where $\mathbb{R}^3$ stands for the 3-dimensional physical space, and $\mathbb{I}$ denotes some 'information space'.

[30]This information base is propagated into the terrain reconstruction thus specifying the terrain representation's *semantics*.

(ii) the actual (geo)metric representation of the TI contained differs at most by a given (accuracy) tolerance in $\mathbb{R}^3$ [31].

Equivalent terrain reconstructions of the same portion of terrain imply supporting datasets equivalent with respect to the information provided to satisfy the rules, since information lost or lacking in the supporting dataset can not be recovered or compensated for by other operations involved in generating a digital terrain representation (such as the subdivision scheme or reconstruction functions). DTMs (i.e., virtual terrain datasets in the sense of definition (4.2)), on the other hand, to be considered equivalent need not only to comprise equivalent terrain reconstructions. Rather, they also need to supply an equivalent set of metainformation $\mathcal{M}$. Since the rules are only elements of $\mathcal{M}$ (rather than $\mathcal{M}$ itself), equivalent terrain surface simulations are not sufficient to guarantee equivalent DTMs.

The following discussion describes an attempt to formalise these qualitative remarks. Section 4.2 identified constructing digital terrain representations as a function of the basic operations discretisation and reconstruction, the latter including subdivision and mathematical surface simulation. Section 4.3 stressed the necessity to drive these operations by application-specific rules in order to yield a well-behaved, unambiguous terrain representation. Given a set of rules prescribing the desired model properties and required model characteristics, *discretisation* involves identification of a finite set of TI sufficient to portray the abstraction of terrain used (and matching the desired modelling scale) and the subsequent capturing of this TI. *Subdivision*, then, structures the data. *Reconstruction*, finally, encompasses the setting of local continuity properties appropriate to satisfy the global continuity properties imposed by the rules[32], as well as selection of an admissible class of reconstruction functions suited to generate a surface honouring the prescribed characteristics[33]. An overview of the main steps involved in constructing a digital terrain representation is given in table 4.1.

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[31] In the presence of well-set up rules prescribing specific metric accuracy requirements, the second condition to equivalence lapses.

[32] By *local continuity properties*, the continuity properties within a region R_i of the domain partition are referred to (i.e., the continuity properties of the individual $\hat{z}_i$, ($i = 1, ..., p$), making up the reconstruction family $\hat{z}(\cdot)$). *Global continuity properties* refer to the continuity properties of the globally assembled reconstruction function $\hat{z}(\cdot)$ across the edges and vertices of the domain partition.

[33] The decision on the range of the reconstruction, that is, the desired uncertainty representation, is also a task of the reconstruction step.

	Digital terrain modelling operation	*Formal formulation*	*Contribution to* $DTM = \{\mathcal{M}, (T_R; \mathcal{D}_T)\}$
Discretisation	*Domain reduction:* Identification of a finite set of TI sufficient to portray the terrain abstraction used.	$T \longrightarrow (z(\cdot)\|_S, TI_{(S,K)}, (S,K), \mathbb{V}, \mathbb{V}_{TI}) =: T_D$ where $TI_{(S,K)}$ must compensate for the information lost by discarding $\mathbb{D} \setminus (S,K)$.	Essential model for data survey.
	Surveying/measuring, based on the essential model T_D.	$T_D \longrightarrow \{\{\boldsymbol{s}_i; \tilde{z}(\boldsymbol{s}_i)\}_{i=1}^{m},\ \{t_j; \tilde{\chi}(t_j)\}_{j=1}^{n' \leq (m+n)}\} =: \mathcal{D}_T$ where $\boldsymbol{s}_i \in \tilde{S},\ t_j \in \tilde{S} \cup \tilde{K}$. $\tilde{z}(\boldsymbol{s}_i)$ denote the measured elevations, $\tilde{\chi}(t_j)$ the TI surveyed, $\tilde{S}$ the actual set of representative points, and $\tilde{K}$ the realisation of K.	Supporting terrain dataset $\mathcal{D}_T$.
Reconstruction	*Subdivision:* Choice of a suitable kind of domain partition. Inference of topological relations.	$\Sigma = \mathcal{S}(\mathcal{D}_T) = \mathcal{S}(\tilde{S}, \tilde{K}, \tilde{z}(\boldsymbol{s}_i), \tilde{\chi}(t_j))$, where $i = 1, ..., m,\ j = 1, ..., n' \leq m + n$.	Subdivision $\Sigma = (V, E, R)$
	Mathematical reconstruction: Simulation of the topographic surface' continuity by means of mathematical functions. The task includes: - Setting of local continuity properties (appropriate to satisfy the global continuity properties imposed by the rules), - decision upon the uncertainty representation (i.e. upon the range $\mathbb{V}$), - selection of an admissible class of reconstruction functions, - parametrisation.	Local - continuity properties: $\hat{z}_i \in \mathcal{C}^k(R_i, \mathbb{V})$ where $0\,(\leq l)\ \leq k \leq \infty$, and l captures the global continuity properties, if any. $\mathcal{D}_T \xrightarrow{\Sigma} \{\hat{z} = (\hat{z}_1, ..., \hat{z}_p) \in \mathcal{C}^l(\mathbb{D}),\ \hat{z}_i \in \mathcal{C}^k(R_i) \mid \forall \boldsymbol{s} \in \tilde{S}\ \ \exists i \in \{1, ..., p\} :\ \hat{z}(\boldsymbol{s}) = \hat{z}_i(\boldsymbol{s}) \stackrel{!}{=} \varepsilon(\tilde{z}(\boldsymbol{s})),\ \mathcal{T}(\hat{z}(t)) \stackrel{!}{=} \mathcal{E}(\tilde{\chi}(t)),\ t \in \tilde{S} \cup \tilde{K}\}$ where $\hat{z}_i,\ i = 1, ..., p$ are the admissible reconstruction functions, $\varepsilon(\tilde{z}(\cdot))$ is a function of the measured data, $\mathcal{T}(\cdot)$ is an operation deriving TI from $\hat{z}$, and $\mathcal{E}(\tilde{\chi}(\cdot))$ is the mathematical encoding of the sampled TI.	Terrain reconstruction $T_R := (\hat{z}(\cdot), \Sigma, \mathbb{V})$

Table 4.1: Main steps involved in generating a digital terrain representation satisfying a given set of rules.

Given a portion of terrain to be modelled, let $\mathcal{A}$ be the set of all its possible discretisations, $\mathcal{P}$ be the set of all possible domain partitions, and $\mathcal{C}$ the set of all admissible reconstruction function spaces[34]. Digital terrain reconstruction, then, consists of the tasks: (i) appropriately selecting one element of each of the above sets, where 'appropriately' refers to the specified rules, and (ii) implementing this selection. At an abstract level, the basic discretisation and reconstruction operations, therefore, may be understood as observables of the digital terrain modelling process, as they generalise the idea of observables introduced in section 2.2 by prescribing how the real-world phenomenon terrain has to be mapped to a formal mathematical object.

Generation of the actual terrain reconstruction is embedded in a frame of *general conditions* pegged by the driving rules. These define, among other things:

- the demanded DTM expressiveness,
- appropriate modelling scale,
- global continuity properties desired,
- range of the reconstruction,
- accuracy requirements, etc.

Within this frame, two kinds of *observables* are identified: modelling choices and parameters. Those observables that are neither 'measurable' by surveying the real terrain nor deducible from the data supplied, will be understood as *'modelling choices'*. Modelling choices, therefore, are not directly induced by the actual terrain but selected with respect to plausibility according to the rules. Hence, they may be equated with the *parameters* introduced in section 2.2. The modelling choices encompass, among other things:

- Identification of the TI to be sampled,
- selection of convenient subdivision schemes,
- selection of *local* continuity properties and selection of an admissible class of reconstruction functions.

The remaining modelling steps – provision of the supporting dataset $\mathcal{D}_T$ and parametrisation of the reconstruction functions via structuring the data

[34] For the sake of completeness, let $\mathcal{V}$ denote the set of all possible reconstruction ranges.

by domain partition – clearly are not modelling choices, because they directly depend on either the actual terrain or the supporting data. These *observables*, in analogy to Casti's (1989, 1992) theory (sketched in section 2.2), may be separated into *inputs* and *outputs* of the modelling process. The objective is to provide a parameterised finite terrain description $\hat{z}(\cdot)$. The field reconstruction $\hat{z}(\cdot)$, therefore, is understood as output of the modelling process. Hence, *digital terrain modelling* may be written in a formal way as a function Φ:

$$\begin{aligned} \Phi_{rules,\,\alpha,\,\Sigma}: \qquad & U \xrightarrow{\mathcal{S}(\cdot)} Y, \\ & \mathcal{D}_T \overset{\Sigma}{\longmapsto} (\boldsymbol{s}, \hat{z}(\boldsymbol{s})), \quad (\boldsymbol{s} \in \mathbb{D}), \end{aligned} \tag{4.3}$$

where U is the input space (with $\mathcal{D}_T = \{\{\boldsymbol{s}_i;\, \tilde{z}(\boldsymbol{s}_i)\}_{i=1}^{m},\ \{t_j;\, \tilde{\chi}(t_j)\}_{j=1}^{n' \leq m+n}\}$), Y is the output space, α is the space of the modelling choices, and Σ denotes the selected subdivision scheme. Notation (4.3) is chosen to explicitly indicate the dependence of the resulting DTM on the general modelling frame pegged by the rules and the modelling choices made.

Based on this framework, an approach is ventured which attempts to answer the question when two representations of the same portion of the earth's surface T are equivalent. Consider two descriptions of T:

$$\Phi_{rules,\,\alpha,\,\Sigma}: U \xrightarrow{\mathcal{S}(\cdot)} Y, \quad \Phi_{rules,\,\hat{\alpha},\,\hat{\Sigma}}: U \xrightarrow{\hat{\mathcal{S}}(\cdot)} Y.$$

If (purely) 'geometric' supporting data (i.e., data points located in 3-dimensional physical space without an enhanced information base) were sufficient to unambiguously specify a finite terrain representation, the answer would be that the two descriptions are *equivalent* if there exists a coordinate change in U such that the description $\Phi_{rules,\,\alpha,\,\Sigma}$ is transformed into $\Phi_{rules,\,\hat{\alpha},\,\hat{\Sigma}}$. Diagrammatically, a bijection $g_{(\alpha,\,\Sigma),\,(\hat{\alpha},\hat{\Sigma})} : U \longrightarrow U$ is sought such that the following diagram commutes:

$$\begin{tikzcd} U \arrow[r, "\Phi_{rules,\alpha,\,\Sigma}"] \arrow[d, "g_{(\alpha,\,\Sigma),\,(\hat{\alpha},\,\Sigma')}"'] & Y \arrow[d, "id"] \\ U \arrow[r, "\Phi_{rules,\hat{\alpha},\,\Sigma'}"'] & Y \end{tikzcd}$$

In this (hypothetical) *case of complete knowledge*, the rules would only need to define questions of uncertainty representation and accuracy demands. Of course, only terrain representations adhering to the same rules (i.e., sharing the same uncertainty representation and accuracy demands) may be equivalent, as indicated implicitly by the notation used.

However, as motivated in section 4.2, terrain reconstruction from such geometric data alone is underdetermined, thus necessitating enhancement of the information available for model specification. Therefore, unless a specific metric can be defined on the 'information dimension' of supporting data and terrain representation, a *qualitative interpretation* of the above diagram must suffice. In this general case of *incomplete knowledge*, the rules must specify the desired model properties. Clearly, it remains true that only terrain representations adhering to the same rules (and thus modelling abstractions of a given terrain that are similar enough) may be *equivalent*. The *identity relation* on the output space, then, means that the terrain reconstructions must agree on information contents and semantics, and that their (geo)metric representation must not differ by more than a given accuracy tolerance (cf. footnote [31] on the second part of this condition). The other part of the diagram is then reasonably interpreted as follows: Assuming a *change in the modelling choices* (e.g., selection of a different subdivision scheme, or of another class of reconstruction functions), the resulting alternative terrain description is interpreted as equivalent to the original one if the change in modelling choices can be 'reversed' by an alternative representation or choice of supporting data such that an identity in the above sense holds on the output space. Or, vice versa, assuming usage of an *alternative set of supporting data*, a respective terrain representation is interpreted as equivalent to the one based on the original data if the change in the supporting dataset can be compensated for by alternative modelling choices in order to preserve identity in the above sense on the output space.

4.5 Review

Reliable DTM application means that the TI derived must be *replicable* and *consistent*, a demand which may not be met unless an explicit and suitable reconstruction of the topographic surface is provided. It was argued that specification of a well-behaved, finite digital terrain reconstruction necessarily requires incorporation of additional knowledge and/or assumptions derived from *application-specific rules* into the modelling process. The importance of conceptual models of terrain acting as pools from which such rules may be gained was pointed out by placing digital terrain modelling in a context of DTM generation and DTM application. The *workflow process* proposed implies that there are more mappings involved in DTM generation than sampling elevations and entering them into a storage device. Rather, also involved are, (i) mapping of the terrain to be modelled to a conceptual reference model and derivation of specifying rules, (ii) sound mapping of the reference model and rules to both mathematical and spatial data models, and (iii) consistent encoding into a computer representation. Consequently, more *factors affecting TI quality* are revealed than measuring uncertainty and uncertainty related to digital storage of the data (as will be discussed in chapter 5).

Reliable digital terrain modelling charges the *discretisation* step with the task of identifying a supporting dataset that provides sufficient information to support mathematical specification of a surface featuring the desired properties and whose resolution complies with the prescribed modelling scale. This task raises questions of *information content* and *scale*. The scale-dependence of DTM's is the topic of chapter 8, where an approach to deal with scale issues in digital terrain modelling is discussed.

The introduction of additional knowledge through the rules and/or enriched supporting data gives reliable digital terrain modelling an 'information dimension' which implies that (geo)metric accuracy alone cannot be sufficient to describe TI quality. Because DTMs represent an abstraction of the true terrain - not the true terrain itself - knowledge of this abstraction - captured by the rules - is indispensable to provide a frame of reference for DTM quality assessment. However, unless a specific metric can be defined on the information dimension that is introduced, the latter defies strictly mathematical considerations, thus limiting corresponding quality statements to a qualitative level.

Chapter 5

Factors Affecting TI Quality

Section 1.3 argued that the notion of *fitness for use* is best suited to provide an overall paradigm for quality management in digital terrain modelling. As stressed repeatedly, fitness for use apportions the responsibilities between providers and users of TI (cf. sections 1.3 and 3.1.3). The production domain is assigned the responsibility to provide explicit and appropriate information on the factors affecting the quality of the TI supplied (along with the respective TI). Such documentation is needed by the TI user, whose responsibility is to ensure the information is only applied to problems where such use is adequate.

This chapter investigates the workflow process involved in reliable TI generation and application with the objective to compile the *major factors affecting* TI *quality*. The objective of this chapter is to prepare the ground for chapter 6, which aims at structuring the context reflecting the relevant characteristics of a DTM - that is, the context required to appraise DTM quality (cf. section 1.2). Within the structure imposed on this context, appendix C will propose metadata elements suitable to exhaustively document TI quality.

Three major factors affecting TI quality are discussed in this chapter, where logical organisation is provided with respect to primary cause. These are:

- *Uncertainty* (discussed in section 5.1), which arises from deficiencies in the individual modelling steps involved in reliable digital terrain modelling;

- *Consistency* (discussed in section 5.2), which is a measure of the integrity of the overall workflow;

- *Validity* (discussed in section 5.3), which is a measure of the genericness of models and data generated as driven by application-specific rules.

5.1 Uncertainty

5.1.1 Causes

As mentioned in the introduction, *uncertainty* arises from *deficiencies in the individual modelling steps* when mapping the real-world phenomenon terrain to a DTM (i.e., to a virtual dataset; section 4.3) and to derived TI. Major causes for the inevitable emergence of deficiencies affecting this mapping were given in section 1.1. These include the terrain's inherently indistinct and indeterminate nature; terrain's property to always manifest detail at a scale larger than the sampling scale; human's inability to measure beyond some limits of precision; and terrain's continuous nature. This continuous nature requires discretisation when sampling terrain data, and, paired with an incomplete understanding of the phenomenon, impedes the terrain's lossless reconstruction.

5.1.2 Sources of Uncertainty

In section 3.3, the conceptual model of IS, which models physical geographical phenomena by the processes of abstraction and representation, and which ascribes GI quality to deficiencies in these processes, was suggested as an appropriate basis to approach quality management in digital terrain modelling. From this point of view, *abstraction* and *representation* are sources of the uncertainty inherent to TI. Taking a close look at the workflow process involved in providing a digital terrain representation reliably fitting the needs of a specific application, enables a more sophisticated differentiation. The following major sources contributing to the uncertainty accrued are then identified (see also figure 5.1 and table 5.1):

Abstraction Abstraction combines the processes involved in both mapping a true terrain to a reference model and specifying a set of rules driving terrain reconstruction and TI extraction. Abstraction always involves *simplification*, in the sense of separating terrain features and properties considered relevant to a specific application from those considered not. Omitting apparently non-relevant parts of the perceived reality introduces some amount of uncertainty (although, in a somehow 'deliberate' way). The notion of the *rules* was introduced in section 4.3 to compensate for the incomplete understanding of the phenomenon terrain. Compensating for incomplete knowledge, the rules must be partly specified based on plausibility reasoning and assumptions - rather than on knowledge - which are, by their nature, uncertain to some degree. In short, *abstraction* seizes control of the model's *information content* and (thematic) *detail richness*, and thus introduces *scale*. In other words, scale is implied by the abstraction process

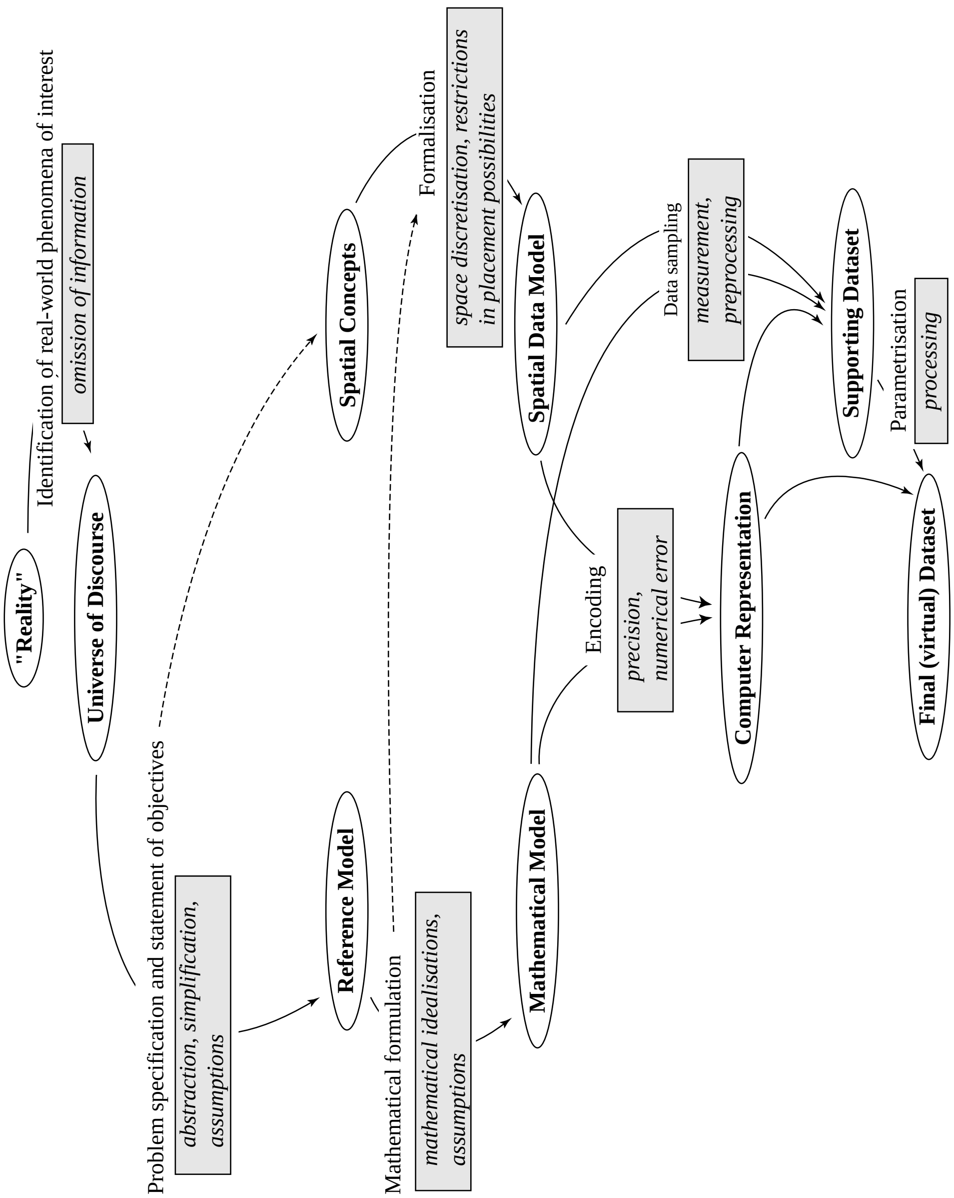


Figure 5.1: Uncertainties arising when mapping a real-world terrain to a DTM (dashed lines illustrate modelling steps confining other steps in the modelling sequence). Rounded boxes denote the individual modelling steps when proceeding from the real terrain to the DTM. Light gray rectangles name the processes causing uncertainty.

which finally decides upon the degree of detail displayed and the degree of detail neglected; the latter always being a cause of uncertainty.

Specification Processes Specification processes provide translation of the reference model to the mathematical and digital world within the guidelines specified by the rules. Mapping an apparently mathematically undetermined phenomenon such as terrain to a mathematical model necessarily involves *mathematical idealisations* (such as interpreting the terrain surface shape as built from cubic Bézier-splines) and possibly mathematical simplifications (such as neglect of higher order terms) as well.

Spatial data models supporting the representation of fields vary well in their *localisation qualities*. A raster, for instance, predetermines the locations of the data samples by prescribing a regularly spaced sampling scheme regardless of the size and location of the forms to be captured, thus preventing (terrain-) specific structures from being accurately modelled. Similarly, contours restrict the data to pre-defined elevations, excluding peaks, pits, and passes from being accurately represented, and cause areas that are not steep to be poorly sampled. In short, mapping of reference models, spatial concepts and rules to highly formalised representations introduces uncertainty through the *mathematical idealisation* involved, as well as by delimiting the *amount of information that is accurately representable*.

Translation to the digital world encompasses specification of the *data types* and their *numerical precision*, as well as of the *numerical methods* to be used in light of the digital implementation of the mathematically specified terrain representation. Although referring to numerical precision may seem far-fetched, precision has in fact notable influence on TI uncertainty. Carter (1992), who investigated the effect of data precision on the calculation of slope and aspect from gridded DTMs, for instance, showed that the precision of an elevation matrix rounded or truncated to the metre is not sufficient to make distinctions in the derived aspects as fine as a single degree. *Encoding*, hence, adds to TI uncertainty by introducing issues of *precision* and *numerical error* into DTM generation, representation and TI derivation.

Survey Supporting data, either directly measured or derived from other data sources, are always tainted with uncertainty stemming mainly from *measurement*, or, in more general terms, from the *surveying process*. Major contributions to the uncertainty accrued arise from:

- The measurement *technique* applied,
- the *instruments* used,
- the *recording media* involved (analog, digital, optical),

- and the operational *skills* of the observer.

If the supporting data do not only carry elevation information, but are attributed further properties characterising the terrain surface to be modelled, these attributes are affected by uncertainty as well. Therefore, through data collection, *accuracy* (both *metric* and *thematic*) enters the uncertainty discussion.

(Pre)Processing Compilation of a supporting dataset useful to a specific application may require some *preprocessing*, including:

- Georeferencing and/or (geometric) corrections;
- transformation, aggregation or classification of the raw data.

Both preprocessing as well as actual model parametrisation cause *propagation* of the uncertainty occurring in the supporting data. Uncertainty propagation is a complex matter influenced by many factors, such as the mathematical idealisations imposed by the mathematical model specified, the limits of possible precision given by the computer representation, and the amount of numerical error introduced by the computational methods and algorithms applied.

5.1.3 Comments on the Sources of Uncertainty

The major sources contributing to the uncertainty affecting TI were identified in the last section, based on careful analysis of the workflow approach to digital terrain modelling presented in section 4.3.3. The validity of the findings achieved when referred to terrain representations resulting from less ideal modelling efforts might be doubted. In particular, their general applicability to terrain representations lacking a (known) reference model[1] or terrain datasets without explicit surface specification might be questioned.

It is argued here that the arguments given for the encoding, survey, preprocessing, and processing steps hold in any case (providing, these steps are carried out at all). Referred to abstraction, the argument is that information is necessarily lost when sampling terrain. Uncertainty, thus, is introduced due to *inevitable* information loss (because reality can not be captured in its entire complexity) instead of due to *intentional* omission of information (for the sake of 'controlled' complexity reduction). Particularly, *scale* is inescapably induced by the sampling process by the spatial resolution of the resulting dataset (section 4.3.1).

[1] In terms of chapter 4, terrain reconstructions lacking the documenting metadata needed to fulfil the requirements to a DTM.

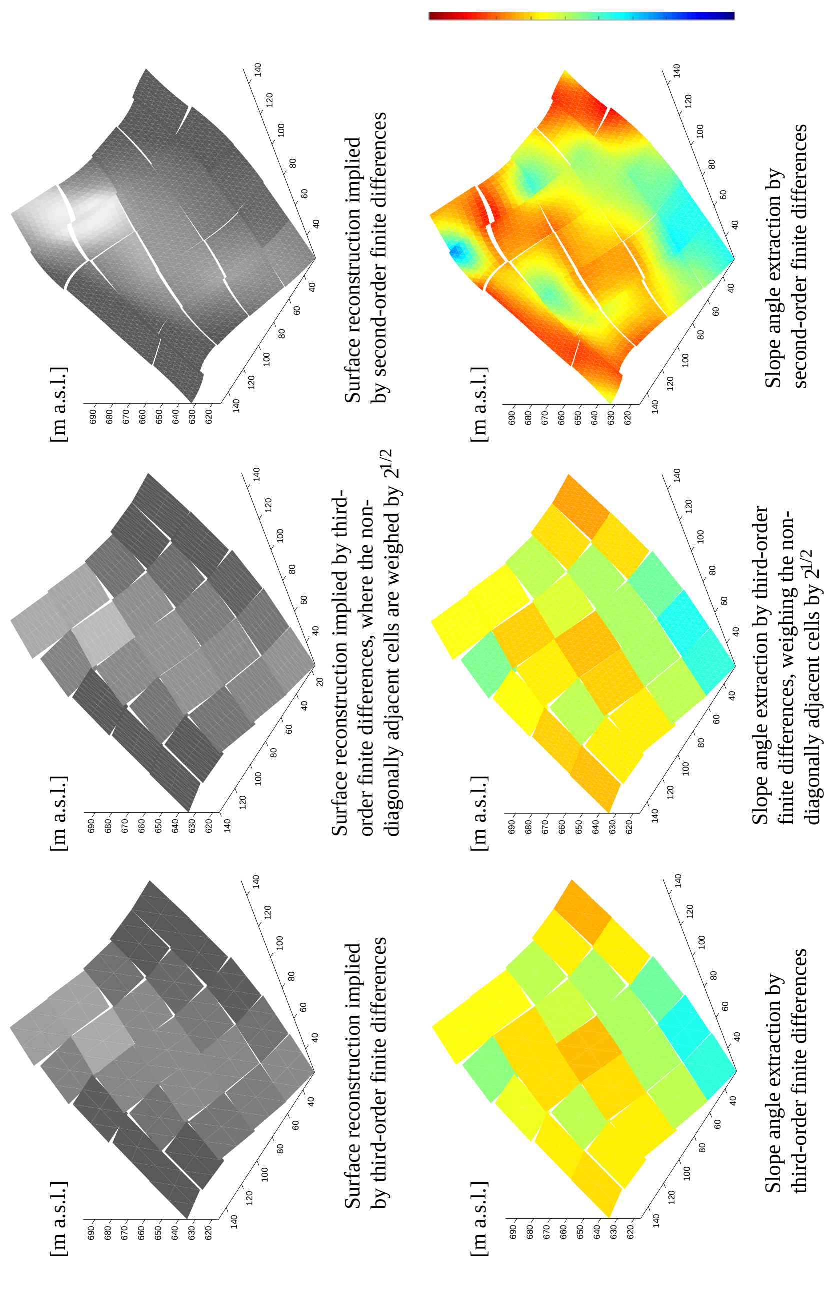


Figure 5.2: Surface reconstructions implied by different methods for slope angle computation (cf. appendix B), exemplified for a fairly rugged terrain (slope angles expressed in degrees; DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

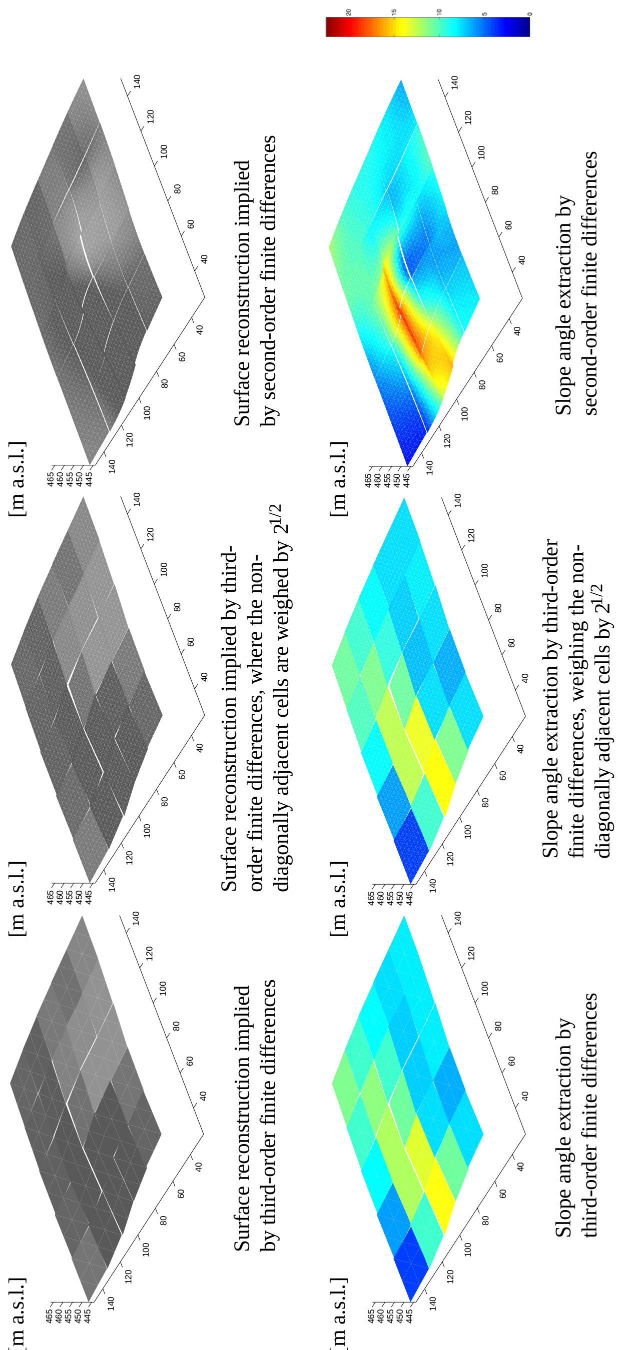


Figure 5.3: Surface reconstructions implied by different methods for slope angle computation (cf. appendix B), exemplified for a rather flat terrain (slope angles expressed in degrees; DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

If no topographic surface is explicitly specified from the supporting data, and no mathematical model is designed at all[2], apparently, there will be no contribution to uncertainty stemming from mathematical idealisation. The catch, however, is that TI derivation always in one form or another relies on implicit assumptions about the topographic surfaces underlying derivation. For instance, gradient derivation (at a raster point) from a RSG using a second-order finite difference method (also known as numerical differentiation; Dozier and Strahler (1983)) renders the gradient of a biquadratic surface through the 3 by 3 neighbourhood of the central point (appendix B.1; cf. slope calculation according to Zevenbergen and Thorne (1987)). Gradient computation by third-order finite differences (Sharpnack and Akin 1969)[3], on the other hand, returns the gradient of a plane fitted to a 3 by 3 neighbourhood of the raster point by least squares (appendix B.3). In figures 5.2 and 5.3, the surfaces implied by different methods for slope angle calculation are illustrated for both a fairly rugged as well as a rather flat terrain. Both terrains displayed are supported by 5 by 5 regularly spaced data points. In the rugged case, elevation ranges from 621.00 to 690.70 m a. s. l.; for the flat terrain, elevation ranges from 446.80 to 465.20 m a. s. l. Surface reconstruction is, therefore, implicitly integrated into TI extraction. Additionally, as set forth in section 4.1, if no topographic surface is explicitly specified, methods to derive different types of TI may be based on different implicit representations of the same terrain, thus affecting consistency of the derivation results (for further discussion of this topic, see sections 4.1 and 5.2.1; see also figures 4.1, 4.2, 5.2, and 5.3. It is, therefore, argued that eventually, in the absence of a (known) reference model and/or an explicit surface reconstruction, the sources of uncertainty are not decreased; they are only hidden better. Because documenting metadata is typically unavailable, detection of occurring uncertainties is less obvious, and enforcing consistency and replicability of the modelling results becomes more demanding.

5.1.4 Types of Uncertainty

While section 5.1.2 uncovered sources contributing to the uncertainty affecting TI quality, this section aims at investigating the *nature* of the uncertainties that are contributed. Analysis of the identified factors limiting digital terrain representation (cf. table 5.1) suggests the distinction of *three types of uncertainty*; namely abstraction, specification and data uncertainty.

[2]That is, in the terminology introduced in chapter 4, if a discrete terrain dataset $\mathcal{D}_T$ is at hand rather than a terrain reconstruction T_R or even a complete DTM.

[3]The popular method for gradient derivation proposed by Horn (1981) is basically a third-order finite difference method with particular weighting factors for the non-diagonally adjacent cells.

Source	*Process introducing uncertainty*	*Uncertainty cause*	*Type*
Abstraction	Omission of information (for the sake of complexity reduction)	Information content, (thematic) detail richness	Abstraction uncertainty
	Introduction of rules on the ground of plausibility reasoning and assumptions (to compensate for incomplete knowledge)	Information base, scale range	
Specification processes	Mapping of both, the reference model and rules, to a highly formalised representation	Information content that can be represented, math. idealisation	Specification uncertainty
	Encoding (i.e., translation to the digital world)	Precision, numerical error	
Survey	Survey of the supporting data (including measurement techniques applied, instruments used, recording media involved, and operational skills of the observer)	Accuracy (both metric and thematic)	Data uncertainty
Preprocessing, processing	Preprocessing required Processing of the data	Uncertainty propagation, subject to precision and numerical error of the (pre)processing methods, within the bounds of the math. idealisation imposed	Data uncertainty

Table 5.1: Sources and types of uncertainty.

Abstraction Uncertainty Abstraction uncertainty (Duckham 1999), as the name implies, arises from *deficiencies in the abstraction process* (section 5.1.2). The amount of abstraction uncertainty accrued is closely related to the desired degree of complexity reduction - that is, to the degree of abstraction and simplification - and thus is a measure of the model's *semantic distance* (Salgé 1995) to perceived reality. A key feature of abstraction uncertainty is the emergence of *scale* issues. Crucially, abstraction uncertainty is effective on a *model level.*

Specification Uncertainty While abstraction uncertainty relates to the process of abstraction, specification and data uncertainty arise from representation. Specification uncertainty renders the *shortcomings of mapping conceptual models*, such as the reference model and spatial concepts, *to highly formalised representations*, such as the mathematical model or the computer representation (section 5.1.2). The specification uncertainty accrued is mainly determined by two factors: first, by the degree of mathematical abstraction and simplification, that is, by the *'semantic distortion'* due to (geo)metric representation; and second, by the effects of *precision* and *numerical error* arising from digital encoding. Specification uncertainty, essentially, is effective on the level of declaration of *variables* and of formalisation of their *relationships* and *geometric descriptions* on the one hand (cf. section 2.3.1), and on the level of *property types* on the other[4] (cf. section 2.3.3).

Data Uncertainty Data uncertainty, finally, is due to *parametrisation* of the model designed by actual data. Data uncertainty arises from the uncertainty inescapably inherent to all *measurements*, possibly propagated by processing of the data. A key feature of data uncertainty is the emergence of *accuracy* (both *metric* and *thematic*). Representation uncertainty, crucially, is effective on the *level of individual data items.*

5.1.5 Concluding Comments on Uncertainty

In section 4.4, an attempt was made to understand the construction of a digital terrain representation formally as function of the basic operations discretisation, subdivision, and reconstruction. Notation (4.3) was introduced which explicitly indicates the dependence of a digital terrain representation on the general modelling frame pegged by the rules and on the modelling choices driving its generation. Merging this attempt to formalise digital terrain modelling with the uncertainty discussion reveals *rules* to in-

[4]From an object-oriented perspective, one would say that specification uncertainty is effective on a class level.

troduce *abstraction uncertainty*[5]. However, by specifying the desired model properties and requirements (such as the desired continuity properties or accuracy demands), the rules themselves also set bounds to the specification and data uncertainty that is tolerable. Within this acceptable scope, *specification uncertainty*, then, remains mainly a function of the *modelling choices* made[6], while *data uncertainty* is introduced into the parameterised model through the *supporting data*. However, the effects acting on specification and data uncertainty are not strictly independent from each other, thus allowing (limited) *trade-offs* between the two[7].

5.2 Consistency

5.2.1 Sources and Types of Consistency

Sources

Placing digital terrain modelling in a context of DTM generation and DTM application revealed the *workflow nature* of reliable digital terrain modelling. This workflow nature of the modelling process gives rise to possibly occurring *inconsistencies*. While each step involved in mapping from the real world to a (virtual) dataset was pictured as contributing its particular uncertainty factor[8] (cf. table 5.1), the introduction of different types of inconsistency is not sequential. Rather, the same type of inconsistency may be repeatedly incurred through the entire modelling chain (cf. table 5.2). The most important difference between the two factors affecting TI quality is, however, that inconsistencies, in contrast to uncertainties, are not inevitable. On the contrary, they may be prevented by careful implementation of the modelling task.

[5]The rules were introduced in section 4.3 to compensate for incomplete knowledge and infinite complexity of the phenomenon terrain by prescriptions always assumptive and normative to some degree. The rules driving digital terrain reconstruction thus inherently introduce abstraction uncertainty.

[6]Typical modelling choices to be made include, for instance, selection of a convenient subdivision scheme, local continuity properties, an admissible class of reconstruction functions (cf. section 4.4), or the data types to be used for implementation (cf. section 5.1.2). Similar tasks arise when mapping a reference model to a mathematical or digital world. Thus, the modelling choices clearly contribute specification uncertainty.

[7]For instance, a sophisticated subdivision scheme may to some degree attenuate the effect of highly inaccurate supporting data on surface derivatives such as slope, aspect, or curvature. In figure 4.9, surface derivatives extracted from the terrain reconstruction based on the constrained triangulation (bottom right) are likely to be more reliable than the ones calculated from the terrain representation based on the Delaunay-triangulation (bottom left), regardless of the accuracy of the supporting data.

[8]This uncertainty factor, then, is possibly propagated through the further mapping, and, adds to the total amount of uncertainty amassed.

Types

Consistency has three different facets, giving rise to three types of consistency:

- *Structural integrity* is concerned with the consistency of a concept, model, algorithm, or dataset in itself (i.e., with questions such as whether contradictions, logical errors, or the like occur). Structural integrity is not a formal issue, as its testing requires a certain information (or knowledge) base. For more detailed discussion of integrity, see the next paragraph.

- *Formal correctness* is concerned with whether code or data are syntactically correct. Being a syntactical issue, formal correctness can be tested by application of formal 'consistency checks'.

While violations of structural integrity may arise at all levels of abstraction, formal correctness is a matter of the computer representation and the data used (both supporting and virtual). However, these two types of consistency are effective on a 'horizontal' level, in that they affect 'modelling products', such as models, algorithms, or data per se.

- *Conformance* is concerned with whether a concept, model, algorithm, piece of code, or data truly serves the specified purpose. Conformance, in this sense, is effective on a 'vertical' level, as it impacts on the integrity of the modelling chain; that is, violations of conformance arise from deficient transition between modelling stages at different levels of abstraction. Conformance has both a content-related (semantic) and a syntactic facet, as will be discussed below.

In the following, the notion of consistency is exposed in more detail, and examples are provided to illustrate the individual factors of consistency (for a summary of the sources and types of consistency, see table 5.2). However, these factors are not strictly orthogonal and can, therefore, not always be strictly delimited from one another.

'Horizontal' Consistency Types On a horizontal level, consistency issues impact on the structural integrity and the formal correctness of models or data (or closed components of them). Two types of horizontal consistency were identified above:

- *Structural integrity*: Problems of missing structural integrity may arise at all levels of abstraction. In particular, the following areas may be affected[9]:

- The *internal logic*: The concept, model, algorithm, specification or data affected are contradictory or logically erroneous.

  For instance, the supporting data are contradictory if a representative point marked as a peak is not a local maximum. A DTM (i.e., a virtual terrain dataset) is logically erroneous if it supplies slope angles computed with a third-order finite differences method, on the one hand, and flow lines derived by searching the lowest neighbour in a 3 by 3 neighbourhood, on the other hand, thus causing flow processes in a direction different from the one of steepest descent (figure 5.4). Rules are contradicting if they, for instance, prescribe that the terrain shall be modelled as a fractal surface and at the same time demand a terrain reconstruction enforcing drainage continuity.
- *Topological relations*: The topological structure (conceptually) imposed is false or contradictory.
- *Functional relationships*: The functional correlations are portrayed in a false or misleading way.

  Think, for instance, of a DTM supplying slope angle computed by using a third-order finite differences method and profile curvature derived from interpolating a biquadratic polynomial to a 3 by 3 neighbourhood (as proposed by Zevenbergen and Thorne (1987)). Profile curvature is the rate of change of slope angle in the gradient direction. However, as computation of slope angle by third-order finite differences renders the slope angle of a plane fitted to a 3 by 3 neighbourhood by least squares (section 5.1.3 and appendix B.3), the curvature value so calculated does not correspond to the rate of change of the apparently corresponding slope angle thus causing functional inconsistency (cf. figures 5.2 and 5.3 which illustrate the differences in the respective slope angles).

- *Formal correctness*: Problems of formal correctness may concern the computer representation as well as both the supporting data and the parameterised DTM. In particular:

  - *Formal integrity*: Code (or pieces of code) or data show syntactical or logical errors, including deficiencies resulting in runtime errors.

[9] For the sake of completeness it shall be mentioned that problems of structural integrity may also affect *temporal relationships* in which case the correctness of temporally ordered events is afflicted.

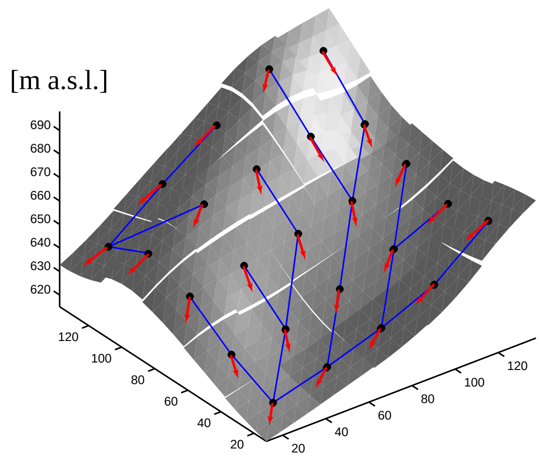


Figure 5.4: Flow lines (blue) running in a direction different from the direction of steepest descent (red). The flow lines are derived by searching the lowest neighbour in a 3 by 3 neighbourhood. The direction of steepest descent is computed by third-order finite differences (DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

  - *Topological relations*: Spatial data structures or topology are built in a faulty way.
  - *Domain consistency*: Sampled or computed data values do not adhere to the specified value domains.

'Vertical' Consistency Types On a vertical level, consistency issues impact on the integrity of the modelling sequence. In other words, inconsistencies may arise from deficient transition between modelling stages at different levels of abstraction. As already mentioned, this affects *conformance* which embodies both, a content-related (semantic) and a syntactic component.

- *Conformance - the semantic facet*: Conformance in this sense addresses the question whether corresponding model elements agree in design at different levels of abstraction. This includes:
  - The model's *information base*: Either the body of information and knowledge once introduced is not properly retained through the modelling process or the information required is not properly provided by the supporting data.

If, for instance, a surface is specified from contours by bivariate-quintic interpolation (via the intermediate step of first triangulateing the contours; see, e.g., Akima (1978)), the resulting terrain is likely to 'overshoot' (Brändli 1991, Schneider 1998). In this case, the information that is implicitly contained in the supporting contour data - that all surface elevations have to lie within the contour interval of neighbouring contours - has been neglected. Another example of semantic conformance is domain subdivision by Delaunay triangulation not honouring breaklines explicitly represented in the supporting data (cf. figure 4.9, bottom row).

– The actual *operationalisation* of the TI supplied: TI derivation may be both structurally coherent and formally correct, whilst not conforming to its specification.

  Consider, for instance, a DTM required to provide horizon. Assume the horizon being calculated uses an algorithm for view shed computation. While the method applied may be structurally consistent and well implemented, the results yielded overestimate horizon, as local horizon is just a subset of the view shed. This is an example of operationalisation inconsistencies in that the *encoded method* (view shed computation) does not derive the TI demanded by the *requirement specification* and *reference model* (horizon). For another instance, recall the example where slope angle was computed by third-order finite differences, and profile curvature is derived according to the method proposed by Zevenbergen and Thorne (1987). While both algorithms may be perfectly coherent in themselves, for the reasons given in the last paragraph, the curvature thus supplied is not a derivative of the slope angle provided. This is an example of operationalisation inconsistency in that the *encoded method* (curvature computation after Zevenbergen and Thorne (1987)) does not yield the TI specified by the *mathematical model* (curvature modelled as derivative of slope angle).

– From a more technical point of view, attention must be drawn to inconsistencies that may be affecting *encoding* and *digital implementation*. So, for instance, when proceeding to the computer representation, it must be made sure that the numerical methods applied are appropriate to digitally implement the mathematical model[10], or that the precision(s) specified does not interfere with accuracy demands.

[10]This is particularly crucial when dealing with TINs.

- *Conformance - the formal facet*: From a formal point of view, issues of conformance may include:
  - *Completeness* is concerned with the question whether, or to which degree, all real-world features that have a conceptualisation are formally preserved through the subsequent modelling steps.
  - *Format consistency* is concerned with the question whether, or to which degree, the format of data, whether sampled or computed, is in accordance with the schema specified by the computer representation.

The notion of consistency, particularly the notion of conformance, honours the *process-related quality concept* (section 1.3). A major task of *quality assurance* in reliable digital terrain modelling, therefore, is the prevention of inconsistencies.

Adequacy Not a type of consistency in the narrow sense of the word, although surely an element of good modelling practice, is the notion of *'adequacy'*. Critical questioning of the modelling steps invoked with regard to their adequacy to achieve the specified objectives[11] is likely to contribute to increasing the reliability of results.

5.2.2 Comments on Consistency

Like the identification of sources contributing to TI uncertainty in section 5.1.2, the inconsistencies possibly affecting digital terrain modelling were discussed by referring to the workflow framework presented in section 4.3.3. Therefore, again, the general validity of the arguments proposed might be questioned when applied to terrain representations not adhering to a known set of rules, or lacking a known reference model or explicitly specified computer representation (or any combination of the above)[12].

It is argued in this regard that inconsistencies effective on a horizontal level (i.e., issues of structural integrity and formal correctness) may affect any terrain representation, as their occurrence does not directly depend on the existence of an elaborated modelling scheme.

Then, even in the absence of a known and elaborated modelling scheme, terrain representation and TI derivation are still products of distinct although interrelated operations (such as data collection[13], possibly required

[11]The approach chosen to model a specific problem may be formally correct but not adequate due to, for instance, exaggerated accuracy or lack of elegance.

[12]In short, when applied to terrain representations failing to meet the requirements to a DTM (in the sense of definition (4.2)).

[13]Where the data may be collected by direct survey, or computed from other data sources.

Modelling stage	*Consistency type*	*Frame of reference*
Reference model - intentional definition	*Structural integrity* of the intentional definition	Logics
	Conformance of the intentional definition to the problem specified and the application model generated	Problem specification, application model
	Completeness of the intentional definition	Problem specification, application model
	Adequacy of the intentional definition with respect to the problem specified	Problem specification
Reference model - requirements	*Structural integrity* of the requirements formulated	Logics
	Conformance of the requirements to the problem specified and the application model generated	Problem specification, application model
	Adequacy of the requirements formulated with respect to the problem specified	Problem specification
Rules	*Structural integrity* of the rules (including basic assumptions)	Logics
	Conformance of the rules to the problem specified and the application model generated	Problem specification, application model
	Conformance of the rules to the intentional definition and the formulated requirements	Intentional definition, requirements
	Completeness of the rules	Logics, problem specification, application model, knowledge available
	Adequacy of the rules with respect to the problem specified and the application model generated	Problem specification, application model
Mathematical model	*Structural integrity* of the math. model	Logics
	Conformance of the math. model to the reference model and the rules	Reference model, rules
	Completeness of the math. model	Reference model, rules
	Adequacy of the math. model to represent the reference model according to the rules	Reference model, rules, problem specification
Mathematical model - spatial data model	*Conformance* of the data model to the spatial concepts (reference model)	Spatial concepts, reference model
	Conformance of the data model to the rules	Rules
	Conformance of the data model to the math. model	Math. model
	Adequacy of the data model to formalise the spatial concepts according to the rules	Reference model, rules, problem specification
	Adequacy of the spatial data model to represent the math. model	Math. model

Modelling stage	*Consistency type*	*Frame of reference*
Computer representation	*Structural integrity* of the computer representation (including numerical methods)	Logics
	Formal correctness of the computer representation	Logics
	Conformance of the computer representation to the math. model	Math. model
	Conformance of the computer representation to the rules (including basic assumptions and demands)	Rules
	Conformance of the computer representation to the spatial data model	Spatial data model
	Completeness of the computer representation	Math. model, spatial data model
	Adequacy of the computer representation to encode the math. model according to the rules (including demands)	Math. model, spatial data model, rules
Survey - data sampling	*Structural integrity* of the survey	Logics
	Formal correctness of the methods involved in compilation	Logics
	Conformance of the survey to the model specified (according to the rules)	Reference model, math. model, rules
	Conformance of the survey to the schema specified	Computer representation
	Adequacy of the survey to meet the demands of the model specified (according to the rules)	Reference model, math. model, rules
Survey - compilation	*Structural integrity* of compilation	Logics
	Formal correctness of the methods involved	Logics
	Conformance of compilation to the raw data (assumed error models, precision, etc.)	Raw data
	Conformance of the compilation to the computer representation (coordinate system, frame of reference, etc.)	Computer representation
	Conformance of the compilation to the rules (accuracy demands, etc.)	Rules
	Completeness of compilation	Logics, computer representation
	Adequacy of compilation processes (calibrations, corrections, etc.)	Computer representation, rules

DTM as virtual dataset - supporting dataset	*Structural integrity* of the dataset *Formal correctness* of the dataset *Conformance* of the dataset to the computer representation (precision, etc.) *Conformance* of the dataset to the rules (accuracy demands, scale range, etc.) *Completeness* of the dataset *Adequacy* of the supporting data to parameterise the computer representation	Logics Logics Computer representation Rules Computer representation, logics Computer representation, rules
DTM as virtual dataset - parameterised model	*Structural integrity* of the DTM *Formal correctness* of the DTM *Conformance* of the DTM to the rules *Completeness* of the DTM *Adequacy* of the DTM with respect to the application model	Logics Logics Rules Computer representation Application model

Table 5.2: Sources and types of consistency.

preprocessing, interpolation or TI extraction methods). Lacking knowledge of the mechanisms behind the operations applied, then, makes *Conformance* harder to be enforced - conformance in the sense of accordance between both the various operations themselves as well as between these operations and the data they are applied on. In this case of lacking or incomplete specifications, possibly occurring inconsistencies are much less obvious and therefore harder to detect. Similarly, detection of data or model *incompleteness* is a tricky task, if respective frames of reference are unsettled.

5.3 Validity

5.3.1 Causes and Types of Validity

Causes

Issues of *validity* arise, first, from the lack of genericness of model and data generation driven by application-specific rules. Second, from the real terrain undergoing (usually slow but) permanent change (over time) while its digital representation is usually a one-time snapshot (in the sense that regular updates are fairly unusual for DTMs).

Types

Semantic Validity The semantic validity of both DTM and TI derivable depends on the degree to which reconstruction of the topographic surface is based on properties intrinsic to the phenomenon terrain. Conversely, validity is affected by the degree to which terrain reconstruction is based on properties depending on the purposes of a specific application. For instance, a hydrologist will probably desire a terrain representation based on a reconstruction scheme enforcing drainage continuity (i.e., a reconstruction scheme possibly avoiding or removing pits that might occur). However, the validity of such a reconstruction scheme might be disputed across information communities. From the point of view of a visualisation application, for instance, such a 'drainage continuous' terrain may look quite unattractive. For purposes of visualisation, terrain may rather be modelled as a fractal surface. While in this case the model may look more realistic, it also features a multitude of sinks. This discrepancy is meant to illustrate that the apparent absence of sinks in real terrain is not a property intrinsic to any terrain. Rather, it is a matter of the purpose of a specific application (and also a matter of scale (Wood 1998b)). Issues like the above seriously impact the validity of DTM application in spatial modelling projects, as conclusions based (solely) on how terrain was algorithmically compressed to provide a finite representation - rather than on terrain-intrinsic properties - do not truly describe the topographic surface itself, but rather describe aspects of our approach to its finite representation. Similarly, validity is affected by the degree to which TI is specified in a closed (or self-contained) form; or, respectively, by the degree to which this TI depends on *empirical thresholds*[14]. Note, however, that TI specification depending on empirical thresholds not only affects the validity of the TI modelled, it also constitutes a considerable source of abstraction uncertainty (as empirical thresholds are clearly used to compensate for incomplete knowledge).

Validity with Respect to (Geographical) Region Clearly, issues of validity with respect to geographical region apply only to unparameterised stages of the digital terrain modelling process (such as the reference model, mathematical model, or computer representation)[15]. Validity with respect to geographical region provides information about the re-usability of the unparameterised specifications by investigating their *applicability to distinct regions* of, for instance, varying climatic conditions or diverse geological subsoils, or to regions shaped by different geomorphological processes such as fluvial or glacial erosion.

Validity across Scales Validity across scales refers to the applicability of sampled data as well as models specified for TI representation and derivation

[14] Take, for instance, the example of delineating a stream network by applying a threshold value to select discretisation units with a high accumulated flow from a previously computed total flow accumulation per discretisation unit.

[15] As data always is uniquely geo-referenced to a specific region.

to scale ranges other than the current sampling or modelling scale. In other words, the *scale effects* introduced through abstraction, sampling and (digital) representation are investigated.

Temporal Validity Issues of temporal validity apply to both supporting data and specified models at all levels of abstraction. Temporal validity is mainly a function of the *dynamics of variability* of the TI represented. In this sense, temporal validity is also concerned with the investigation of the influence of possible seasonal or even diurnal variation (seasonal variation, for instance, may be observed in connection with modelling of glacial structures and processes, diurnal variation may occur in connection with modelling of coastal regions).

5.3.2 Comments on Validity

Again, issues of validity were discussed on the basis of the ideal case of all steps involved in reliable digital terrain modelling having been carried out and explicitly specified. Therefore, implications of modelling steps may be left out or whose specification may not be generally available are only briefly discussed.

The view taken in this regard is that the types of validity identified apply in a straightforward manner to whatever is known about a given digital terrain description. Semantic validity always remains a function of the degree to which properties intrinsic to the topographic surface are described, rather than artifacts of its reconstruction. However, with decreasing knowledge of the semantics of a terrain representation, reliable assessment of its validity gets trickier. As mentioned in sections 5.1.2 and 5.1.3, the processes of abstraction, sampling and representation inevitably give rise to the emergence of scale. Therefore, whenever the application of any data, sampled or derived, or model specification, to a scale range other than the original sampling or modelling scale is intended, the preservation of the corresponding validity needs to be clarified. Similarly, because updates of both data (sampled or derived) and models are fairly infrequent, questions of temporal validity always persist.

As soon as derivation of TI from any given terrain representation is envisioned, to ensure consistent and reliable results, the validity (in the sense of consistent applicability) of both, the methods and data used together with the abstraction and modelling assumptions underlying them, needs clarification - regardless of the explicit availability of corresponding knowledge. As already mentioned, the less such knowledge is available, the harder this clarification of validity issues will get.

5.4 Review

In light of comprehensive documentation and effective TI quality handling, the following results of this chapter can be summarised. Quality of digital terrain representations was found to be affected by a complex system of given, generated and propagated factors acting on metric, attribute and semantic levels. Distinguished were *three major factors affecting TI quality*. These are:

- *Uncertainty* arises from deficiencies in the individual steps of the DTM generation and TI derivation processes. Uncertainty, hence, combines aspects of both a *fitness for use* and a *product-related* understanding of quality (cf. section 1.3) and thus essentially contributes to the integration of the latter quality concept into the overall discussion.
- *Consistency* is a measure of the integrity of the digital terrain modelling workflow. Particularly, the notion of 'vertical' consistency types pays regard to the *process-related* quality concept and hence may be interpreted as an aspect of *quality assurance*.
- *Validity* is a measure of the *genericness* of a given DTM. Validity questions the re-usability of data (both sampled and virtual) and models supplied for applications other than the ones originally envisioned and thus provides information linked to the idea of *fitness for use*.

The challenges of developing a comprehensive and well-structured framework for TI quality description are manifold. The identified factors affecting TI quality display a rich diversity with respect to both the information component concerned with (geometric description, thematic description, and semantics) as well as the level acted upon, the latter ranging from individual data items to the entire DTM.

Chapter 6

Metadata Components for TI Quality Description

This and the next chapter, together with appendix C, present a metainformation framework for organising and using knowledge about TI quality. The objective of this chapter is to suggest *metadata components for exhaustive TI quality description* and to establish *principles for TI quality reporting.* Chapter 7 will present an (object-oriented) conceptual model for metadata representation that integrates the metadata components suggested in this chapter and the metadata elements described in appendix C into a structure conforming to the DTM definition proposed in section 4.3.

The chapter first identifies the *metadata components* necessary for comprehensive TI quality description. Then, the basic *principles for TI quality reporting* are discussed. The chapter is inspired by many sources, including ISO/TC211 (1999), CEN/TC287 (1996), Open GIS Consortium (1999a,b), Federal Geographic Data Committee (1998a), Duckham (1999), Brändli (1991). The chapter concludes with a discussion of how the proposed structures fit into the ISO Standard 19113 Geographic Information - Quality Principles (ISO/TC211 1999), regarded as the standard most likely to carry through.

6.1 Metadata Components

Chapter 5 identified three major factors affecting TI quality - uncertainty, consistency, and validity - arising from the workflow process involved in reliable TI generation and application (discussed in section 4.3.3). The considerations exposed imply that a context for TI able to reflect the factors likely to affect its quality must provide means to make explicit the various mappings involved in DTM generation and TI production.

Besides such aspects stemming from the *workflow perspective* taken in chapter 5, comprehensive metainformation must account for a few further factors influencing TI quality, namely:

- The *history* of models and data, including the life-cycle of the data through the various processing steps to its present form.
- *Technical issues* arising from the actual digital implementation.
- *Miscellaneous questions*, such as contact information for users or information on access.

Accommodation of all these requirements may be accomplished by means of *three primary metadata components*:

- *Modifiers* (discussed in section 6.1.1 and in appendix C.1) make explicit the mappings involved in the digital terrain modelling workflow (cf. section 4.3.3) and thus provide part of the *quality norm* (Brassel et al. 1995).
- *Descriptors* (discussed in section 6.1.2 and in appendix C.2) provide informative data explaining the supplied TI with regard to its history, validity and expressiveness, technical implementation, or availability.
- *Quality elements* (discussed in section 6.1.3 and in appendix C.3) measure the actual performance of the supplied TI.

6.1.1 Modifiers

Modifiers serve to make explicit the mappings involved in reliable digital terrain modelling by:

- Informing about the linkage between the real terrain (or, in general, the real world) and its modelled abstraction, thus rendering explicit the UoD, reference model, rules, and spatial concepts.
- Making explicit the mappings between this abstraction and the model specification, exposing the mathematical model, spatial data model and computer representation.

Modifiers, therefore, serve to make explicit links which might otherwise have been lost in the processes of abstraction and specification. Consequently, modifiers are clearly *normative*. They may be used by TI producers (and possibly also TI users) to provide the formal product specification. Thus, they are part of the *quality norm* (Brassel et al. 1995).

A modifier is a recognised normative aspect of metainformation. Modifiers are grouped into *modifier elements*, which in turn comprise one or

Modifier	*Modifier element*	*Modifier attribute*
Abstraction modifier	Abstractive information	Abstraction Information base Dictionary
Specification modifier	Supporting property type specification	Mathematical specification Supporting type representation Geometry specification Sampled attribute specification
	Virtual property type specification	Mathematical specification Virtual type representation Geometry specification Virtual attribute specification Stored function

Table 6.1: Modifiers, modifier elements and their associated modifier attributes.

more *modifier attributes* (see figure 6.1). The parts comprising a modifier element provide the mechanism for reporting modifiers.

Table 6.1 summarises the modifiers, their modifier elements and associated modifier attributes identified as essential to exhaustively expose the workflow process involved in reliable TI generation. The parts comprising a modifier element are shown in figure 6.1 and discussed in more depth in the next section. Appendix C.1 provides selected details and examples on the above suggested modifiers.

Parts Comprising a Modifier Element

- *Qualifying section*, restricting:
  - The *reporting scope*: Modifier elements are only meaningful when referring to specific dataset levels (for an explanation of dataset levels, please refer to section 6.2.1). The reporting scope is used to restrict the scope of some modifier element to the acceptable dataset levels at which it may be applied.
  - The *cardinality*: Defines the multiplicity (or cardinality) of a modifier element's relationship with a dataset level.
- *Declarative section*, describing:
  - The *modifier scope*: Used to declare the dataset level or reporting group[1] to which a modifier element is applied.

[1]The term 'reporting group' stems from the ISO Standard family on geographic in-

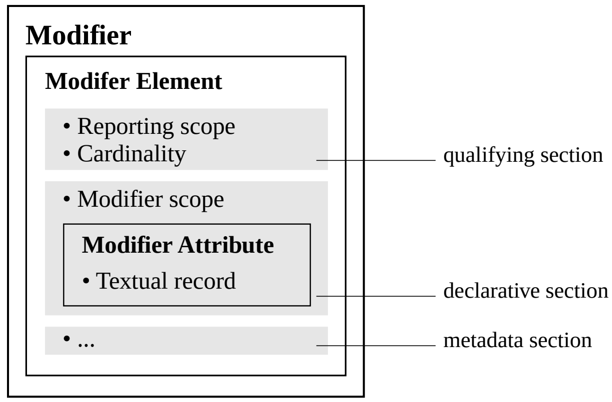


Figure 6.1: A modifier and its parts.

 - The *modifier attribute*: Description field containing one or more entries that define and report the individual attributes of a modifier element. Modifier attributes thus form the core part of modifier elements. A modifier element may comprise one or more modifier attributes.

- *Metadata section*: A modifier element itself may be attributed further metadata either to provide documentation of the modifier element (for instance, the rationale for its creation or its originally envisioned application) or to report its consistency.

A Modifier Example: The Abstraction Modifier

Modifier An **abstraction modifier** (cf. appendix C.1.1) is designed as a modifier that supports or informs the linkage between the real terrain and the abstraction, thus exposing a DTM's UoD, reference model, rules, and spatial concepts[2].

Modifier Element An **abstraction modifier** comprises the modifier element **abstractive information** (see table 6.1), which, basically, is a collection of modifier attributes recording a model's semantic distance from

formation (ISO/TC211 1999) and denotes a collection of data located within a (possibly virtual) dataset sharing common aspects. After ISO/TC211 (1999) common aspects may include data collection criteria or a shared original source.

[2]An **abstraction modifier**, in this sense, is very similar to the concept of "abstractive uncertainty" introduced by Duckham (1999, 2001).

perceived reality resulting from the process of abstraction. **Abstractive information**, hence, addresses *abstraction uncertainty* (discussed in sections 5.1.2 and 5.1.4). As abstraction uncertainty is, crucially, effective on a model level, the *reporting scope* of **abstractive information** is set to the entire, possibly virtual, dataset, thus ensuring that the entire dataset is based on the same abstraction. Consequentially, its *cardinality* is set to 1 (that is, the modelled terrain shall have only one abstraction). The modifier element **abstractive information** may be attributed further *metadata*, such as:

- **Lineage** (cf. appendix C.2.1), particularly, **source**, **domain characteristics**, and **update**, which serve to document the modifier element's history.

- **Conformance** (cf. appendix C.3.2), particularly, **completeness**, which reports on the completeness and comprehensiveness of the **abstractive information**.

- **Structural integrity** (cf. appendix C.3.2), which documents the consistency of the modifier element.

Modifier Attribute The modifier element **abstractive information** comprises three modifier attributes (see table 6.1), namely:

- **Abstraction**, which is a textual description of:

  - The *modelled TI* in terms of its properties, its conceptualisation (spatial, functional, etc.), and of its relationships.
  - The modelling *scale*.

  The attribute **abstraction** records the modelled TI's distance to perceived reality resulting from the process of abstraction[3]. It basically exposes the *reference model*, combined with elements of the *rules* (for a detailed discussion of the concept of rules, see section 4.3).

- **Information base**, which is a textual exposition of:

  - The *body of knowledge* input to the terrain reconstruction, as well as the *assumptions* it is based upon.
  - The *thematic resolution* (expressed by the 'amount' of TI modelled, that is, the number of TI structures conceptualised) and hence the model's (semantic) expressiveness.

[3]It thus lends from the notion of an *"abstraction modifier"* as proposed by CEN/TC287 (1996).

  - The *degree and magnitude of detail* displayed (as implied by the modelling scale).

  In this sense, the **information base** may be interpreted as a subset of the *rules* (section 4.3).

- **Dictionary**, which is a textual definition of all the terms used in the abstraction and specification steps. It must provide sufficiently rich definitions and examples so that there will be no ambiguity concerning the meaning of the terms used.

Examples and selected details on the modifiers listed in table 6.1 are provided in appendix C.1.

6.1.2 Descriptors

Descriptors are metadata elements providing *informative* data comprehensively explaining the TI with regard to, for instance, its history, its expressiveness, its purpose, or its usage. Descriptors are grouped into *descriptor elements* which, in turn, comprise one or more *descriptor attributes* (see table 6.2). The parts comprising a descriptor element provide the mechanism for reporting descriptors.

Table 6.2 summarises the descriptors and their associated descriptor elements that were identified as necessary to provide comprehensive TI documentation on an *informative* level. The parts comprising a descriptor element are illustrated in figure 6.2 and discussed in more detail in the next section. Appendix C.2 provides selected details and examples on the above proposed descriptor elements.

Parts Comprising a Descriptor Element

- *Qualifying section*, restricting:
  - The *reporting scope*: Descriptor elements are only meaningful when referring to specific dataset levels (for an explanation of dataset levels cf. section 6.2.1). The reporting scope is used to restrict the scope of a descriptor element to the acceptable dataset levels at which it may be applied.
  - The *cardinality*: Defines the multiplicity (or cardinality) of a descriptor element's relationship with a dataset level.
- *Declarative section*, describing:
  - The *descriptor scope*: Used to declare the dataset level or reporting group to which a descriptor element is applied.

Descriptor	*Descriptor element*
Lineage	Source Domain characteristics Survey Preprocessing Transformation Conversion Update
Purpose descriptor	Purpose
Usage descriptor	Usage
Validity	Semantic validity Validity across scales Temporal validity
Model expressiveness	Modelling scale Model explicitness Thematic resolution Domain consistency properties
Technical quality descriptor	Technical quality

Table 6.2: Descriptor and associated descriptor elements.

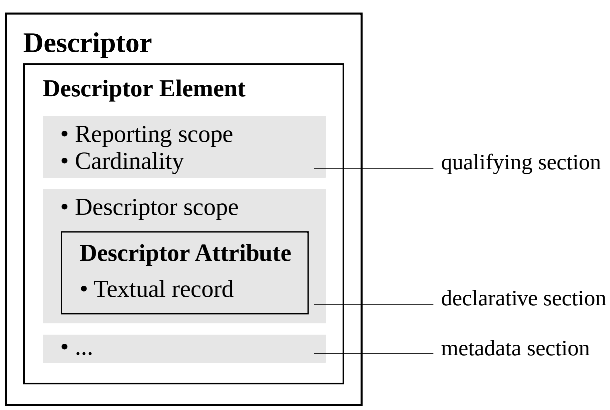


Figure 6.2: A descriptor and its parts.

- The *descriptor attribute*: Description field containing one or more entries that define and report the individual attributes of a descriptor element. Descriptor attributes thus form the core part of descriptor elements.

- *Metadata section*: A descriptor element itself may be attributed further metadata, mainly to provide documentation of its consistency.

A Descriptor Example: Model Expressiveness

Descriptor The descriptor **model expressiveness** (cf. appendix C.2.5) is designed as a collection of descriptor elements aiming at documenting the properties of the provided TI which critically impact on the DTM's expressiveness.

Descriptor Element The descriptor **model expressiveness** comprises the descriptor elements **modelling scale**, **model explicitness**, **thematic resolution**, and **domain continuity properties** (see table 6.2). All of them may be attributed further *metadata*, such as **conformance** (cf. appendix C.3.2), and particularly **completeness**, which checks whether there are any mandatory entries missing within the provided **model expressiveness** elements and their associated descriptor attributes.

Descriptor Attribute The descriptor attribute of the element **modelling scale** consists of a textual description of:

- The DTM's *scale*, and

- the degree and details of the DTM *generalisation* (hence indicating the degree of detail represented).

The descriptor attribute of the element **model explicitness** documents, for instance, the ratio of explicitly represented to implicitly derived TI.

Thematic resolution embraces a descriptor attribute, which is a textual recording of the variety of topographic structures that are explicitly modelled.

The descriptor attribute of the element **domain continuity properties**, finally, provides a textual description of the geometry for which TI of a given property type is available (slope angle from a gridded DTM, for instance, may be available across the entire model, at the grid points only, or along the original contours).

Examples and selected details on the descriptors listed in table 6.2 are provided in appendix C.2.

6.1.3 Quality Aspects

Quality elements are metadata components which measure the actual performance of the provided data (TI, in the present case). Quality elements may be used by TI producers to measure how well the provided TI or its underlying models meet the criteria set forth in their specification.

Quality elements are recognised aspects of metainformation which can somehow be measured. A quality subelement is a quality element component describing a certain facet of that quality element[4] (figure 6.3). The parts comprising a quality subelement provide the mechanism for reporting quality information.

Table 6.3 summarises the quality aspects, their quality elements and associated quality subelements required to comprehensively report the performance of a DTM and derived TI with respect to the criteria set forth in their specification. The parts comprising a quality subelement are illustrated in figure 6.3 and discussed in more detail in the next section. The design proposed for the quality elements and their associated quality subelements is based on the structure for *"quality elements"* suggested by the ISO Standard 19113 Geographic Information - Quality Principles (ISO/TC211 1999). Appendix C.3 provides selected details and examples of the quality elements and their associated subelements listed in table 6.3.

Parts Comprising a Quality Subelement

- *Qualifying section*, describing:
  - The *reporting scope*: Quality subelements are only meaningful when referring to specific dataset levels (cf. section 6.2.1). The reporting scope is used to restrict the scope of some quality subelement to the acceptable dataset levels at which it may be applied.
  - The *metric scope*: Some quality subelements may only be meaningful when the information to which they refer is of a particular type or metric. By means of the metric scope the scope of a quality subelement can be restricted to defined types or metrics (Duckham 1999).
  - The *cardinality*: Defines the multiplicity (or cardinality) of a quality subelement's relationship with a dataset level.
- *Declarative section*, describing:
  - The *quality scope*: Used to declare the dataset level or reporting group to which a quality subelement is reported.

[4]That is, quality elements may be understood as quality subelement collections.

Quality aspect		*Quality element*	*Quality subelement*
Representative quality	*Accuracy elements*	Metric attribute accuracy	Absolute accuracy Relative accuracy Polyhedral value accuracy ...
		Non-metric attribute accuracy	Value-typed attribute accuracy Classification accuracy ...
		Shape fidelity	Slope fidelity Curvature fidelity Drainage structure fidelity ...
	Information content	Supporting information content	Randomness Dimensionality Sampling resolution and pattern Redundancy Information elements provided
		Information Substance	Scale Reported precision Data persistence Support
Consistency		Structural integrity	Logical integrity Topological integrity Functional integrity Temporal integrity
		Formal correctness	Formal consistency Data structure consistency Domain consistency
		Conformance	Information persistence Operationalisation consistency Encoding consistency Specification consistency Format consistency Completeness

Table 6.3: Quality aspects, quality elements and their associated quality subelements.

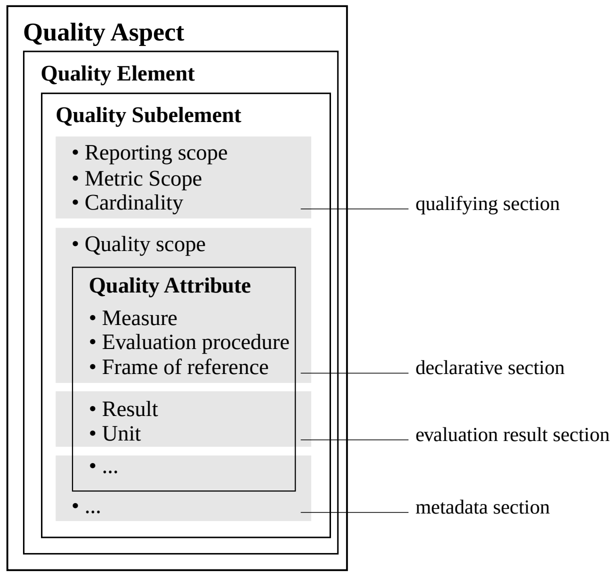


Figure 6.3: A quality subelement and its parts.

- *Metadata section*: A quality subelement itself may be attributed further metadata, mainly to provide *lineage documentation*.

Quality subelements are attributed one or more *quality attributes*, which themselves are made up of a declarative section, an evaluation result section, and a metainformation section (figure 6.3):

- *Quality attribute - declarative section*, defining:
  - The *measure*: The quality measure may name and describe briefly the type of test or evaluation being applied to the quality scope.
  - The *evaluation procedure*: Designed to describe or reference documentation describing the methodology used to apply a measure to a quality scope.
  - The *frame of reference*: Identifies the frame of reference chosen for applying a measure to the declared quality scope.
- *Quality attribute - evaluation result section*, reporting:
  - The *result*: Value or set of values resulting from applying a quality measure to a quality scope.
  - The *unit*: Specifies the type of value used to report the result.

- *Quality attribute - metadata section*: A quality attribute itself may be attributed further metadata documenting or assessing its *consistency* (in particular, the consistency and adequacy of its measure and evaluation procedure).

A Quality Element Example: Shape Fidelity

Quality Element **Shape fidelity** (cf. appendix C.3.1) is designed as a collection of quality subelements that serve to assess the similarity in the shape of some digitally represented linear or areal TI with a reference shape accepted as true. In figure 6.4, for instance, the contour picture (bottom right) indicates a similar surface shape for the terrains reconstructed by cubic Bézier splines applied over a constrained triangulation and by bilinear interpolation of regularly spaced data (top row). Whereas the terrain built using cubic Bézier splines over a Delaunay triangulation (bottom left) shows a different surface shape, particularly along the (apparently interrupted) drainage channel in the left half of the picture.

Quality Subelement Examples for quality subelements of **shape fidelity** include **slope fidelity, curvature fidelity, or drainage structure fidelity** (cf. table 6.3). All of them may be attributed further *metadata*, such as **lineage** (cf. appendix C.2.1) and particularly **source**, to document the subelement's history.

Quality Attribute Subelements of **shape fidelity** may, for instance, be assessed by comparison of surface networks (Pfaltz 1976, Wolf 1990). So, a *measure* for **curvature fidelity** may be defined as the existence, the number or percentage of deviating nodes and edges when comparing the surface network derived from the DTM to be assessed with the surface network computed from the 'true surface shape'. Another measure of **curvature fidelity** could be the degree of topological similarity or geometric conformance of the outlines of convex and concave regions derived from the terrain to be evaluated and the 'true surface', respectively (see figure 6.5).

Drainage structure fidelity may, for instance, be assessed by investigating the degree of similarity or conformance of drainage network topologies. Other measures for **drainage structure fidelity** include the degree of similarity or conformance (on a geometrical level) of catchment areas computed from the DTM to be assessed and from the 'true terrain surface', respectively.

To such quality attributes, further *metadata*, such as **structural integrity**, **formal correctness** or **conformance** (appendix C.3.2) can be associated, mainly to report on the consistency of the *quality evaluation*

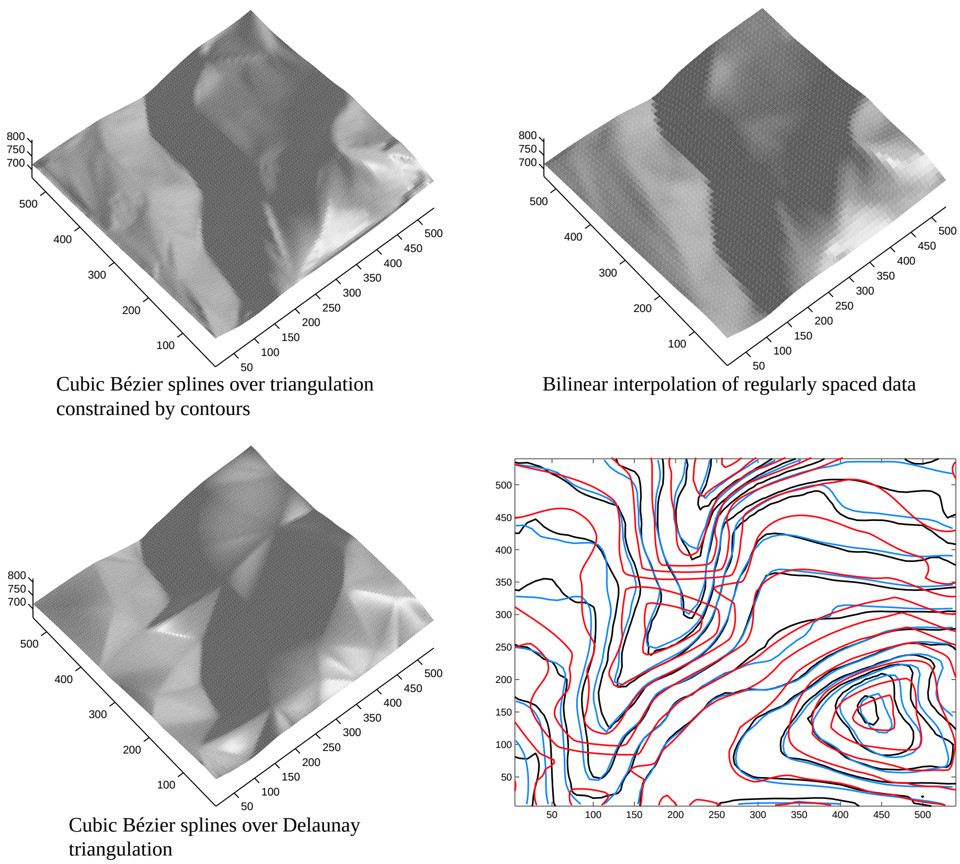
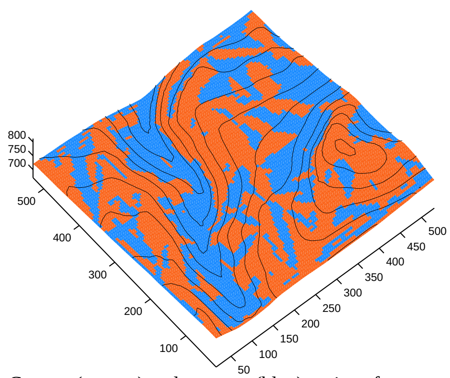
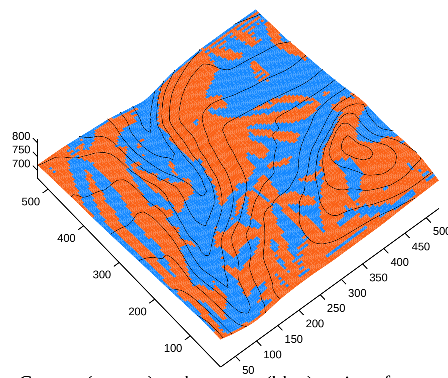


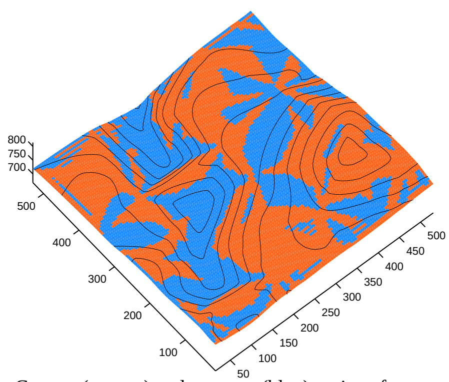
Figure 6.4: Shape fidelity. The bottom right picture shows the contours (with 10 m contour intervals) for the cubic Bézier spline surface specified over a constrained triangulation (black), the bilinearly interpolated regularly spaced data (blue), and the cubic Bézier surface specified over a Delaunay triangulation (red).



Convex (orange) and concave (blue) regions from
cubic Bézier splines over constrained triangulation



Convex (orange) and concave (blue) regions from
bilinearly interpolated regularly spaced data



Convex (orange) and concave (blue) regions from
cubic Bézier splines Delaunay triangulation

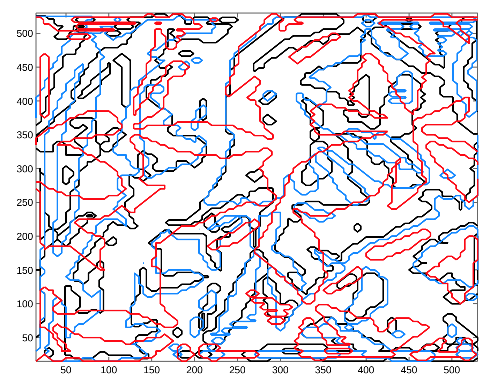


Figure 6.5: Curvature fidelity. The bottom right picture shows the borders between the convex and concave regions (in black for the case of a cubic Bézier spline surface specified over a constrained triangulation; in blue for the case of bilinear interpolation of regularly spaced data; and in red for the cubic Bézier surface specified over a Delaunay triangulation).

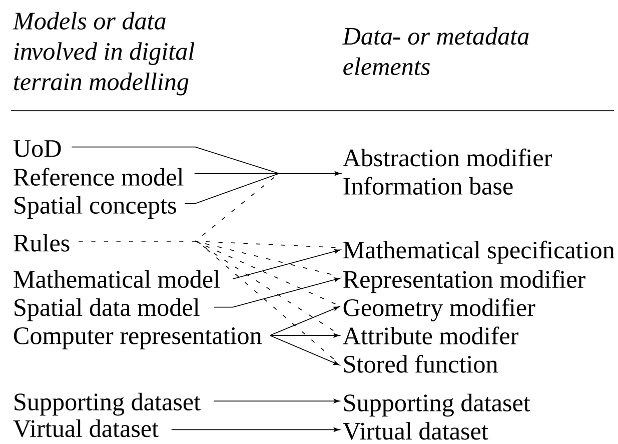


Figure 6.6: Mapping of the models and data involved in the digital terrain modelling process to data- and metadata elements. The straight black lines visualise the actual mappings, which are themselves to some degree guided by the rules, as indicated by the dashed lines.

procedures. That is, they can be used to describe the methods utilised to apply a *quality measure*.

6.2 Reporting Principles

6.2.1 Hierarchical Reporting

DTM Structure

The discussion presented in sections 2.3.1, 2.3.3 and 4.3.3 identified the following models involved in a digital terrain modelling sequence (shown in the first column of figure 6.6): UoD, reference model and spatial concepts, mathematical - and spatial data model, computer representation, supporting dataset, and finally the virtual dataset. By means of the metadata components proposed in the last section, these steps in the modelling sequence can be mapped to either the (ordinary) dataset or the metadata set (completing the ordinary dataset to form a DTM conforming to definition 4.2). These are the data and metadata elements appearing in the second column of figure 6.6, which potentially act as modifier, descriptor or quality scopes.

Taking an 'atomistic' view, supporting and virtual datasets may be broken down further. A supporting *dataset*, then, may be understood as

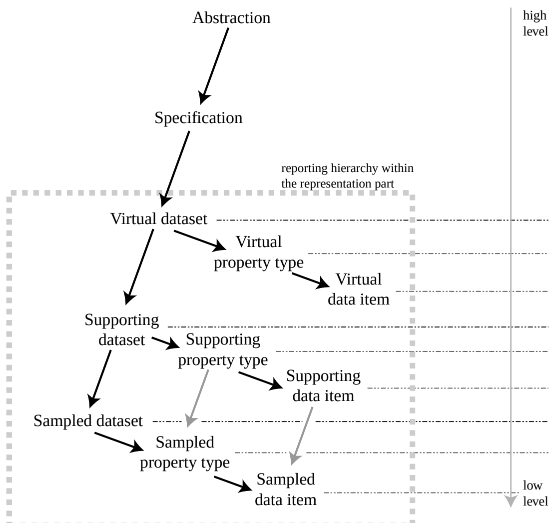


Figure 6.7: Dataset levels or reporting hierarchy.

composed of supporting *property types*[5] which are, in turn, collections of supporting *data items* (or supporting *properties*, single occurrences in any case); virtual datasets may be decomposed analogously. In view of reporting metainformation, a dataset can then be structured into levels as depicted in figure 6.7. The distinction between supporting and sampled data in figure 6.7 is made in view of the discussion in chapter 7 and is intended to mean the following: While *sampled data* exclusively consists of sampled elements such as spot heights, *supporting data* may comprise sampled and a reconstructed components, such as contours built from a sampled point sequence by interpolation. *Virtual data*, finally, may be purely virtual (such as slope angles derived from an elevation field). Figure 6.7 may also be denoted a *reporting hierarchy* (as described in the next section).

The Hierarchical Principle for Reporting Metainformation

The amount of metadata being recorded is dependent on the number of modifier, descriptor and quality scopes deemed applicable. To address the problem of redundancy of metadata, it is suggested to report metainformation in accordance with the *hierarchical reporting principle* presented in ISO/TC211 (1999).

[5]Remember, property types denote variables mapped to a digital representation (as discussed in section 2.3.3).

According to this principle, metainformation is whenever possible reported at a virtual dataset level (or at the highest possible dataset level). Metainformation for further dataset levels or reporting groups is typically reported only when it differs from metainformation already reported at a higher level. Hence, metainformation reporting starts at the uppermost possible level and then works its way down the reporting hierarchy depicted in figure 6.7.

6.2.2 Two-fold Metainformation Reporting for Virtual TI

The idea behind defining a DTM as a VDS (definition 4.2 in section 4.3) was to enable a TI user to obtain the TI requested at the location needed by supplying virtual data for the properties or locations not sampled. Consequently, a basic body of metainformation describing the same detail for both supporting and virtual data must be provided[6]. However, the quality of virtual or derived TI that is made available through a stored function is determined by both:

- The quality of the supporting data it is derived from, and
- the quality of the derivation or stored function applied.

Comprehensive quality description, therefore, must not only include assessment of the derivation result (i.e., of the virtual TI), but also evaluation of the stored function applied. To this end, the *stored function descriptors* exposed in appendix C.2.7 were introduced. Stored function descriptors document the reliability and performance of stored functions in terms of their numerical and structural stability and of the artifacts that may have been caused.

While the concepts of artifacts and numerical stability probably do not need further discussion, the importance of *structural stability* to VDSs in general, and DTMs in particular, is perhaps worth further explanation. The concept of structural stability basically says that in the absence of convincing reasons to the contrary, 'good' models of physical form or process should be *stable* (or *robust*) with respect to small disturbances in their input quantities (Casti 1992)[7]. Mathematically, such small disturbances

[6]For instance, an elevation value shall be attributed the same kind of accuracy information no matter if it was sampled or interpolated. The distinction between sampled and interpolated data, in this case, may be provided by the subelement **data persistence** of the proposed representative quality element **information substance** (cf. appendix C.3.1).

[7]*Structural stability* is an important concept within the philosophy of science in general. The notion of *replicability* which says that the same experiment must give the same result under the same conditions, is a fundamental concept in modern science. However, in the form stated above, the idea is just an ideal one, because it is never possible to generate exactly the same conditions by abandoning all external factors even in the most carefully

are modelled by perturbation functions. Structural stability, then, is the insensitiveness of the mapping or family of mappings describing the model of physical form or process to these perturbation functions. Applied to digital terrain modelling, the concept of structural stability implies that questions such as "is a family of functions mathematically reconstructing a terrain surface insensitive to measurement uncertainty and thus structurally stable?" have to be answered (Wolf 1991).

To illustrate the impact, a very simple example lent from Wolf (1991) is provided[8]. Consider first the functions $f(x) = x^2$ and $f_p(x) = x^2 + \varepsilon x$, with εx representing a perturbation function. For the derivatives of $f(x)$ and $f_p(x)$, it holds:

$$\begin{aligned} f(x) &= x^2 & \qquad f_p(x) &= x^2 + \varepsilon x \\ f'(x) &= 2x & f_p'(x) &= 2x + \varepsilon \\ f''(x) &= 2 & f_p''(x) &= 2. \end{aligned}$$

By setting the first derivatives to zero, solving the resulting equations with respect to x and examining the second derivatives, the *critical points* of the two functions are obtained:

$$\begin{aligned} &2x = 0, \text{ thus } x = 0 & \qquad &2x + \varepsilon = 0, \text{ thus } x = -\tfrac{\varepsilon}{2} \\ &f''(0) = 2, \text{ thus } f''(0) > 0 & &f_p''(-\tfrac{\varepsilon}{2}) = 2, \text{ thus } > 0. \end{aligned}$$

These calculations indicate that the perturbation function moves the resulting minimum from $x = 0$ to $x = -\frac{\varepsilon}{2}$ in a way depending smoothly on ε, while ε is an arbitrary small number. However, the type of the critical point (minimum) as well as the structure of the graph of $f(x)$ in a surrounding of $x = 0$ are not affected by the perturbation (figure 6.8). Therefore, the function f is *structurally stable* at $x = 0$.

Next, consider the functions $g(x) = x^3$ and $g_p(x) = x^3 + \varepsilon x$, with εx representing again a perturbation function. For the derivatives of $g(x)$ and $g_p(x)$, it holds:

$$\begin{aligned} g(x) &= x^3 & \qquad g_p(x) &= x^3 + \varepsilon x \\ g'(x) &= 3x^2 & g_p'(x) &= 3x^2 + \varepsilon \\ g''(x) &= 6x & g_p''(x) &= 6x. \end{aligned}$$

Determining the critical points of $g(x)$ and $g_p(x)$ leads to:

$$\begin{aligned} &3x^2 = 0, \text{ thus } x = 0 & \qquad &3x^2 + \varepsilon = 0, \text{ so } x_{1,2} = \pm\sqrt{\tfrac{-\varepsilon}{3}} \\ &g''(0) = 0 & &g_p''(\pm\sqrt{\tfrac{-\varepsilon}{3}}) = \pm 6\sqrt{\tfrac{-\varepsilon}{3}}\ . \end{aligned}$$

designed experiment (Wolf 1991) - needless to mention the difficulties the problem of replicability poses to a science like geography. To address the problem, the idealised concept has been weakened by tolerating small changes in the conditions under which an experiment is carried out provided these changes do not affect the result significantly. To rephrase it, "what we really expect is not that if we repeat the experiment under precisely the same conditions we will obtain precisely the same results, but rather that if we repeat the experiment under approximately the same conditions we will obtain approximately the same results. This property is known as structural stability ..." (Saunders 1982).

[8]For the sake of simplicity, the example is kept restricted to functions of a single variable.

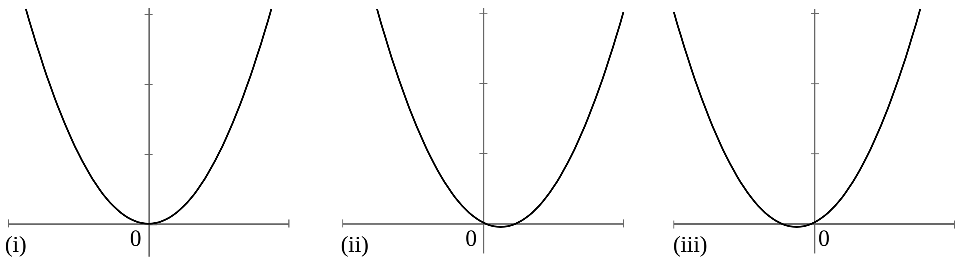


Figure 6.8: Graphs of the function $f_p(x) = x^2 + \varepsilon x$ for (i) $\varepsilon = 0$, (ii) $\varepsilon < 0$ and (iii) $\varepsilon > 0$.

These calculations yield an interesting result: While the function $g(x)$ has a degenerate critical point a $x = 0$, $g_p(x)$, which is obtained from $g(x)$ by adding the perturbation term εx, has no critical points for positive ε but has two critical points for negative ε, namely a local minimum at $x_1 = \sqrt{-\frac{\varepsilon}{3}}$ and a local maximum at $x_2 = -\sqrt{-\frac{\varepsilon}{3}}$. Therefore, the function g shows an *irregular* and *unstable* behaviour. Figure 6.9 depicts graphs of the function $g_p(x) = x^3 + \varepsilon x$ for different values of ε.

For the purposes of digital terrain modelling, a reconstruction function showing a sensitive behaviour to small disturbances or uncertainties in the supporting data like the above example function g is rather un-edifying. However, it is undoubtedly critical to a TI user to know whether or not the stored functions applied are structurally stable. Therefore, documentation of stored functions is an important principle when providing (comprehensive) metainformation for virtual TI.

6.2.3 Metainformation on Metainformation

Section 4.3.3 identified digital terrain modelling as a workflow process not always easy to keep track of. Likewise, getting a general idea of all factors actually emerging in the modelling process and affecting the quality of the finally produced TI is not an easy task. To address this problem, the quality component **consistency** (discussed in appendix C.3.2) as well as most of the descriptors (explained in detail in appendix C.2) are designed to be applicable to both data and metadata. The potential of these metadata elements to be applied to other metadata elements, particularly to the modifiers **abstraction modifier** and **specification modifier** (explained in appendix C.1), allows for assessment and documentation of the resulting 'intermediate modelling products' at different levels of abstraction. Like-

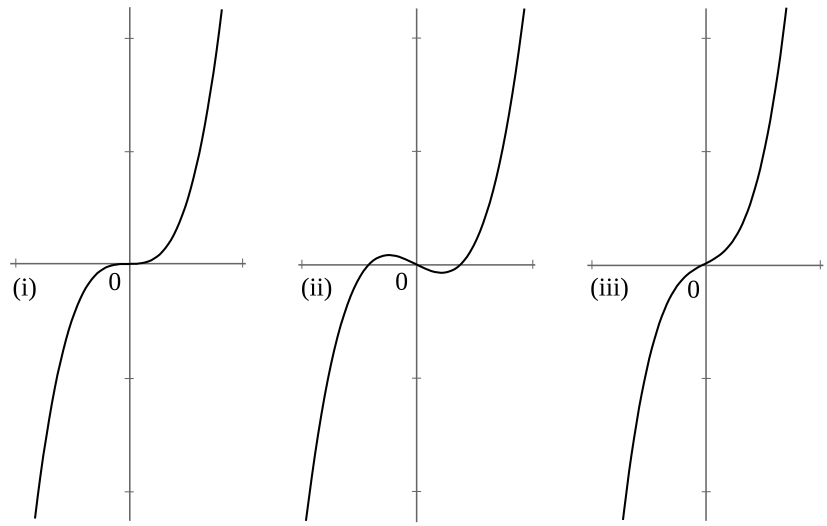


Figure 6.9: Graphs of the function $g(x) = x^3 + \varepsilon x$ for (i) $\varepsilon = 0$, (ii) $\varepsilon < 0$ and (iii) $\varepsilon > 0$.

wise, the transitions between them during the progression of the modelling effort may be evaluated. The principle of enabling reporting of metainformation on metainformation thus provides a useful tool of *quality assurance*.

6.3 Relation to the ISO Standard 19113

The concepts reported in this chapter are, basically, designed to extend the concepts provided by the ISO Standard family on geographic information, particularly the Standard part 19113 *Geographic Information - Quality Principles* (ISO/TC211 1999). The attempt to arrange this work in a fashion somehow compatible to the ISO initiative was based on the expectation that, amongst all the data quality standards recently presented (for an overview, see section 3.1), the ISO contribution is the most likely one to carry through - especially thanks to its close collaboration with the OpenGIS Consortium. Correspondence between the concepts outlined in this chapter and the ones presented by the ISO Standard family include:

- The quality models underlying both attempts are compatible.

- The concept of the metadata component *quality (element)* may be identified with the concept of *quality elements* suggested by the ISO

Standard 19113. Likewise, the concept termed *descriptor* here may be interpreted as *quality overview element* according to ISO 19113.

- The *principle of hierarchical reporting* of metainformation is lent from ISO 19113.

Despite the close correspondence, the concept presented in this chapter extends the ISO Standards by providing four substantial improvements:

- Aspects of quality *absent* in the Standards are included, namely:
  - Measures of a dataset's *information content* (given by the representative quality elements **supporting information content** and **information substance** (appendix C.3.1), as well as by the descriptor **model expressiveness** (appendix C.2.5));
  - descriptors for reporting *technical quality*; and
  - a structure to support the concept of *validity*.
- The concept of *modifiers* is introduced to enable integration of the abstraction and (product) specifications into the dataset. The processes of abstraction and specification essentially influence the semantics of the resulting data. Therefore, their recounting is inevitable for comprehensive TI documentation.
- Concepts, structures and reporting principles for recording metadata are provided describing not only supporting data but also *virtual data*. This contribution is crucial to the reporting of metainformation documenting field-type phenomena such as terrain.
- A tool for *quality assurance* is added by allowing reporting of metainformation on metainformation.

It must be pointed out however, that the focus of the two works is somehow different. In this chapter, the metadata components essential for exhaustive documentation of TI are stressed, rather than where and how the metadata is to be stored. The latter question is even left open intentionally to avoid limitations in a metadata set's expressive range due to the inflexibility inherent in any standard.

6.4 Review

This chapter presented *metadata components* necessary for comprehensive TI quality description as well as basic *principles for TI quality reporting*. As far as the metadata components are concerned, the focus was definitely on comprehensiveness rather than on orthogonality. In other words, the elements of metadata exposed are likely not to be orthogonal. **Information substance**, particularly its subelement **scale**, for instance, clearly impact on **metric attribute accuracy**. However, comprehensiveness is deemed as more important than orthogonality.

It is a TI producer-oriented perspective on reporting metainformation reflected in this chapter and in appendix C. The definition and grouping of the elements of metadata suggested in appendix C is, to a great extent, guided by the TI production process. The next chapter adopts a more user-oriented view. It presents a conceptual model for metadata storage, where the metadata elements are grouped according to the kind of information provided to the TI user.

Chapter 7

The Pluggable Terrain Module

In this chapter, the concept of the *Pluggable Terrain Module* (PTM) is introduced. Basically the PTM attempts to

- *realise the concepts enabling reliable digital terrain modelling* discussed in chapter 4,
- and *store metadata* documenting TI quality by means of the metadata components proposed in section 6.1 and presented in more detail in appendix C.

The chapter begins by discussing the motivation to propose the PTM concept. Section 7.2 outlines the advantages of a modular approach to realise the concepts for reliable digital terrain modelling suggested in chapter 4 and to store an extensive set of metadata documenting TI conforming to the quality principles proposed in chapter 6. The core part of the chapter is then dedicated to the presentation of the internal PTM design.

7.1 Motivation

Reliable DTM application means that the TI derived must be replicable and consistent (cf. sections 1.4.2 and 4.1). Section 4.1 argued that consistency between different types of TI can not be guaranteed, unless the TI is derived from an explicitly pre-specified *terrain reconstruction*. Consequently, it presents a challenge to reliable digital terrain modelling to provide an explicit and suitable reconstruction of the topographic surface. Yet, DTMs are mostly represented as discrete datasets lacking explicit handling of the terrain's continuous nature.

DTMs always (implicitly) model at a certain *scale range* which is determined by the scale range of the sampled data supporting the terrain representation. This scale range is implicitly assigned to all TI derived from the DTM (section 4.3.1). Scale is manifested by the degree of detail displayed and the degree of detail neglected, the latter always being a cause of uncertainty. Scale, thus, critically affects TI quality (section 5.1.2). Yet, scale is usually not considered by current approaches to TI quality documentation (notable exceptions are provided in Ackermann (1980), Tempfli (1980), Li (1993b)). Also, tools for DTM analysis with consideration of scale are usually not provided.

The importance of *TI quality* issues to spatial modelling applications and spatial decision making is theoretically well known. So, for instance, the sensitivity of TI to be derived from digital terrain representations to the uncertainty in the supporting data has been investigated (Fisher 1992, Lee et al. 1992, Veregin 1997). Yet, most DTMs do not provide the tools for inclusion of extensive metainformation. This problem involves two aspects:

- The mappings involved in DTM generation are poorly documented. In other words, documentation of the *abstraction* modelled by a DTM and usually also its *specification* are most often lacking. Hence, ambiguous DTM interpretation may occur (given, for instance, a grid elevation matrix, it may not always be clear whether a point-grid or a cell-grid is represented, or whether there are any forms of implicit interpolation).

- There exists poor or no documentation on the *algorithms* available for DTM analysis. Hence, no evidence is provided on how the analysis results are affected by the methods used (Bernhard and Weibel 1999, Wise 1998)[1].

7.2 A Modular Approach to Reliable Digital Terrain Modelling

Monolithic GISs do not and can not provide the functionality and structures required to realise the concepts for reliable digital terrain modelling presented in chapter 4, nor can they store an extensive set of metadata documenting TI conforming to the quality reporting principles proposed in chapter 6. Therefore, it follows that it would be more advantageous to process digital terrain representations outside a GIS.

The OpenGIS Consortium presents an approach to distributed information technology – the so-called "Pluggable Computing Model" (Bühler and

[1] Wise (1998), for instance, illustrates how different GIS packages (providing functionality for DTM analysis) and even different algorithms within one GIS package generate significantly different results for the seemingly well-defined problems of generating a gridded terrain representation from contours and subsequent aspect calculation.

McKee 1998) – that forms a perfect foundation for this intention. One major component of this concept is the "Pluggable Tool". For every task in a computing environment a separate Pluggable Tool is implemented, each containing the necessary data and algorithms. All tools posses a well-defined and broadly accepted interface through which they communicate with each other and with other components of the computing environment.

Martinoni and Schneider (1999) used the notion of the Pluggable Tool to develop the concept of the *"Pluggable Terrain Module"* (PTM). Designed as a software module in the sense of the Pluggable Tools concept, the PTM is equipped with a standardised interface through which other software components, for instance a GIS, may query the module's content. The PTM itself is responsible for the access to the data, provides the methods for reliably modelling and analysing the terrain surface, manages quality information, and supplies exhaustive documentation of the TI provided.

The PTM concept addresses the limitations listed in section 7.1 affecting many of today's common DTMs by:

- *Simulating* a *continuous surface* with the help of supporting data and appropriate terrain reconstruction and TI derivation methods, all hidden inside the module. The module is, therefore, in accordance with the notion of a VDS (section 4.3). The PTM thus exhibits the conditions necessary to realise a digital terrain representation conforming to the DTM definition 4.2 given in section 4.3.

- Recognising a terrain representation for being a function of, amongst others, *scale*. To account for this, 'scale-aware' techniques for terrain representation are provided, based, for instance, on wavelet analysis (a topic discussed in chapter 8).

- Designing the module to actually be a *conceptual model for metadata storage* conforming to the principles exposed in the last chapter. Hence, first of all, documentation of both abstraction and specification of the modelled TI is ensured and integrated into the (virtual) dataset. Second, accomplishing the principle of two-fold reporting of metainformation on virtual TI (discussed in section 6.2.2) documentation of the available algorithms is ensured.

7.3 The PTM Design

7.3.1 Storage Model [2]

Analysis of the metainformation comprehensively documenting the TI reveals that it provides three different kinds of information:

- Information on the *quality performance* of the supplied TI;
- *specification information*, specifying data models and structures as well as the stored functions applied;
- *explanatory information* which, on the one hand, exposes the abstraction underlying the modelled TI and, on the other hand, provides informative data about the model in general.

This tripartite structure is carried over to the PTM, which is designed to consist of *three components*, namely of:

- The *property set*,
- the *specification schema*,
- the *identification schema*.

The Property Set

The terrain module basically is a collection of spatial elements, or properties. These so-called *geospatial properties* consist of geometry and possibly also carry non-spatial attributes. Geospatial properties may supply supporting data, in which case they are termed *sampled or supporting geospatial properties* (see section 6.2.1 on the distinction between sampled and supporting data). Availability of geospatial properties at arbitrary locations or for properties not sampled is ensured through persistently stored functions, so-called *virtual geospatial properties*. The storage model for geospatial properties is set up to conform with the reporting hierarchy depicted in figure 6.7. Being a VDS, the PTM is inherently object-oriented. A class diagram for the *geospatial property storage model* is shown in figure 7.1. The graphical notation used consistently throughout this research is based on that of Rumbaugh et al. (1991); the primitives for the class diagrams are shown in figure 7.2.

Geospatial properties possess distinct *quality properties* in terms of what was introduced as **representative quality** and **consistency** in section 6.1.

[2]The actual implementation of the PTM concept is the core task of ongoing research work carried out by Daniel Wirz (Wirz 2001). Therefore, only the *model* for metadata storage is briefly discussed here.

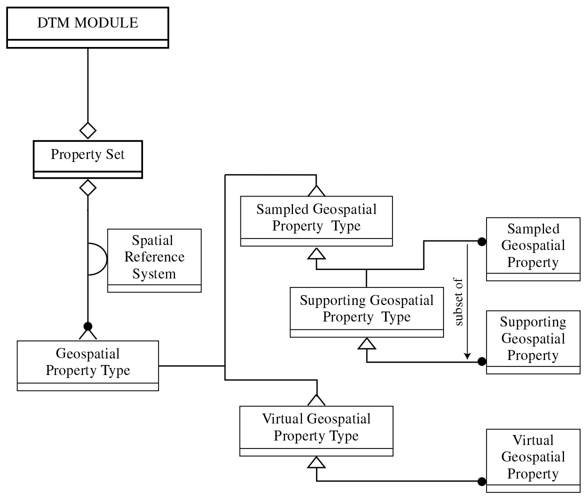


Figure 7.1: Geospatial property storage model. The property set basically is an aggregation of geospatial properties (either sampled and supporting or virtual). The geospatial properties are organised in geospatial property types, that is, classes of geospatial properties with common characteristics (cf. the DTM structure displayed in the reporting hierarchy depicted in figure 6.7).

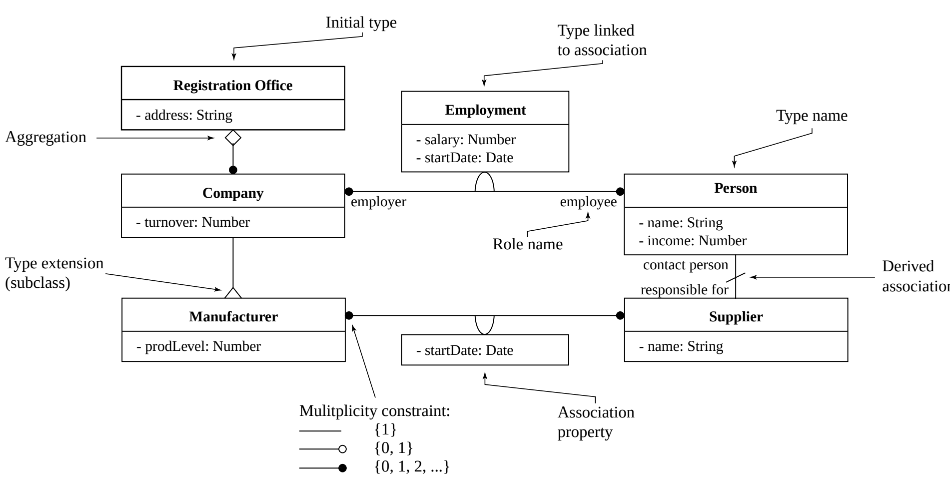


Figure 7.2: Class diagram conventions (after Cook and Daniels (1994)).

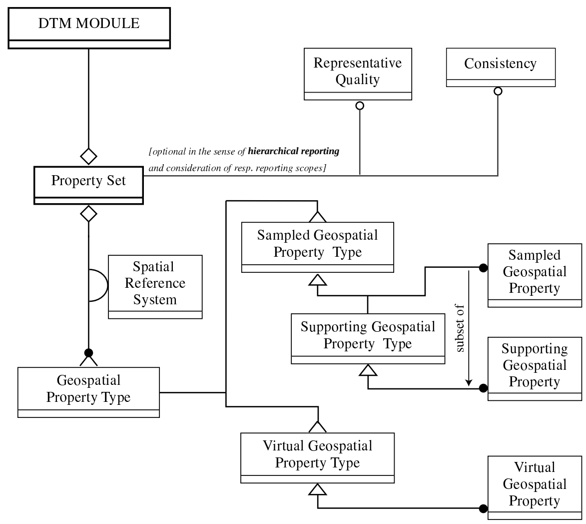


Figure 7.3: Storage model for geospatial properties and their associated quality properties.

In the storage model, therefore, **representative quality** and **consistency** both result in associations of the property set (figure 7.3). The inheritance structure of the class diagram ensures that all geospatial properties (i.e., each individual geospatial data item) carry their own representative quality and consistency information (except, of course, cases where the reporting scopes are restricted to higher dataset levels; for specification of the reporting scopes of the subelements of **representative quality** and **consistency**, see appendix C.3). However, according to the *principle of hierarchical metainformation reporting* presented in section 6.2.1, individual quality subelements at low levels of the reporting hierarchy need to be overwritten only if they differ from the ones reported at higher dataset levels.

Figure 7.4 illustrates the general structure of the class **quality**, the super-class of **representative quality** and **consistency** (cf. section 6.1.3). The class **quality** has one method, **get_quality()**. Since all geospatial properties (i.e., the entire property set) have associated **representative quality** and **consistency** objects, information on representative quality and consistency of each geospatial data item can be assessed by accessing the corresponding overwritten **get_ quality()** method.

Acting at the level of the individual geospatial properties, representative quality and consistency are tightly coupled to the actual data. Therefore,

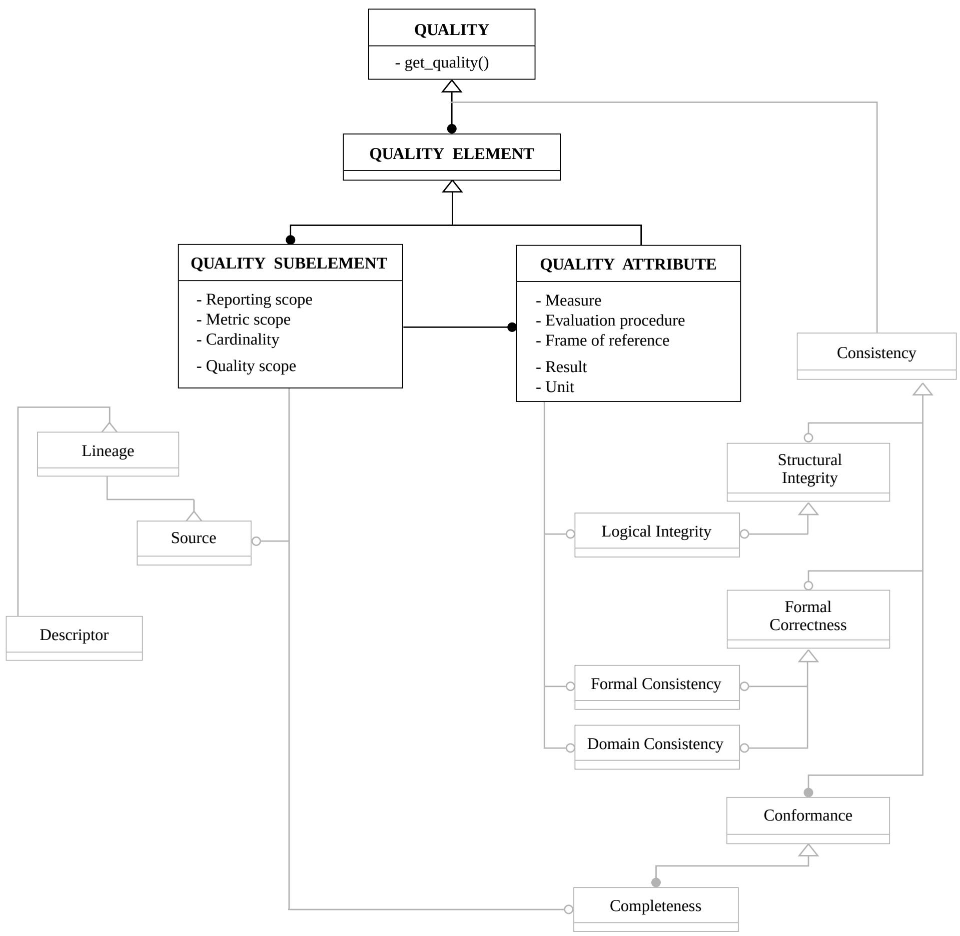


Figure 7.4: Class design for the metadata component **quality**. Portrayed in gray is the metadata that might be used to document a quality subelement's lineage and completeness or to assess the consistency of a quality attribute.

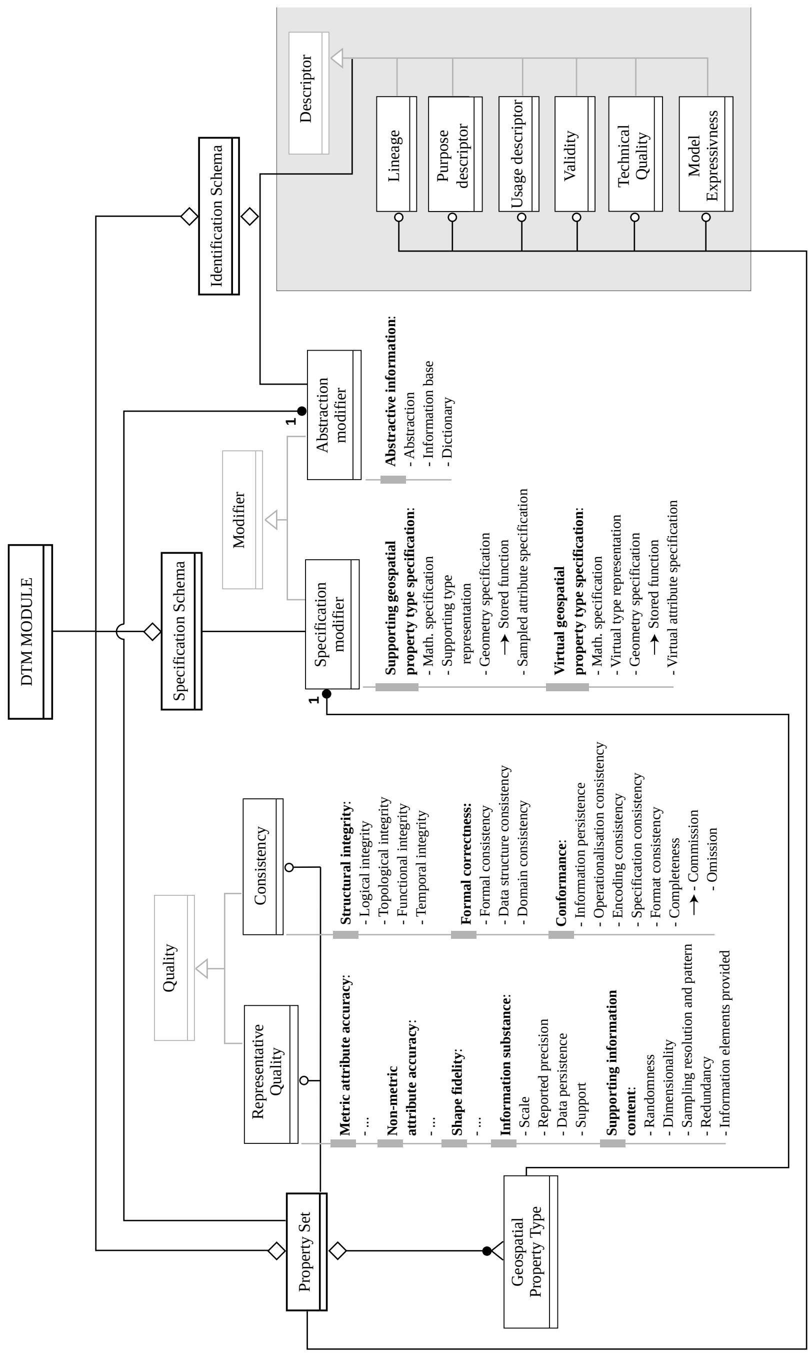


Figure 7.5: Overview of the metainformation content provided in the PTM. For the sake of clarity, a zoom into the the descriptor part of the identification schema (shaded box) is deferred to figure 7.6.

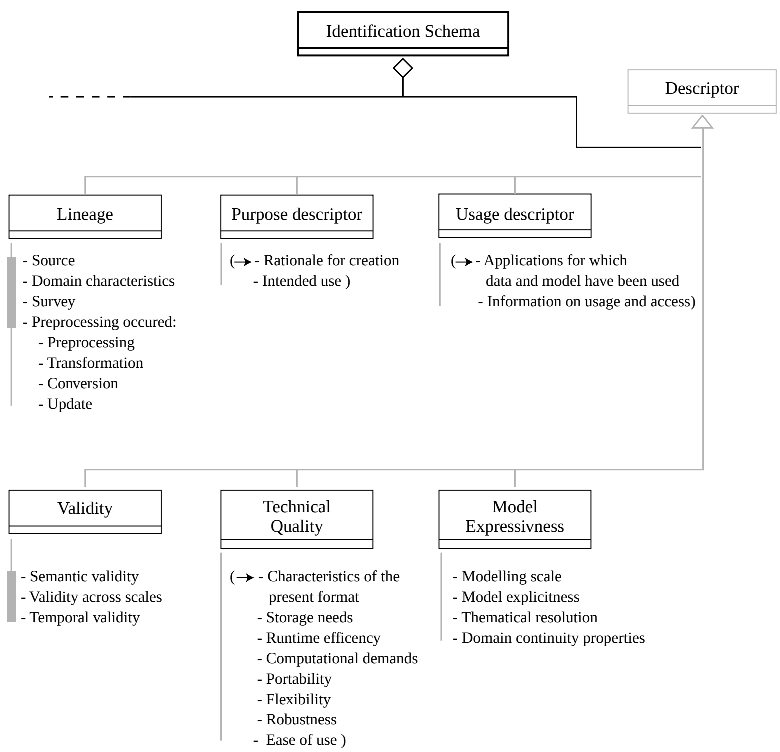


Figure 7.6: Overview of the metainformation supplied by the descriptors aggregated in the identification schema. The figure provides a zoom into the shaded box of figure 7.5.

they are conceptualised as a part of the *property set*, as indicated in figure 7.11. They may be interpreted as *quality properties* providing information on the data's *accuracy*, as well as on its *information content*. Hence, they distinguish the information content of the supporting dataset, supplying the information input available for terrain reconstruction and (virtual) TI derivation, from the information substance of the entire virtual dataset (see the column depicting **representative quality** in figure 7.5).

The Specification Schema

The specification schema exposes the exact specifications of both supporting and virtual terrain data. The metadata component defined to this end in section 6.1 was the modifier **specification modifier**, comprising the modifier elements **supporting geospatial property type specification** and **virtual geospatial property type specification**. They thus make up the specification schema (figure 7.7), providing documentation of a geospatial property type (see also the column illustrating **specification modifier** in figure 7.5):

- *Mathematical specification* (basically documenting its mathematical model);
- *Discretisation scheme* and *spatial data model*, made explicit through the modifier attributes **supporting type representation** or **virtual type representation**, respectively (cf. appendix C.1.2);
- *Geometry*;
- *Non-spatial attributes*, determined by the **sampled attribute specification** or **virtual attribute specification**, respectively, specifying, amongst others, the exact property name/value type pairs.

The specification schema fulfils two tasks. First, it captures the uncertainty introduced through mathematical idealisation and the limits of possible precision, that is, the uncertainty introduced through mapping the continuous real-world phenomenon "terrain" to a finite computer representation (for a detailed discussion of this topic, please be referred to section 5.1.2). Second, information on specification is addressed to TI users who, for some reason, are interested in low-level details of the actual PTM implementation (e.g., because they need to extend the functionality for TI derivation at their disposal). In this sense, it lends from the "feature schema" of the "project world" in Bühler and McKee (1998).

As illustrated in figures 7.8 and 7.9, the class diagram for the modifier **specification modifier** provides *stored functions* to specify continuous geometries and virtual attributes. These stored functions are documented by

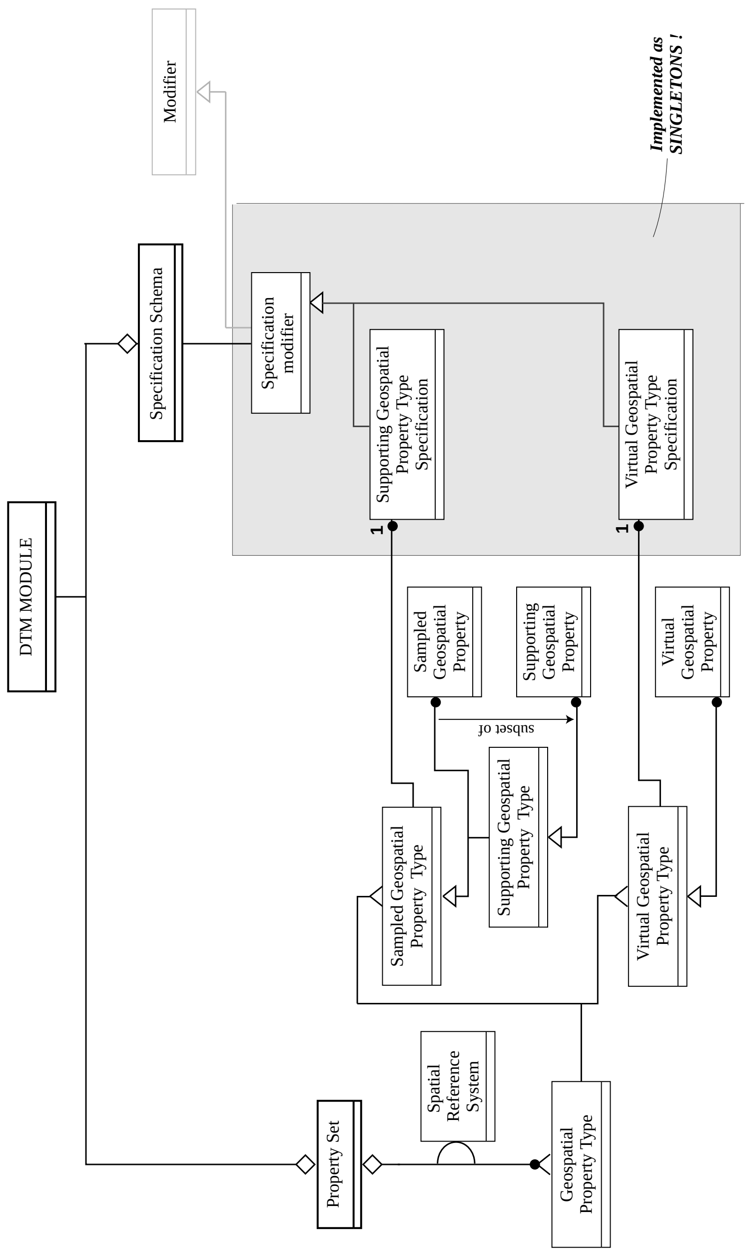


Figure 7.7: Overview of the specification schema. For the sake of clarity, the quality elements discussed in the previous paragraph are neglected in the diagram.

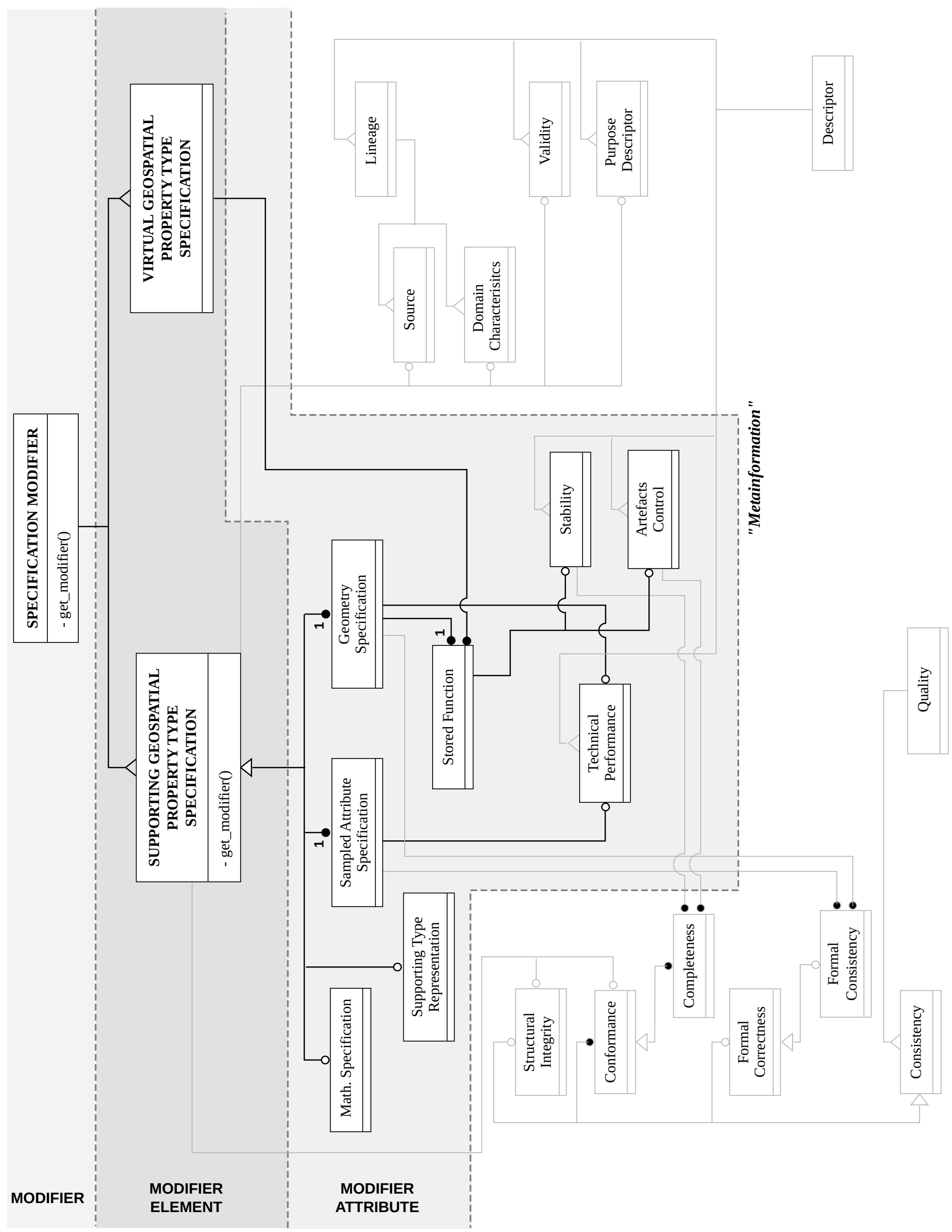


Figure 7.8: Class diagram for the modifier element **supporting geospatial property type specification**. Stored functions are provided to specify continuous geometries.

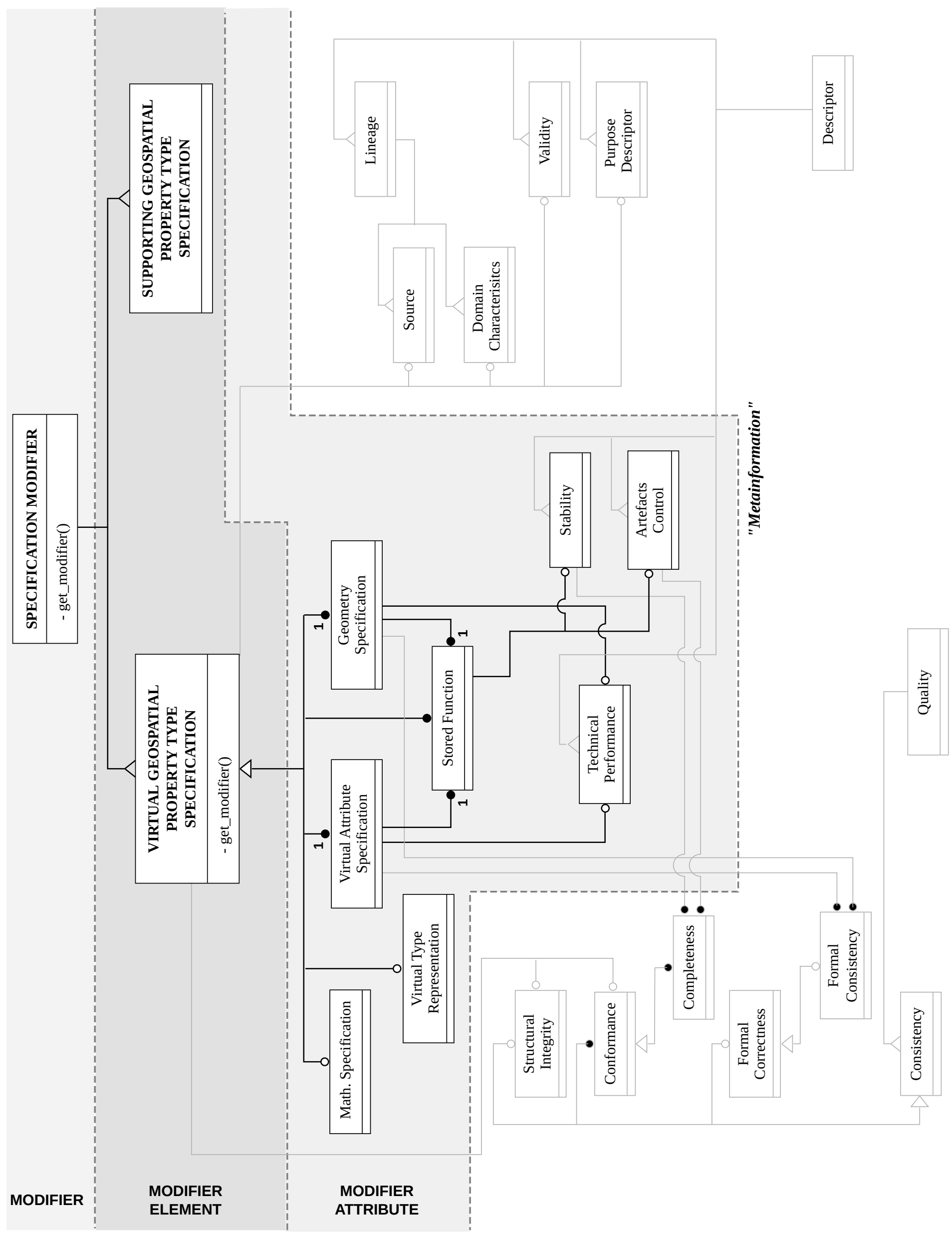


Figure 7.9: Class diagram for the modifier element **virtual geospatial property type specification**. Stored functions are provided to specify continuous geometries and virtual attributes.

so-called *stored function descriptors* (cf. appendix C.2.7), thus providing the structure to realise the *principle of two-fold metainformation reporting* for virtual data (presented in section 6.2.2).

Documentation of the specification was conceptualised as acting on a property type or class level. Therefore, in the storage model a specification modifier is associated with each geospatial property type. To ensure geospatial property types having only one specification, the modifier **specification modifier** may be designed as a *singleton* (Gamma et al. 1995)[3]; see also figure 7.7. Again, a geospatial property type's specification can be assessed by accessing the corresponding overwritten **get_modifier()** method of the **modifier** super-class (figure 7.10).

The Identification Schema

The identification schema serves two purposes: On the one hand, it explains the TI supplied by exposing its abstraction and the information base its derivation is founded on. On the other hand, it provides informative data on usage and availability or on the PTM's technical quality. To this end, the identification schema aggregates the modifier **abstraction modifier** and the TI descriptors discussed in section 6.1.2 and presented in appendix C.2 (see figure 7.11).

The modifier **abstraction modifier** was introduced to maintain the linkages between the real terrain, the UoD and the respective conceptual models on which the TI modelling is based upon. These links might otherwise have been lost in the process of abstraction. As discussed in appendix C.1.1, modifier **abstraction modifier** provides documentation on the *abstraction*, on the *body of knowledge and information* that the modelling is based upon as well as on the *nomenclature* used (see the column depicting **abstraction modifier** in figure 7.5). Abstraction uncertainty and **abstractive information** were defined as effective at a dataset level (sections 5.1.4 and 6.1.1). Carried over to the PTM storage model, this is realised by associating an element of the modifier **abstraction modifier** to the property set. To guarantee that a property set has just one abstraction, the modifier is designed as a *singleton* (Gamma et al. 1995), as indicated in figure 7.11.

The **descriptors** aggregated in the identification schema document the module in view of (see also figure 7.6):

- Its *lineage*,
- its *purpose* (i.e., of the rationale for its creation or its intended use),

[3]Singleton design patterns are applied when at most one instance of a class shall be allowed and a global point of access for the single instance of the class is desired.

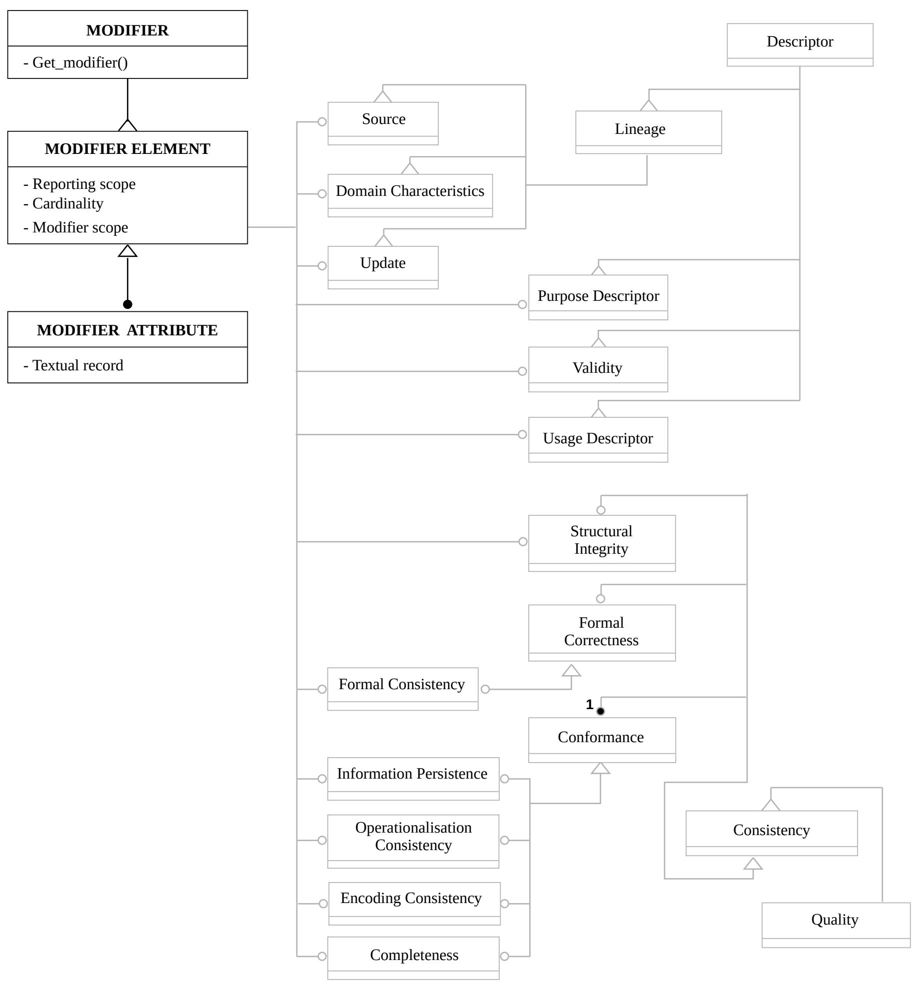


Figure 7.10: Class design for the metadata component **modifier**. Portrayed in gray is the metadata that might be associated with a modifier element for its documentation or to report its consistency.

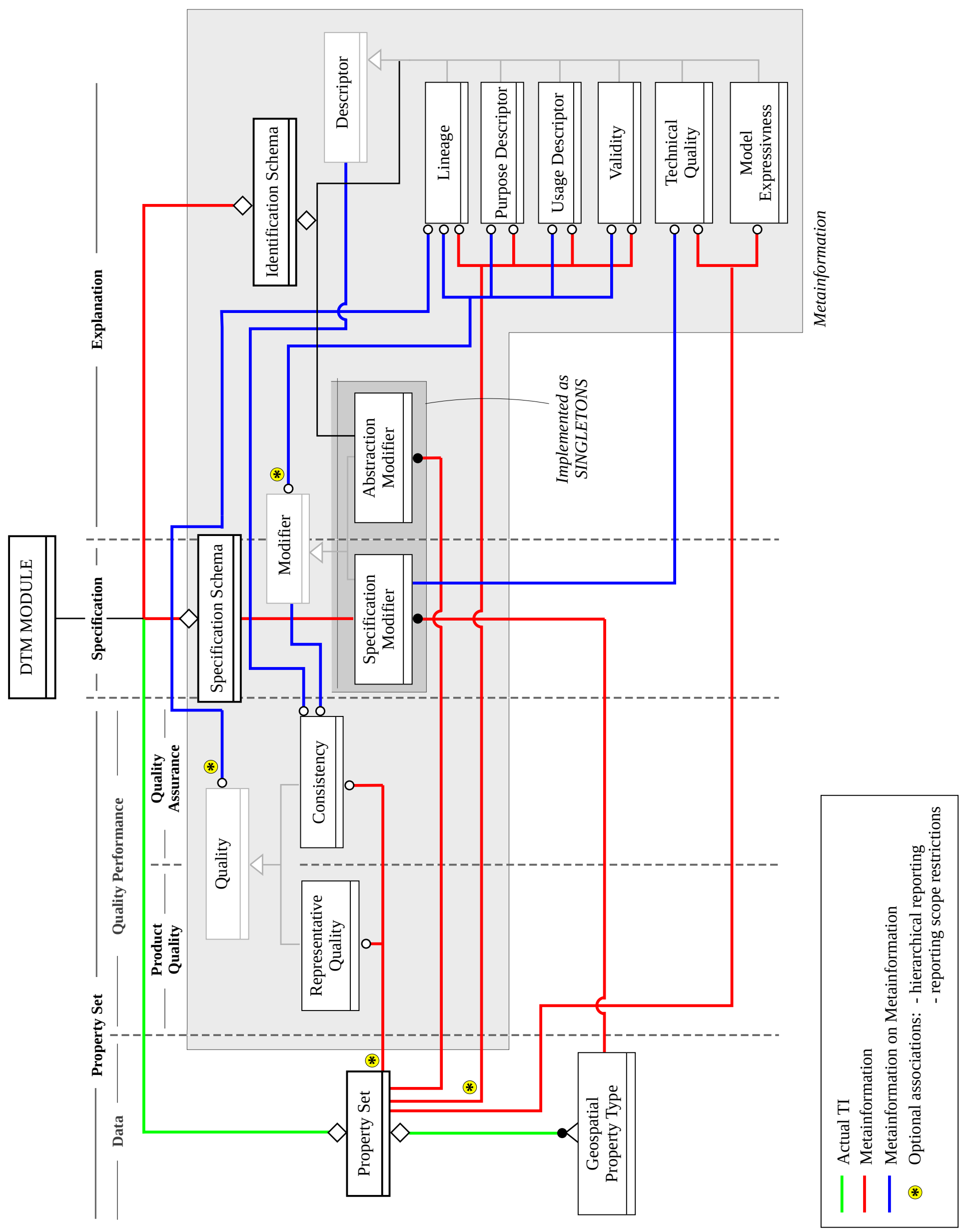


Figure 7.11: Overview of the PTM storage model.

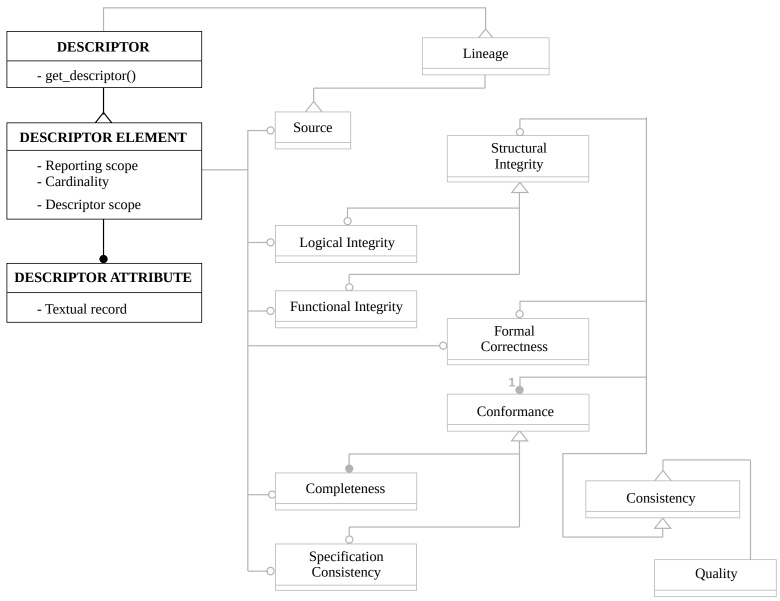


Figure 7.12: Class design for the metadata component **descriptor**. Portrayed in gray is the metadata that might be associated to a descriptor element, mainly to provide documentation of its consistency.

- its *usage* (including information on usage and access),
- its *validity*,
- its *expressiveness* (including clarification of the modelling scale, of the model's explicitness, and of its thematic resolution),
- and its *technical quality*.

The inheritance structure of the class diagram depicted in figure 7.1 allows each geospatial property to have its own descriptors (except, of course, cases in which the descriptor's reporting scope is restricted to higher dataset levels). Descriptors, however, are subject to the *principle of hierarchical reporting*, meaning that they only need to be overwritten at low levels of the reporting hierarchy (cf. section 6.2.1 and figure 6.7) if they differ from the ones reported at higher dataset levels. The general structure of the metainformation component descriptor illustrated in figure 7.12 shows that a mechanism to access a geospatial data item's individual descriptor information is provided by the corresponding overwritten method **get_descriptor()**.

The identification schema is a crucial element for the *communication* between TI producers and users. It contains an explicit documentation of the abstraction the DTM implementation is based upon. It communicates the intended meaning, that is, the *semantics* of the terrain representation. This metainformation section is, thus, essential to determine the usability of the abstraction underlying a digital terrain representation with respect to a specific purpose, because it forms an essential information source for comparing the application-specific *rules* with the implemented terrain representation.

7.3.2 Metainformation on Metainformation

Figures 7.4, 7.10 and 7.12 depict the general structure of the modifiers, descriptors and quality elements used in the PTM storage model for reporting metainformation. As explained in section 6.2.3, the idea of applying metainformation on metainformation is to provide a mechanism to support *quality assurance*. Figure 7.11 indicates this by further subdividing the part of the property set reporting *quality performance* into a component documenting *product quality* and a component dealing with *quality assurance*.

7.4 Review

This chapter presented the basic concept behind a Pluggable Terrain Module (PTM), together with a corresponding *storage model* (figure 7.11). The PTM enables *realisation of the concepts for reliable digital terrain modelling* discussed in chapter 4 by simulating a continuous terrain surface with the help of supporting data and terrain reconstruction methods, all hidden inside the module. However, the PTM encompasses more than the terrain surface description itself. Based on stored (and thus persistent) functions, it also embodies *derivable TI*. By embedding the functionality needed for terrain analysis, it provides an application with proper tools thereby preventing it from choosing inappropriate approaches to handle TI. Hence, the PTM contributes to *improved consistency* in digital terrain modelling. The module also exhibits the flexibility to allow selection of data structures and terrain analysis algorithms in view of specific application needs without restrictions on other software components.

The PTM concept is basically a *conceptual model for storage of metadata* and for documenting TI quality using the metadata components proposed in section 6.1 (see figure 7.11). The storage model thus provides a mechanism for structuring the wealth 'zoo' of metadata elements available according to the kind of information they intend to provide to a TI user.

Finally, the PTM conforms to section 4.3, where a DTM was defined as $DTM := \{\mathcal{M}; (T_R; \mathcal{D}_T)\}$, in that:

- The *quality performance*, the *specification* and the *explanation* parts (cf. figure 7.11) provide the required set of metadata $\mathcal{M}$;
- $\mathcal{D}_T$ is represented by the *sampled* and *supporting geospatial properties*;
- T_R is hidden in the specification schema in the form of stored functions; however, it manifests itself in the *virtual geospatial properties*.

Chapter 8

Towards an Understanding of Scale in Digital Terrain Modelling

The process of abstraction takes control of a terrain model's information content and detail richness and thus introduces issues of *scale*, as discussed in section 5.1.2. Data sampling and digital representation of data additionally contribute to the emergence of scale effects. DTMs, therefore, always model at a certain *scale range*. The scale dependence of a digital terrain representation is obvious. Speaking in cartographic terms, at larger scales more detail can be represented, while at small scales just an overall shape of the topography is captured. This implies, for instance, that even under the assumption that a set of supporting terrain data is precise and error-free, every terrain representation based on these data may incorporate some amount of uncertainty. Deviations between elevations portrayed in a DTM and the corresponding 'true' surface, hence, are not introduced entirely through inaccurate measurement or bad interpolation and they do not necessarily express a poor overall model quality. Rather, they may be caused by the detail neglected at the chosen modelling scale and therefore may be considered as 'deliberate' to some degree. Scale, therefore, critically affects both TI quality itself as well as TI quality evaluation. The latter is affected in the sense that in view of a specific application, TI accuracy needs to be assessed with respect to a terrain representation at the requested scale rather than by comparison with 'true' values. However, scale is rarely considered by current approaches to TI quality documentation.

Not only elevation at arbitrary locations, but rather TI in general is implicitly assigned the scale range of the DTM it is derived from (Wood (1996b); see also section 4.3.1). For these reasons, the topographic surface may be interpreted as a function $z(\cdot)$ not only of location, but also of scale[1]. Yet, tools for DTM analysis with consideration of scale are not usually provided.

This chapter aims at contributing to techniques which allow

- one to *associate scale ranges* to terrain surface features; and
- *controlled* terrain surface simplification (that is, controlled information reduction).

'Controlled', in this context, means that in view of a specific application, topographic structures assigned a scale finer than the envisioned modelling scale shall be preserved provided they are considered relevant by the application. The final objective of the chapter is to contribute a technique suitable for

- *'scale-aware' DTM analysis* hence enabling *scale-dependent terrain characterisation*; and
- *estimation of the uncertainty* introduced through representing terrain at a specific scale.

Some authors such as Garbrecht and Martz (1994) perceive terrain scale as DTM resolution. Their approach, however, studies the effects of sampling rather than the actual scale properties of the topographic surface (for an interesting discussion of this topic, see Gallant and Hutchinson (1996)). This chapter aims at presenting a method which deals with scale directly, that is, which attempts to define the available terrain information independent of the data density. Of course, the information content of a DTM depends on the supporting data insofar as these data define the maximum information content. The chapter discusses how, by treating wavelength as equivalent to scale (Gallant and Hutchinson 1996), the *wavelet transform* provides an overall mathematical framework for striving for the stated objectives.

Section 8.1 illustrates by example what a wavelet transform actually does, and what kind of information it may provide. The focus, however, is definitely on qualitative explanation. For a rough background on the wavelet transform, please refer to appendix D. The wavelet chosen for the research reported in this chapter is discussed in section 8.2. In section 8.3, scale dependencies of the topographic surface are detected by making use of wavelet analysis for *multiscale terrain characterisation*. Section 8.4 presents

[1] That is, the topographic surface may be mathematically described by (cf. footnote [3]):

$$(id, z(\cdot)) : \mathbb{D} \times \mathbb{R} \longrightarrow \mathbb{D} \times \mathbb{V}, \quad \boldsymbol{s} \mapsto (\boldsymbol{s}, z(\boldsymbol{s}, a)),$$

where $\boldsymbol{s}$ denotes a location in some planar two-dimensional domain $\mathbb{D}$ and a is a scale parameter.

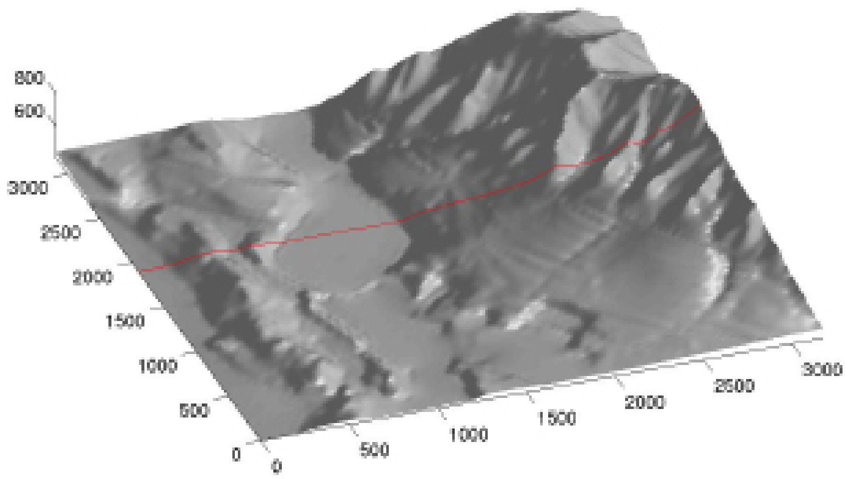


Figure 8.1: The study area (units in m; DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

a method for *selective filtering* of digital terrain data in the wavelet domain. Selective filtering in the wavelet domain serves two purposes. First, it enables decoupling of the degree of detail displayed in the terrain model from the DTM mesh size (that is, from its discretisation unit) thus supplying a basis for *'scale-aware' DTM analysis.* Second, it provides a basis for the *estimation of the uncertainty* introduced by representing terrain at different levels of scale, as discussed in section 8.4.4.

All the results shown in this chapter were computed for a study area of 3.2 km by 3.2 km situated near Zürich, Switzerland, portrayed in figure 8.1. The area was chosen because it displays fairly accentuated topographic structures as well as smooth hills and flat areas. The resolution of the original data is 25 m.

The wavelets used for this research were implemented in MATLAB©, version 5.3.11. Matrix operations are remarkably fast in MATLAB©, which is very important for computationally complex algorithms such as the wavelet transform. A further advantage of MATLAB© is its capability of visualising meshes and surfaces in a simple but powerful way.

8.1 Why Wavelets?

Figures 8.2 and 8.3 illustrate what a wavelet transform actually does, and what kind of information it may provide. While in figure 8.2 a profile

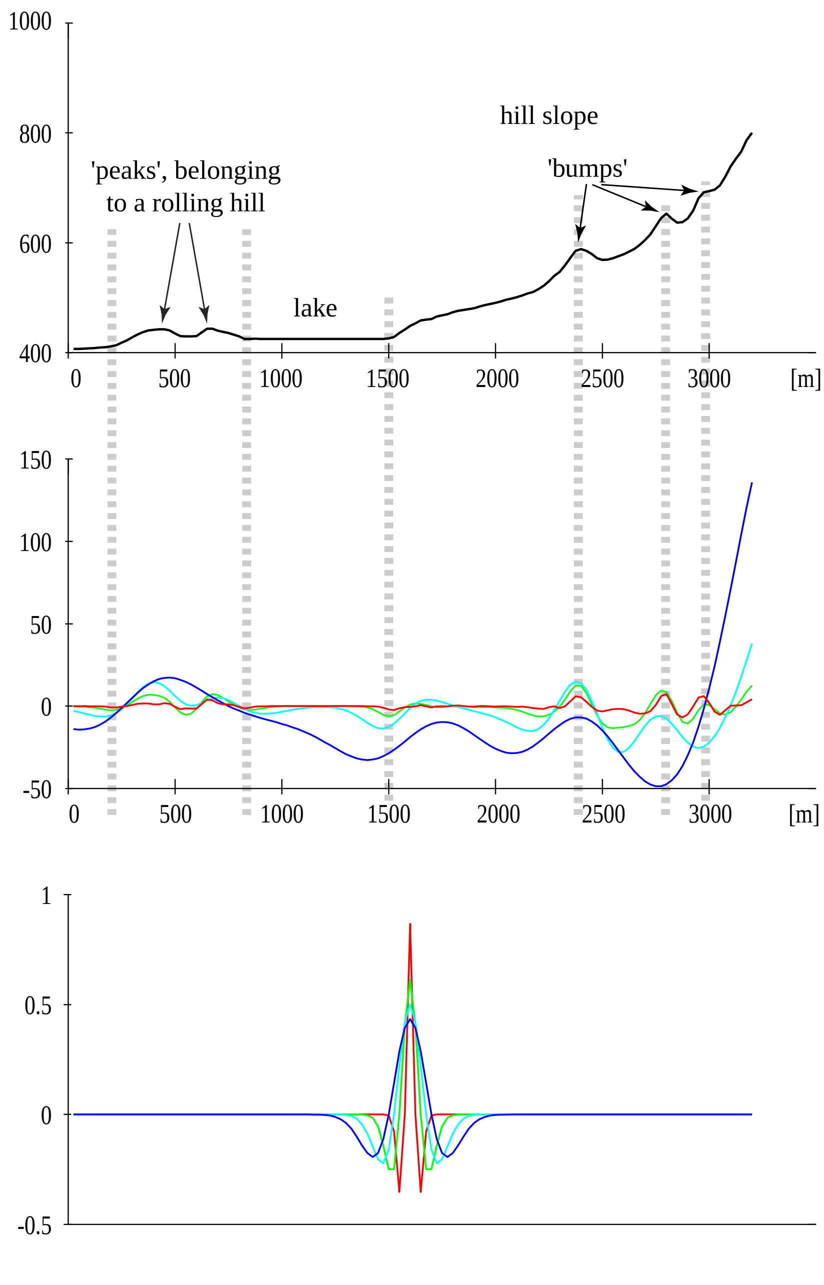


Figure 8.2: Discrete wavelet transform of the profile marked in red in figure 8.1. Top: The Profile. Middle: Its discrete wavelet transform, for the dilation parameters 2 (red), 4 (green), 8 (cyan), and 16 (blue). Bottom: Correspondingly dilated versions of the Mexican hat wavelets used.

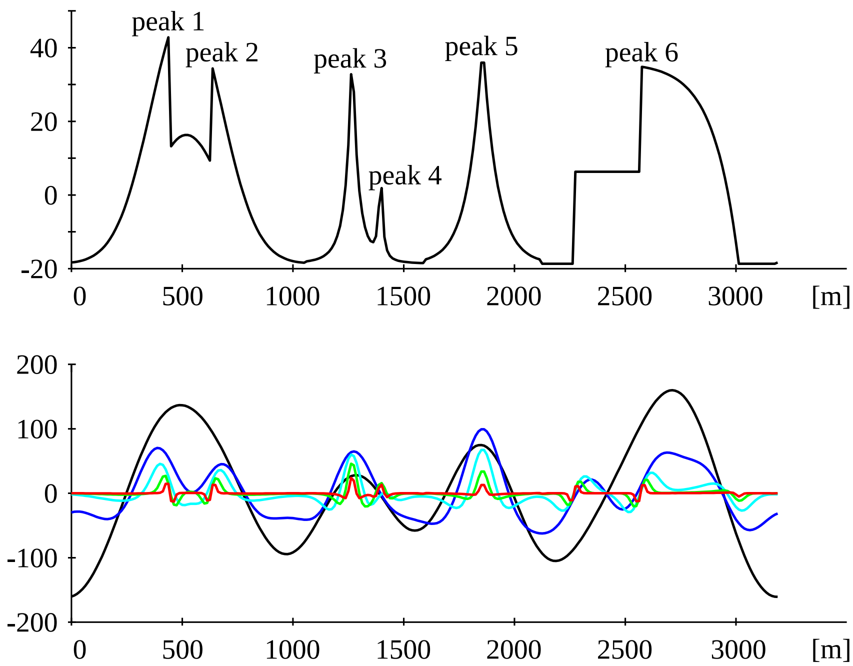


Figure 8.3: Discrete wavelet transform of a synthetic profile designed for illustration purposes. Top: The synthetic profile. Bottom: Its discrete wavelet transform, for the dilation parameters 2 (red), 4 (green), 8 (cyan), 16 (blue) and 32 (dark blue).

through the study area (marked red in figure 8.1) is depicted together with its wavelet transform at the four finest levels of scale, figure 8.3 shows a synthetic signal together with its wavelet transform at the five finest levels of scale. In both cases, the wavelet transform is computed using a so-called *Mexican hat wavelet*. The Mexican hat is introduced in the next section; for an illustration of the scale classification and terminology used, please refer to section 8.2. The profile view was chosen for the sake of clarity, as meaningful visualisation of the two-dimensional wavelet transform is a tricky task. So, what can be deduced from a wavelet transform?

- *Interpretation of wavelet coefficients and levels of scale*: The level of scale for which a wavelet coefficient is computed gives an estimate of the wavelength of the components of the analysed signal in the respective scale (for detailed discussion, see appendix D.3.2). An estimate of the wavelength of the landscape features for the current example is provided in table 8.1. According to the feature sizes reported there, the wavelet coefficients for scale level 1, for instance, report on the occurrence of landscape components at size of approximately 111 m (in any direction).

- *Amplitude*: The amplitude of a wavelet coefficient indicates how closely *correlated* the wavelet is with the corresponding section of the signal. The greater the amplitude modulus, the higher the *similarity* (appendix D.3.1). Note, for instance, the two peaks at the left end of the profile in figure 8.2. They cause corresponding peaks in the wavelet coefficients for the levels of scale 2 and 3, indicating that terrain components of corresponding size (cf. table 8.1) recognisably occur when the terrain is 'looked at' at the levels of scale 2 and 3. At scale level 4, the two peaks can not be distinguished anymore. Rather, they are likely to be perceived as belonging together and forming one feature. The same effect can be seen in figure 8.3 where, for instance, peaks 1 and 2 can not be distinguished anymore at scale level 5, or where peaks 3 and 4 are perceived as forming one feature at the levels of scale 4 and 5. Likewise, the lake in figure 8.2 is (more or less) recognised as a single structure only at scale level 4. The three finer scales, particularly the levels of scale 2 and 3, detect the sharp edge occurring at the right boundary of the lake as an individual landform component rather than as the lake as a whole.

- *Amplitude modulus*: The greater the amplitude modulus, the better the correlation of the signal with the correspondingly scaled wavelet (appendix D.3.1). Comparison of the amplitude moduli across the scales indicates which level of scale is most relevant or best fits to the corresponding section of the signal. In figure 8.3, for instance, peaks 1 and 2 together or the entire complex of peak 6 form features best correlated to the level of scale 5. Whereas, the scale level most relevant to the feature composed by peaks 3 and 4 is scale level 4. Peak 3 by itself, finally, best fits to the levels of scale 2 and 3.

- *Multiscale differential operator property*: The Mexican hat is a second-order multiscale differential operator. That is, it provides information on the curvature properties of the analysed signal (cf. appendix D.3.3). Positive wavelet coefficients indicate convex regions, while negative coefficients describe concave areas (consequently, zero crossings suggest the occurrence of inflection points). However, the occurrence of significant extrema within a sequence of positive or negative coefficients indicates variation in curvature characteristics across adjacent scales. For instance, at scale level 4, the whole profile, from the second peak on the left to its almost right end, is perceived as forming a concave feature. However, the behaviour of the wavelet coefficients for scale level 4 in the right half of the profile indicates the occurrence of convex bumps at scale level 3. The same phenomenon can be observed for the scale levels 3 and 2 and the right end of the profile.

The above listing does not claim to be complete. It is rather an attempt to give an impression of how a wavelet transform actually looks and why it may be beneficial to digital terrain modelling. The remainder of the chapter is concerned with sketching how such information can be exploited to investigate *scale dependencies* of the terrain surface, and assess the *relevance of topographic forms* at different levels of scale.

8.2 Selection of a Wavelet

The concept of multiresolution analysis sketched out in appendix D.2.3, particularly its property of projecting signal details at different levels of scale onto orthogonal spaces and its associated fast wavelet transform, sounds very appealing for revealing scale dependencies of terrain surfaces. However, formulation of the two-dimensional wavelet transform as the tensor-product of two one-dimensional wavelet transforms, one in the x and one in the y direction, overemphasises these two cardinal directions. This effect can clearly be seen in figure 8.4. The decomposition in figure 8.4 is computed using a 'Symmlet' wavelet with 2 vanishing moments (depicted in figure 8.5), that is, a second-order multiscale differential operator with a support of 4 units (appendix D.3.3). Symmetry, orthogonality, compact support and smoothness are desired properties of wavelets used for approximation purposes. However, they can not be met all at the same time by a single wavelet. So, for instance, it can be shown that orthogonal wavelets may not be symmetric. The Symmlet is one of the most symmetric orthogonal wavelets. Yet, it still displays clear artifacts in the cardinal directions. The fast wavelet transform entails two other major problems:

- It imposes strictly *dyadic scaling* of the dilation parameter. This might be a quite coarse discretisation step for the purpose of accessing terrain representation at different levels of scale, especially when the resolution of the original data is not very high.

- A *rotation parameter* could help to detect features more accurately than wavelets restricted to horizontal, vertical and diagonal directions. However, the tensor-product wavelets do not allow introduction of an additional degree of freedom.

For these reasons, the nice features of multiresolution analysis were sacrificed in favour of working with a discrete two-dimensional wavelet-frame (appendix D.2.2), using a so-called *'Mexican hat'* wavelet (shown in figure 8.6). The two-dimensional Mexican hat is given by:

$$\psi_{mh}(\boldsymbol{x}) := \frac{1}{\sqrt{2\pi}} \, (2 - |\boldsymbol{x}|^2) \, e^{-\frac{1}{2}|\boldsymbol{x}|^2}. \tag{8.1}$$

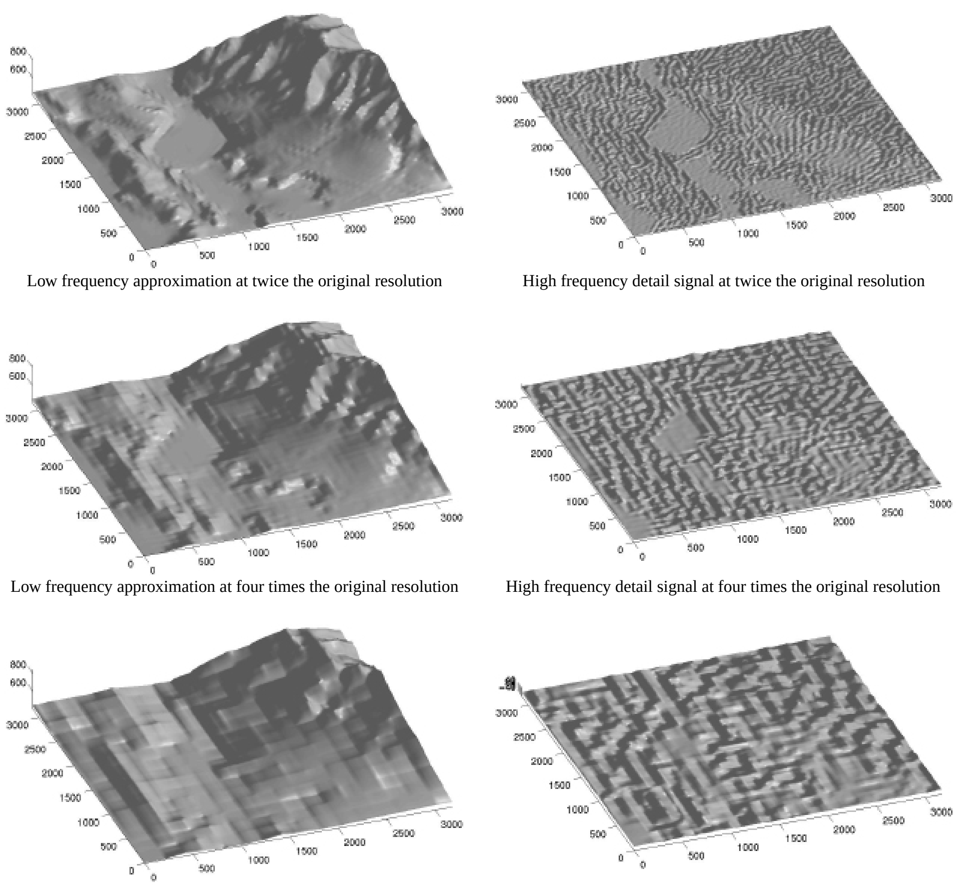
It is proportional to the second derivative of a Gaussian. Hence, it is also a *second-order multiscale differential operator* and therefore good for investigating curvature characteristics of the analysed terrain. It is constructed by rotating a one-dimensional Mexican hat around its middle axis and is thus completely symmetrical. Generally, advantages of using redundant wavelet frames are:

- Scaling of the dilation parameter can be freely chosen.

- A rotation parameter may be easily introduced to support investigation of selected directions (of course, this makes only sense when working with a-symmetric wavelets).

These advantages come at the expense of orthogonality and compact support. The Mexican hat, theoretically, does not have compact support. It was nonetheless chosen as the analysing wavelet within the scope of this research because curvature is thought to be a more conceivable property for terrain characterisation than other derivatives (Heckbert and Garland 1997, Absolo et al. 2000, Wood 1996a, Mahler 2001). Thanks to its fast decay, the impact of the Mexican hat's infinite support does not preponderate. For practical purposes, it is said to have an 'essential support' of 8 units.

Significance of the Wavelet Coefficients The level of scale of the wavelet coefficients gives an estimate of the wavelength of the components



Low frequency approximation at twice the original resolution — High frequency detail signal at twice the original resolution

Low frequency approximation at four times the original resolution — High frequency detail signal at four times the original resolution

Low frequency approximation at eight times the original resolution — High frequency detail signal at eight times the original resolution

Figure 8.4: Terrain approximation using a 'Symmlet' wavelet with 2 vanishing moments (units in m; DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

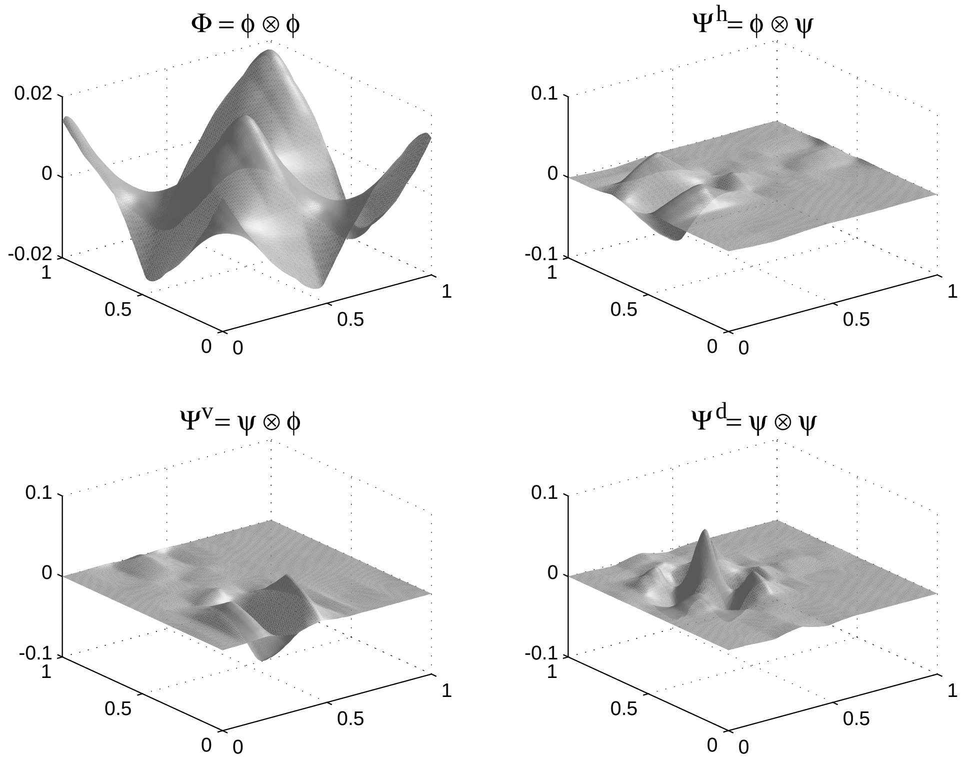


Figure 8.5: Two-dimensional Symmlet with 2 vanishing moments. The scaling function (top left) and the three wavelets capturing the details in horizontal, vertical and diagonal direction (cf. discussion in appendix D.2.3) are portrayed.

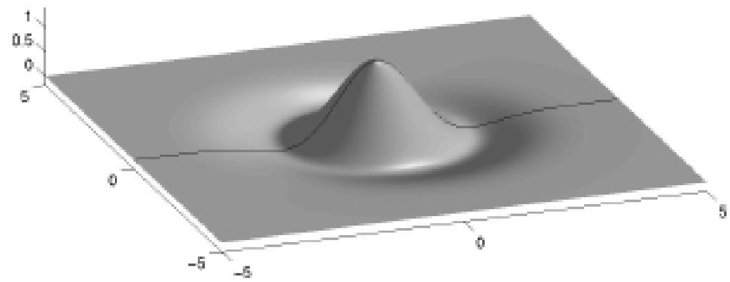


Figure 8.6: 'Mexican Hat' wavelet. The black line shows the one-dimensional version of the Mexican hat.

	Feature Size
Scale 1	111 m
Scale 2	222 m
Scale 3	444 m
Scale 4	889 m
Scale 5	1777 m

Table 8.1: Estimate of the wavelength of landscape features.

of the analysed signal in the corresponding scale (for a detailed discussion of the topic, see appendix D.3.2). Hence, when the analysed signal is a terrain dataset, the scale of a coefficient provides an estimate of the wavelength of the landscape components in the corresponding scale. Table 8.1 identifies the approximate size of the principal landforms in the scale classification used here. The principal feature sizes are determined following the corresponding procedure sketched in appendix D.3.2. However, as suggested by Mahler (2001), the scales of landforms can only be represented in a fuzzy way. Thus, table 8.1 can only provide a rough guideline.

8.3 Multiscale Terrain Characterisation

This section provides a few examples of how wavelet techniques may be used for multiscale terrain characterisation. The focus is entirely on qualitative illustration. Mathematical details, where required, are provided in section 8.4.3.

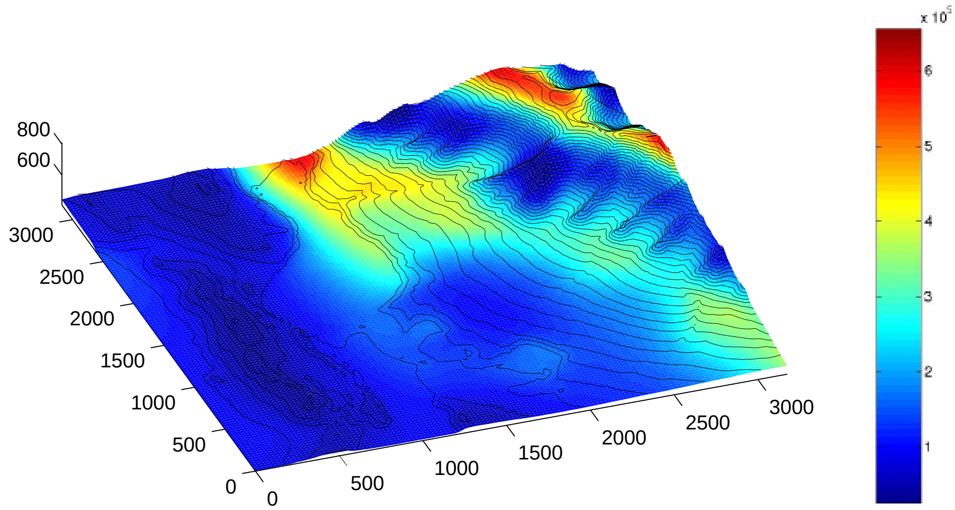


Figure 8.7: Spatial distribution of the total energy of the wavelet representation of the study area (units in m; DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

8.3.1 Energy Conservation

Appendix D.1 mentions the *energy conservation* property of the wavelet transform. This property states that every wavelet coefficient is a measure of the fraction of energy provided by its basis function. High energy regions, then, may be interpreted as regions of high information density (Blatter 1998), while low energy regions are more monotonous in terms of 'signal content'. Figure 8.7 plots the total energy provided by expanding the study area in a Mexican hat wavelet basis for each location in the spatial domain. The correlation between high energy areas and areas of high curvature is quite evident. Figure 8.8 isolates the energy fraction provided by each level of scale (from 1 to 5) for each location in space. Note, however, that for the sake of clarity the colour range is not standardised. The figure gives a first impression of the areas and features of particular relevance at the distinct levels of scale. See figures 8.2 and 8.3 to get an idea of how the wavelet transform reacts sensitively to sharp transitions and changes in terrain curvature. Note the high frequency coefficients created at the lake's shoreline in figure 8.2.

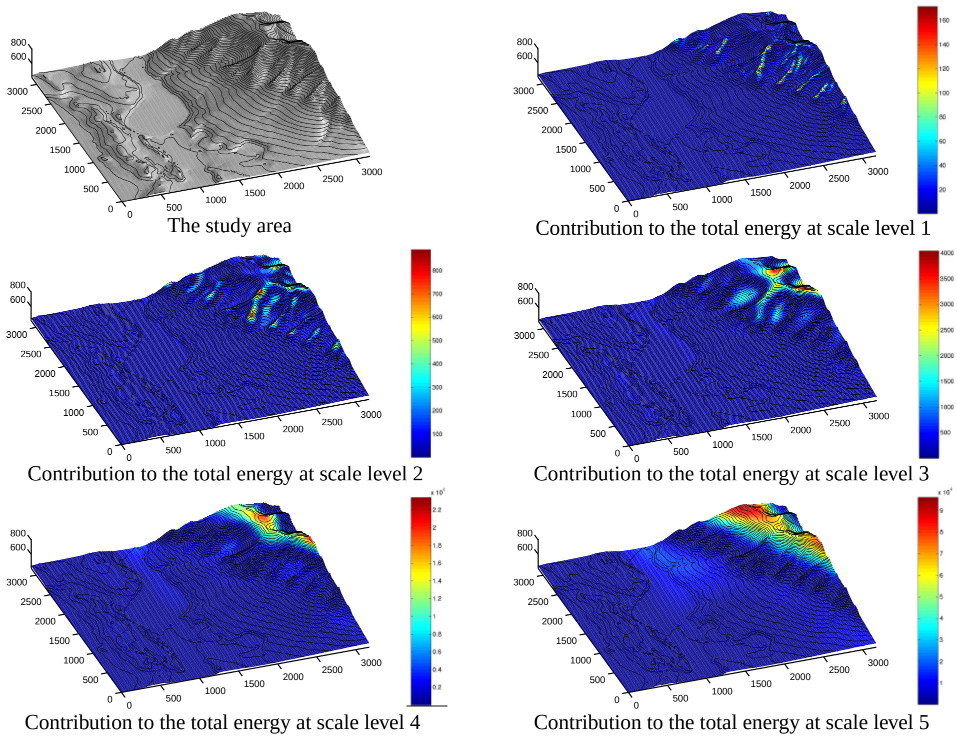


Figure 8.8: Contribution to the total energy of the wavelet representation of the study area for the 5 finest levels of scale (units in m; DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

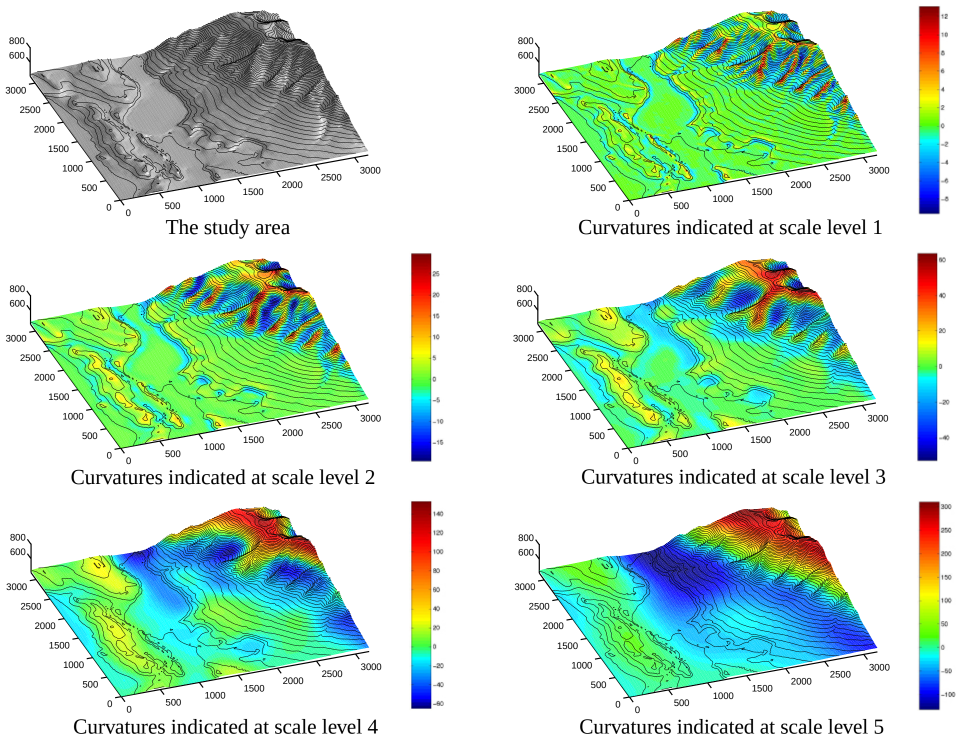


Figure 8.9: Curvatures detected at scale levels 1 through 5 (units in m; DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

8.3.2 Multiscale Terrain Characterisation at Different Levels of Scale

Convex and Concave Regions

Figure 8.9 gives an overall picture of the curvature trends detected at scale levels 1 through 5. It is interesting to note that at the finest scale convex structures are predominant. Concave regions are mostly concentrated on the ravines in the rugged portion of the site. On the third scale level regions of similar curvature cluster very nicely. Note particularly the accurate identification of the rolling hills on the western shore of the lake. At the coarser scales, finally, the evidently small scale convex structures are no longer recognised. Instead, the basin as a whole is perceived as a concave structure.

Edges, Ridge- and Channel-like Structures

Locally extreme wavelet coefficients indicate regions of high information (or energy) density. In the current case, these are regions of locally extremely accentuated curvature, such as ridge or channel-like structures, or sharp edges. Discrete wavelet frames allow searching for local extrema along all the discretised directions. This property can be exploited to detect selected directions in space, along which the information (in terms of signal content) is concentrated. Figures 8.10 and 8.11 depict the regions causing significant local wavelet extrema for two selected areas of the study area and the first three levels of scale. The example in figure 8.10 shows that while wavelet coefficients indicating areas of locally maximum curvature are detected along the rolling hills at all three levels of scale, the degree of their connectedness and hence the distinctness of the resulting structure line increases with increasing level of scale.

In order to correctly interpret these results, it is important to recall that any point in the spatial domain can only be located within the uncertainty bound imposed by the transform in wavelet domain, and vice versa (appendix D.1). The wavelet coefficients, therefore, can only indicate the spatial location of important features somewhere within these uncertainty bounds, but they can never provide their precise localisation in space. This is also the reason why, for instance, the areas likely to feature ridge- and channel-like structures or sharp edges identified in figures 8.10 and 8.11 look more like buffers than like sharp structure lines.

8.3.3 Multiscale Feature Persistence

The results obtained when characterising terrain at a variety of scales as in the above examples can clearly be combined to investigate the behaviour and persistence of structures identified across the investigated scales. Figure 8.12, for instance, illustrates the effect of scale on the convexity and concavity of the study area when proceeding from finer to coarser scales. However, only the type of curvature (convex or concave) is considered without accounting for the varying degree of curvature.

Another example for multiscale feature detection or characterisation is portrayed in figure 8.13, where the behaviour of the edges and ridge- and channel-like structures detected in the last paragraph is visualised across the investigated scales. Tracking of identified features across different levels of scale is accomplished by testing if the occurrence of a corresponding extreme wavelet coefficient repeats itself amongst the corresponding wavelet bases at the next coarser or finer scales[2].

[2]When performing such a test, attention must be paid to the fact that the applied discrete wavelet transform using a Mexican hat wavelet provides a redundant description

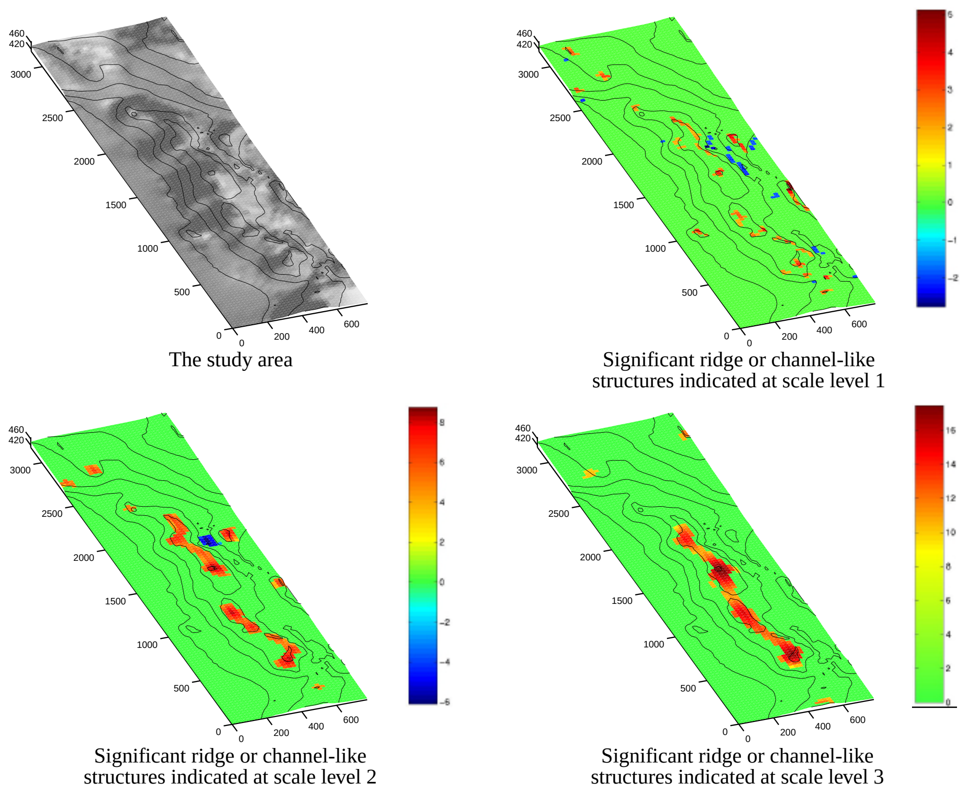


Figure 8.10: Significant edges or ridge- or channel-like structures detected at the three finest levels of scale in quite smooth terrain (units in m; DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

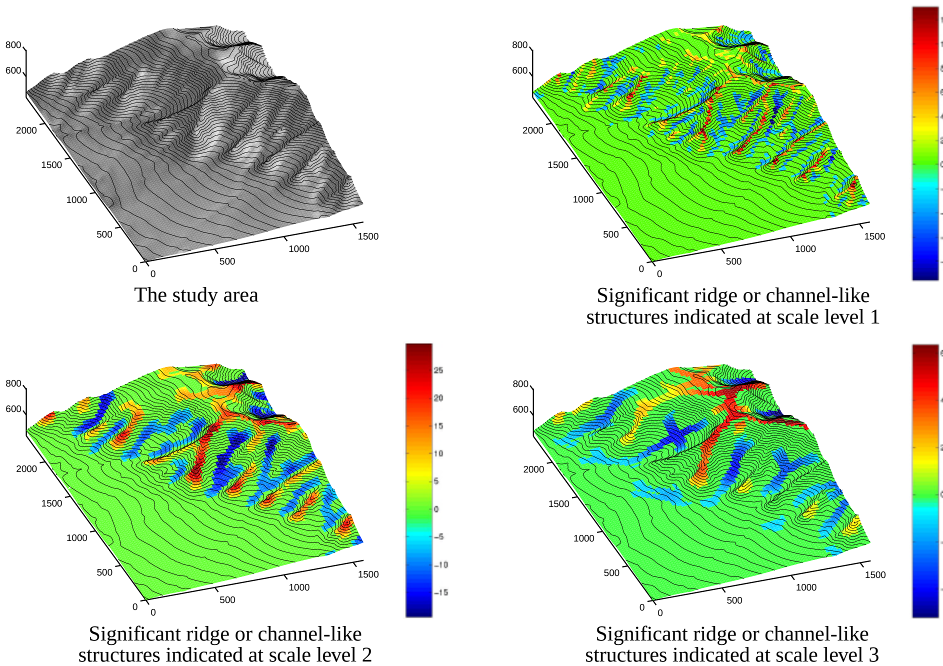


Figure 8.11: Significant edges or ridge- or channel-like structures detected at the three finest levels of scale in fairly rugged terrain (units in m; DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

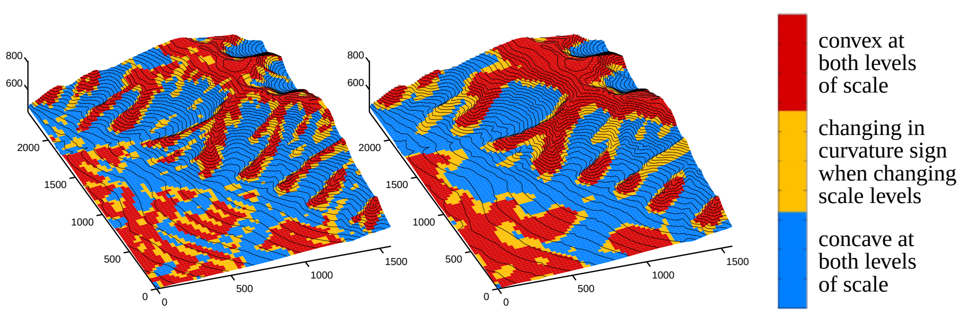


Figure 8.12: Persistence of curvature properties across the scale levels 1 and 2 (left picture) and 2 and 3 (right picture). Units are expressed in m (DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

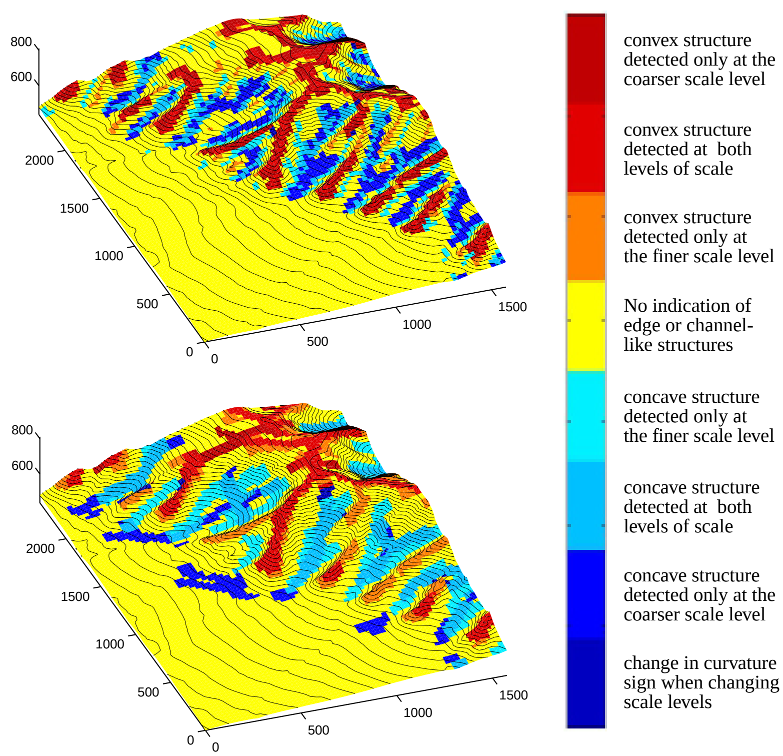


Figure 8.13: Persistence of edges and ridge- and channel-like structures across the scale levels 1 and 2 (top) and 2 and 3 (bottom). Units are expressed in m (DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

8.3.4 Final Remarks

All the examples provided in this section were computed using a Mexican hat wavelet recursively dilated by a factor 2. However, a 'zoom' step of 2 may provide a rather rough discretisation of scale, especially when the original resolution of the investigated data is not very high. Generally, in order to remedy this situation without having to sacrifice too much of the freedom in choosing the analysing wavelet and its width in the frequency domain, a method introduced by Grossmann et al. (1989) can be adopted which uses different *"voices"* per zoom. This amounts to using several different wavelets $\psi^1, ..., \psi^n$ that have slightly staggered frequency localisation centres, and to look at the frame $\{\psi^{\nu}_{m,n} : \; m, n \in \mathbb{Z}, \; \nu = 1, .., n)\}$. Then, for every dilation step, n different scale levels are found (corresponding to the frequency localisations of $\psi^1, ..., \psi^n$), all translated by the same offset. A choice favoured by Grossmann et al. (1989) is to take 'fractionally' dilated versions of a single wavelet ψ (although the ψ^i, $i = 1, ..., \nu$, then, have different norms):

$$\psi^{\nu} = 2^{-\frac{\nu-1}{N}} \, \psi(2^{-\frac{\nu-1}{N}} \boldsymbol{x}).$$

The effect of this approach is illustrated in figure 8.14; its impact on delineation of terrain curvature is visualised in figure 8.15.

8.4 Terrain Filtering in the Wavelet Domain

This section, finally, aims at detecting and assessing effects caused by representing terrain at specific scales. Clearly, this requires the ability of actually representing terrain at different scales, what finally amounts to *(controlled) reduction of the information* displayed.

8.4.1 Global Filtering by Thresholding Scale

In a first approach to reduce the displayed information the concept of an inverse wavelet transform is exploited, allowing the reconstruction of the original terrain surface by superposition of the individual scale components. The terrain is first expanded into wavelets and subsequently reconstructed considering only details up to a specific *threshold scale*. This corresponds

of the original terrain (appendix D.2.2). In other words, the Mexican hat bases are not orthogonal. On the contrary, the wavelets overlap. Solely testing the wavelets concentrated around exactly the same spatial location as the significant wavelet is therefore not sufficient. An algorithm for tracking selected or identified features across several scales must comprise testing of all the wavelets whose spatially essential support overlaps the location around which the wavelets capturing the feature of interest are concentrated.

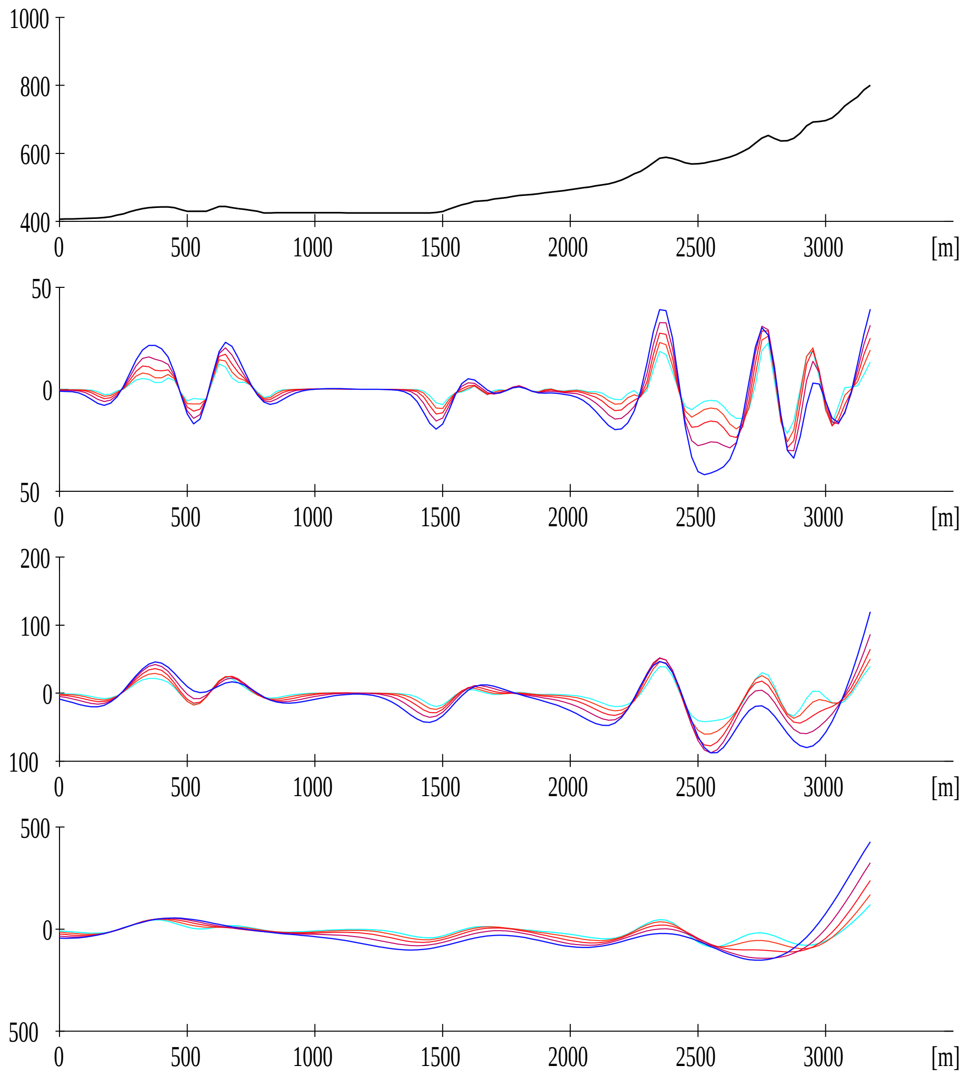


Figure 8.14: Discrete wavelet transform of the profile marked in red in figure 8.1 computed with 4 voices per octave. Top: The Profile. The other three subplots depict the first three octaves (from top to bottom). Plotted in cyan and blue are the lower and upper octave bounds, respectively, given by the dyadic wavelet transform. The lower bounds of an octave each correspond to the upper bound of the previous octave and the upper octave bounds correspond to the lower bounds of the subsequent one; note however, the different scaling of the ordinate. For each octave, then, the voices are plotted (from light to dark red; units in m).

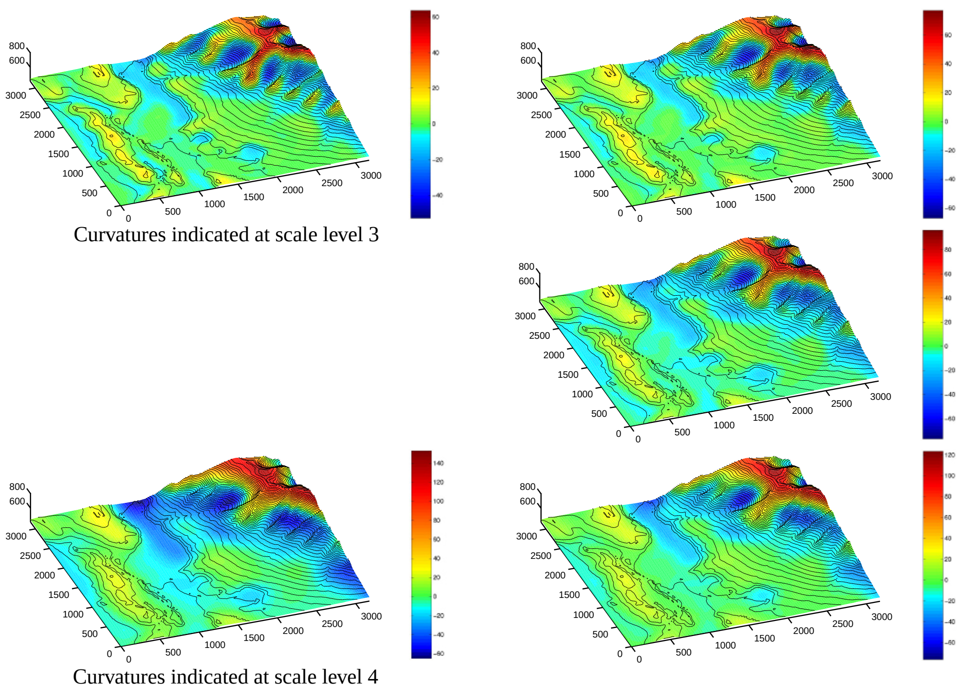


Figure 8.15: Curvatures detected between the scale levels 3 and 4, using 4 voices. Plotted on the left are the curvatures detected by dyadic scaling for the scale levels 3 and 4. On the right, the curvatures for the three intermediate voices are depicted (units in m; DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

$$\boldsymbol{DTM} \xrightarrow{\text{DWT}} \boldsymbol{W_\psi DTM} \xrightarrow{\text{thresholding}} \boldsymbol{W_\psi DTM^{th}} \xrightarrow{\text{inv. DWT}} \boldsymbol{DTM^{th}}$$

Figure 8.16: Rough scheme for the global filtering approach (where the superscript 'th' stands for thresholded).

to *weighting* the wavelet transform according to the following scheme:

$$\{w_{m,\boldsymbol{n}}\} \quad := \quad \begin{cases} w_{m,\boldsymbol{n}}, & \text{if } a_0^m > a_{th}, \\ 0 & \text{otherwise}, \end{cases}$$

where $w_{m,\boldsymbol{n}}$ denote the wavelet coefficients, a_0 is the (fixed) zoom step, and a_{th} is the threshold scale (for the notation used when referring to the discrete wavelet transform, please see appendix D).

The threshold scale a_{th}, then, defines the scale of the filtered surface approximation. The discarded wavelet coefficients manifest the information lost in the 'down scaled' DTM (cf. section 8.4.3). Figure 8.16 depicts the rough scheme of global filtering by a threshold scale.

The results of the approach are visualised in figures 8.17, for suppressing the fine scale details at twice, four and eight times the original DTM resolution. When looking at the results produced, it becomes evident that the task of representing terrain at a specific scale may not be accomplished by such a simple approach. Major reasons identified for the disappointing results are:

- Terrain representation at a specific scale does not entail strict suppression of features at finer scales. It rather means that the major focus is put on the selected scale. Yet, significant landscape forms need to persist into the representation, even if they belong to a finer scale than the given threshold scale. Note, for instance, figure 8.1 where the sharp transition between the boundary of the lake and the surrounding terrain, particularly at the lake's eastern shore, is depicted. This sharp transition creates high frequency terrain components related to a scale finer than the threshold scale (as indicated in the profile in figure 8.2). Strictly dropping all high frequency wavelet coefficients thus destroys the shape of the lake, an effect definitely adverse to what might be qualified as a 'good' terrain representation at the envisioned scale.

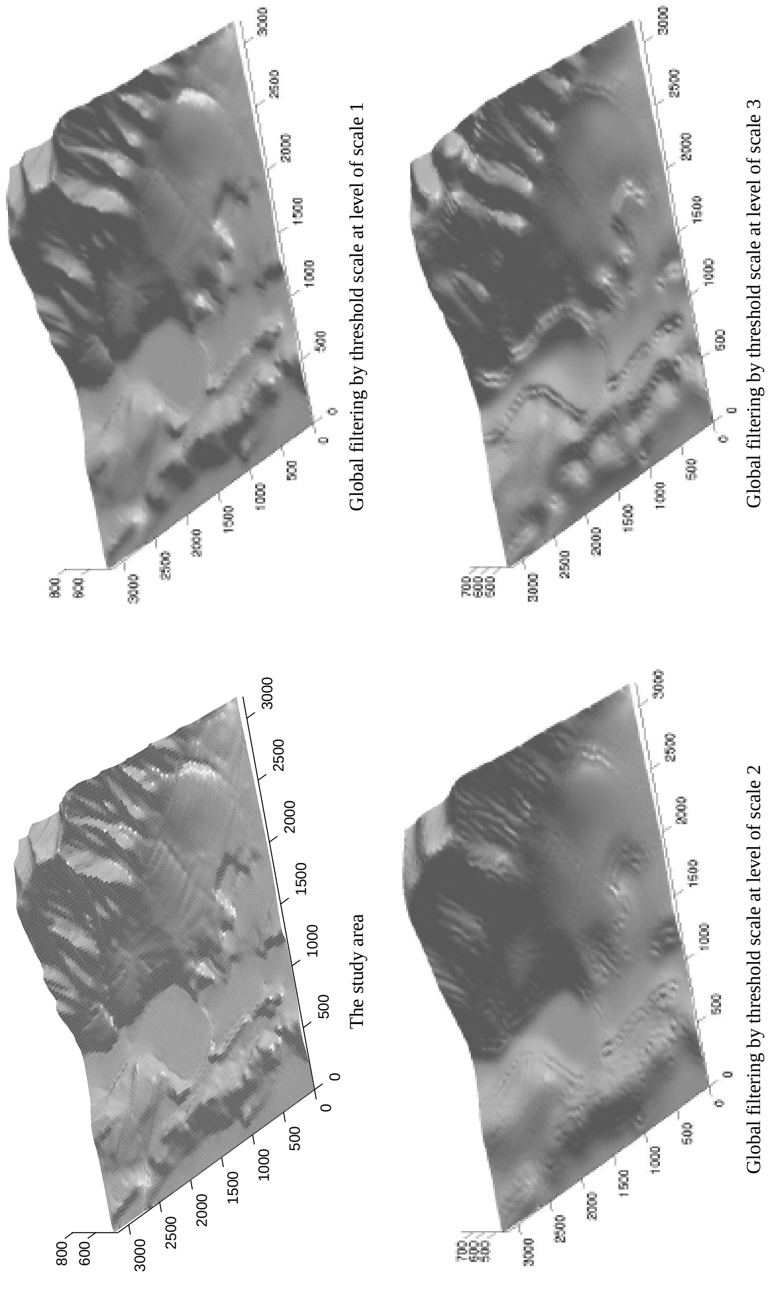


Figure 8.17: Global filtering by threshold scale at levels of scale 1, 2, and 3 (units in m; DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

- 'Good' terrain representation at the envisioned scale, basically, means reducing the amount of information (i.e. removing fine scale features of the surface) by retaining the range of the entire DTM. Global wavelet filtering as described above, however, eliminates fine scale details by clearly averaging the terrain model, thus reducing its range. In other words, (considerable) smoothing of significant landscape forms is to be avoided, while elimination of fine scale structures and perhaps noise (in smooth areas) is encouraged. The objective is to retain both form and crispness of features prevailing in the scale of interest.

- Finally, Weibel (1992) outlines *selection/elimination*, *simplification*, *combination*, *displacement* and *emphasis* as (cartographic) generalisation processes involved when altering a DTM's scale from finer to coarser levels. These processes all somehow have an *intentional* component and are applied selectively to selected regions or structures of the terrain to be simplified. Global terrain filtering by strictly suppressing all the wavelet coefficients belonging to a scale finer than the threshold scale, on the contrary, is a highly un-selective approach to surface simplification.

8.4.2 Selective Filtering in the Wavelet Domain

Integrating Concepts of Cartographic Generalisation into Wavelet Analysis

A remedy to the problems arising with global filtering approaches may be provided by making use of one of the major strengths of the wavelet transform which has not been exploited so far: its *localisation property*. The local support of the wavelet functions allows localisation of them in both the spatial and the wavelet domains, however always within an uncertainty bound (related to Heisenberg's uncertainty principle; cf. appendix D.1). This important property enables an elegant control of the local level-of-detail of the filtered terrain representation, in that wavelet coefficients belonging to scales finer than the threshold scale may be *weighted depending on their significance for the terrain shape* rather than being strictly dropped (i.e., all being assigned zero weights). The basic procedure is roughly sketched in figure 8.18.

To realise *selective filtering*, it is suggested to integrate *concepts of cartographic generalisation* (Weibel 1992) into the filtering process by exploiting the localisation property of the wavelet transform. Of particular interest are the processes of *selection/elimination* and *emphasis*:

- *Selection/Elimination*: The localisation property provides a means of discriminating between specific regions of the analysed data. Feature

$$\boldsymbol{DTM} \xrightarrow{\text{DWT}} W_\psi \boldsymbol{DTM} \xrightarrow{\text{weighting}} W_\psi \boldsymbol{DTM}^{w} \xrightarrow{\text{inv. DWT}} \boldsymbol{DTM}^{w}$$

Figure 8.18: Rough scheme for selective filtering in the wavelet domain (where the superscript 'w' stands for weighted).

elimination or retention, then, is achieved by intentionally dropping or retaining the localised coefficients.

- *Emphasis*: Emphasis is also achieved by identification and localisation of regions to be emphasised and subsequent intentional weighting of their corresponding wavelet coefficients (in an over proportional way). Emphasis may also be achieved by enforcing accurate reconstruction of selected topographic features through all levels of scale or by considerably smoothing adjacent regions not particularly emphasised.

As far as the remaining generalisation processes proposed by Weibel (1992) are concerned, *simplification* is a concept inherent to wavelet techniques, so it does not require explicit integration. On the contrary, selective filtering in the wavelet domain is rather an attempt to seize control of the occurring simplification. Operations such as *combination* or *displacement* may take place unintentionally, when applying (linear) filters to a wavelet transform. An example of a combination of two adjacent peaks is given by Watson and Jones (1993) who report on the occurrence of phantom peaks between two neighbouring maxima. However, enforcing and/or intentional application of combination and displacement are not investigated further here.

Establishing the weighting factors

Factors for weighting the wavelet coefficients at scales finer than the threshold scale honouring the principles of *selection/elimination* and *emphasis* may be provided in two ways: By deriving weighting coefficients from *a priori knowledge*, or by deriving weighting factors by *analysis* of the wavelet coefficients.

Wavelet Weighting Based on a priori Knowledge Weighting factors for wavelet coefficients belonging to levels of scale finer than the threshold scale may be derived from *a priori* knowledge about significant TI structures such as critical points[3], hydrological networks or shorelines. Determination of weighting matrices is trivial, provided *a priori* knowledge is available, and therefore not discussed further here. However, it is important to recall the redundancy of the discrete wavelet frames and the uncertainty principle bounding localisation of features identified in space in the wavelet domain. Consequently, it has to be ensured that weights are provided for all the wavelet coefficients concerned.

Wavelet Weighting Based on Analysis of the Coefficients Establishing weighting factors for the wavelet coefficients does not necessarily require the availability of *a priori* knowledge. Rather, information coded in the wavelet coefficients themselves may be exploited to this end. A corresponding procedure for computing a *basic weighting scheme* is proposed here. The procedure comprises the core steps to be performed when attempting to selectively filter terrain in the wavelet domain with inclusion of limited or no *a priori* information. The resulting basic weighting scheme, then, may be *arbitrarily extended* or *combined with available a priori knowledge* depending on the purposes or needs of a specific application.

Computation of the basic weighting scheme involves three steps, two of which are actually terrain-specific and one is rendered necessary by the properties of the wavelet transform. The two *terrain-specific steps* are *spatial clustering* of the wavelets to be weighted in function of the fraction of the total energy they provide for each location in the spatial domain and *detection of local extrema* in the wavelet coefficients along selected profile directions. In what follows, the three steps are described conceptually; a corresponding example illustrating them is provided in section 8.4.5.

- *Wavelet clustering*: Figure 8.7 depicts the total energy provided for each location in the spatial domain when expanding the study area into wavelets. Grouping the wavelets to be weighted into clusters of different 'total energy levels' provides a basis for subsequent filtering of regions of different information density to a different degree. So, for instance, smoothing of significant structures may be avoided, while at the same time allowing considerable smoothing of fine scale structures in rather monotonous areas.

- *Detection and thresholding of local extrema*: Local extrema of the wavelet transform at a certain scale and orientation indicate that features of extreme curvature or sharp edges occur in the analysed terrain at corresponding scale and direction (see also section 8.3.2). Suppressing coefficients causing local extrema results in smoothing the affected

[3]Gerstner and Hannappel (2000) point out that preservation of a DTM's drainage properties requires inclusion of all critical points, such as pits, peaks and passes, into the filtered model.

features. That is, the *degree of smoothing* of sharp and fairly accentuated structures can, to some degree, be controlled by the number or the amount of locally extreme coefficients suppressed. Retaining too many coefficients, on the other hand, results in a rather moderate degree of simplification. So, the art consists of finding an *appropriate threshold* for each wavelet cluster at each level of scale in order to threshold the detected wavelet transect extrema. That is, the aim is to retain only those whose magnitude exceeds the chosen threshold.

- *'Smoothing' the weighting scheme*: From a digital terrain modelling point of view, the basic weighting scheme could be complete after clustering the wavelets, detecting the local transform extrema, and deciding which of them to retain. However, working with such 'binary' filters (i.e., filters either dropping a wavelet coefficient or retaining it as is), yields a filtering result in which the retained features generally stand out in an 'unnatural' way from the filtered terrain (see figure 8.19). The reason for this effect is that by setting to zero all the wavelet coefficients to be suppressed, the 'extremality' of the retained coefficients is exaggerated relative to the other coefficients. Also, the regularity of the wavelet transform in the neighbourhoods of the retained coefficients is dramatically decreased. As a consequence, sharp edges are likely to arise in the filtered data. This effect can be attenuated by applying a *weighting function*, for instance a Gaussian weighting, to a neighbourhood of the wavelet coefficient to be retained, in order to support the regularity of the wavelet transform. The extent of the neighbourhood to which the weighting function is applied is most conveniently chosen as a function of the given level of scale, that is, of the given wavelet dilation.

The three core steps of selective filtering in wavelet domain discussed above provide a *basic weighting scheme* sufficient for terrain filtering according to the principles of selective filtering. Nonetheless, the basic weighting scheme may be arbitrarily extended if required or desired by a specific application. For instance, it may be combined with weights stemming from *a priori* knowledge, such as the inclusion of critical points to guarantee the persistence of drainage properties. Or the weighting scheme may be enhanced by adding further weighting layers serving specific purposes, such as ensuring that selected features are preserved over a range of scales.

An example of terrain filtering according to the described procedure will be presented in section 8.4.5. In order to quantitatively assess the performance of an applied filter and estimate the uncertainty introduced, some mathematical details are first required, and they are provided in the next section.

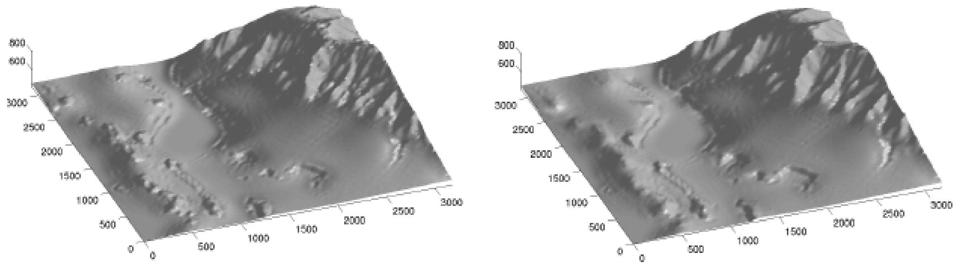


Figure 8.19: The effect of 'smoothing' the basic weighting scheme. Left: Terrain reconstruction at scale level 2 from the 'un-smoothed' weighting scheme. Right: For comparison, the corresponding reconstruction after applying a Gaussian weighting function to the basic weighting scheme (see also figures 8.23 to 8.25). Units are expressed in m (DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

8.4.3 Mathematical Details of Selective Filtering

Terrain Reconstruction from the Discrete Wavelet Transform

In the discrete case, a reconstruction formula does not generally exist as it does with formula (D.11) for the continuous case (see appendix D.2.1). Reconstruction of a function f from its discrete wavelet transform must, therefore, be accomplished by some other means. The problem is, however, that the discrete transform is known only for a finite number of scales $a_0^m < a_c$. Then, to recover the original function f, a complement of information corresponding to $\mathcal{W}f_{m,\theta,[n_1,n_2]}$ for the scales $a_0^{m+1} > a_c$ is needed. Mallat (1998) showed how this can be obtained for the one-dimensional case by introducing a scaling function ϕ that is an aggregation of the coarse scale wavelets (for details on how to construct ϕ, please be referred to Mallat (1998)). Extending the theory to two dimensions, a *low frequency approximation* of f at scale a is obtained as follows:

$$Lf(a,\theta,\boldsymbol{b}) = \langle f(\boldsymbol{x}), \frac{1}{a}\,\phi_\theta(\frac{\boldsymbol{x}-\boldsymbol{b}}{a})\rangle = f * \phi^*_{a,\theta}(-\boldsymbol{b}).$$

The original function f, then, amounts to:

$$f(\boldsymbol{x}) = c_\psi^{-1}\int_0^{a_c}\int_0^{2\pi}\mathcal{W}f_{a,\theta}(\cdot) * \psi_{a,\theta}(\boldsymbol{b})\,d\theta\,\frac{da}{a^3} \;+\; \frac{1}{c_\psi a_c^2}Lf_{a,\theta}(\cdot) * \phi_{a_c}(\boldsymbol{x}). \quad (8.2)$$

An *inverse wavelet transform* is implemented by discretising the integrals 8.2. Suppose that $a_0^K = 2$ is the finest scale. Since $\frac{da_0}{a_0^3} = \frac{d\ln a_0}{a_0^2}$ and since the discrete wavelet transform is computed along an exponential scale sequence $\{a_0^k\}_k$ with logarithmic increment $d\ln s = \ln a_0$, one obtains:

$$f[\boldsymbol{x}] = \frac{\ln a_0}{c_\psi} \sum_{k=K}^{J} \frac{1}{(a_0^j)^2}\,\mathcal{W}f[a_0^k, \theta, \cdot] * \psi_{k,\theta}[\boldsymbol{x}] \;+\; \frac{1}{c_\psi a_0^{2J}}\,Lf[a_0^J, \theta, \cdot] * \phi_\theta[\boldsymbol{x}]. \tag{8.3}$$

If a rotation invariant wavelet such as the Mexican hat is used, the wavelet transform does not depend on the angle θ and (8.3) simplifies to:

$$f[\boldsymbol{x}] = \frac{\ln a_0}{c_\psi} \sum_{k=K}^{J} \frac{1}{(a_0^j)^2}\,\mathcal{W}f[a_0^k, \cdot] * \psi_k[\boldsymbol{x}] \;+\; \frac{1}{c_\psi a_0^{2J}}\,Lf[a_0^J, \cdot] * \phi[\boldsymbol{x}]. \tag{8.4}$$

Detail Signal at Scale a_0^k

It follows from equation (8.3) and the properties of the Fourier transform that the fraction of f concentrated at scale a_0^k and angle θ can be equated with:

$$\Delta_{a_0^k} f = \frac{\ln a_0}{c_\psi}\,\frac{1}{a_0^k}\,\widehat{\mathcal{W}f}_{k,-\theta} \cdot \widehat{\psi_{-\theta}[a_0^k\omega]}. \tag{8.5}$$

And hence:

$$f[\boldsymbol{x}] = \sum_{k=K}^{J} \Delta_{a_0^k} f \;+\; \frac{1}{c_\psi a_0^{2J}}\,Lf[a_0^J, \theta, \cdot] * \phi_\theta[\boldsymbol{x}]. \tag{8.6}$$

Again, for rotation invariant wavelets equation (8.6) simplifies to:

$$f[\boldsymbol{x}] = \sum_{k=K}^{J} \Delta_{a_0^k} f \;+\; \frac{1}{c_\psi a_0^{2J}}\,Lf[a_0^J, \cdot] * \phi[\boldsymbol{x}]. \tag{8.7}$$

8.4.4 Estimates for the Uncertainty Introduced

The objective of this chapter is to provide local quantification of the uncertainty related to representing terrain at a specific level of scale. Such local uncertainty estimates can be obtained by directly accessing the fraction of the represented terrain concentrated at scale a_0^k in the spatial domain from equation (8.5). It follows (for the rotation invariant Mexican hat) that:

$$f[\boldsymbol{x}] = \sum_{k=K}^{J} \Delta_{a_0^k} f[\boldsymbol{x}] \;+\; Lf[\boldsymbol{x}],$$

where $Lf[\boldsymbol{x}]$ denotes the low frequency approximation of f.

Now, let $\mathcal{W}f^s_{k,\,[n_1,n_2]} := \mathcal{W}f_{k,\,[n_1,n_2]} - \mathcal{W}f^w_{k,\,[n_1,n_2]}$ be the difference between the original and the weighted wavelet coefficients. Due to the additivity of convolution and integral, it follows from (8.6) that:

$$\Delta_{a_0^k} f[\boldsymbol{x}] = \Delta_{a_0^k} f^w[\boldsymbol{x}] \;+\; \Delta_{a_0^k} f^s[\boldsymbol{x}], \quad K \leq k \leq J, \tag{8.8}$$

for each investigated scale a_0^k. Therefore, the original function f can be expressed as:

$$\begin{aligned} f[\boldsymbol{x}] &= \sum_{k=K}^{J} \Delta_{a_0^k} f[\boldsymbol{x}] \;+\; Lf[\boldsymbol{x}] && (8.9) \\ &= \left[\sum_{k=K}^{J} \Delta_{a_0^k} f^w[\boldsymbol{x}] \;+\; \sum_{k=K}^{J} \Delta_{a_0^k} f^s[\boldsymbol{x}] \right] \;+\; Lf[\boldsymbol{x}]. && (8.10) \end{aligned}$$

Equation (8.10) says that the uncertainty related to representing the terrain at a specific scale a_0^k can be locally addressed by directly accessing the fraction $\Delta_{a_0^k}$ of the terrain (the actually analysed function f) constituted by the suppressed (totally or partially) wavelet coefficients at each scale a_0^k, $K \leq k \leq J$, in the spatial domain.

8.4.5 Experiment and Evaluation

In this section, the *selective terrain filtering* approach is applied to the study area. First, it is discussed how the *basic weighting scheme* was computed.

Configuring the Basic Weighting Scheme

Wavelet Clustering Clustering of the wavelets to be weighted is based on an analysis of the total amount of energy resulting for each location in space when expanding the terrain into Mexican hat wavelets. Figure 8.20 plots the values of *total energy* provided for each location in space (see also figure 8.7) in sorted and ascending order. Based on this plot, the wavelets were grouped into four clusters C1 to C4, where C1 refers to the lowest level of energy (i.e., lowest information density) and C4 to the highest (i.e., highest information density; as already indicated in figure 8.20), respectively. Figure 8.21 spatially depicts the clustering, while table 8.2 provides some summary statistics.

Detection and Thresholding of Local Extrema There exists a fair amount of literature on wavelet thresholding. However, the most part of it focuses on preferably lossless data compression, usually based on a multiresolution analysis. Due to the different objectives for thresholding and

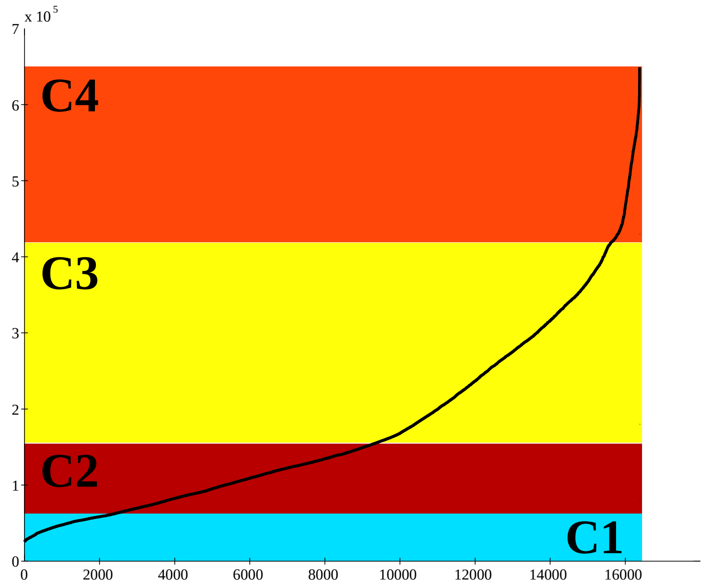


Figure 8.20: Total energy values provided by the wavelet representation of the study area in sorted and ascending order. Based on these total energy values the wavelets are grouped into four clusters C1 to C4.

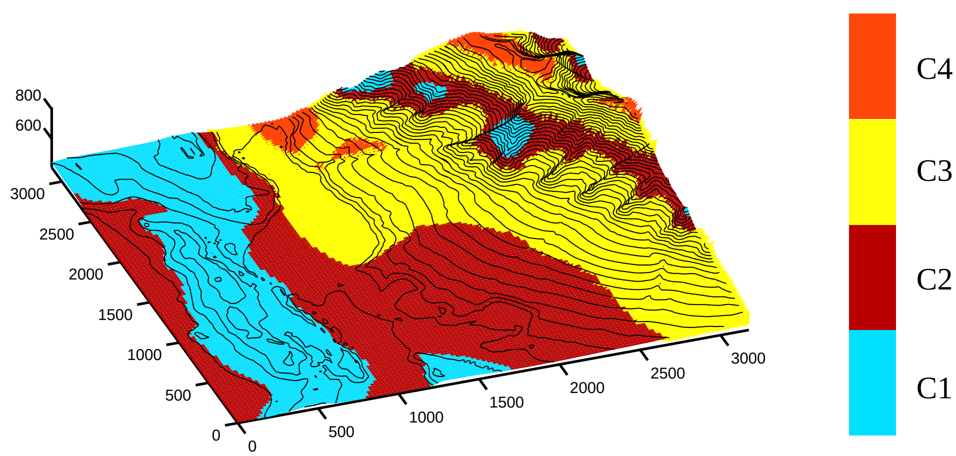


Figure 8.21: Wavelet coefficient clusters (all units are provided in m; DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

	Cluster definition ($E(\boldsymbol{x})$: Energy provided at location $\boldsymbol{x}$ in space domain)	*Percentage of area covered*	*Amount of energy provided*	*Percentage of total energy provided*
C1	$0 \leq E(\boldsymbol{x}) \leq 0.8\,10^5$	23.25	$2.17\,10^8$	7.54
C2	$0.8\,10^5 < E(\boldsymbol{x}) \leq 1.8\,10^5$	40.20	$8.19\,10^8$	28.52
C3	$1.8\,10^5 < E(\boldsymbol{x}) \leq 4.3\,10^5$	33.03	$1.55\,10^9$	53.85
C4	$E(\boldsymbol{x}) > 4.3\,10^5$	3.52	$2.9\,10^8$	10.90

Table 8.2: Summary statistics for the wavelet clusters.

Level of scale	***Threshold***	*Mean (of locally extreme wavelet coeff. moduli)*	*Number of coefficients retained*	*Percentage of coefficients retained*
1	**1.88**	0.94	405	10.63
2	**5.44**	2.72	347	9.11
3	**12.50**	6.25	377	9.90

Table 8.3: Summary statistics for wavelet thresholding in cluster C1.

the different approaches applied (discrete wavelet transform and multiresolution analysis), the results found in the literature did not prove very useful in the context of this research.

Hence, the thresholds were also determined by *statistical analysis* of the corresponding wavelet coefficients. In the example reported here, a basic weighting scheme was derived for the levels of scale 1, 2 and 3. For this example it was found that for each cluster and each of the investigated levels of scale, retaining all the detected wavelet extrema whose amplitude modulus amounts to at least twice the mean of the respective coefficients still yielded acceptable results. However, bearing in mind that the clustering was introduced to enable filtering of areas of varying information density to a different degree, the two 'high energy' clusters C3 and C4 were thresholded using the mean at scale levels 3 (for C3) and 2 (for C4) and using 0.5 times the mean for level of scale 3 for cluster C4, respectively. Figure 8.22 visualises the moduli of the locally extreme wavelet coefficients sorted in ascending order together with the chosen thresholds for each cluster and all the investigated levels of scale, while tables 8.3 to 8.6 give the exact *threshold values* as well as some summary statistics.

Smoothing the Weighting Scheme The weighting scheme resulting from clustering, wavelet maxima detection and thresholding was 'smoothed'

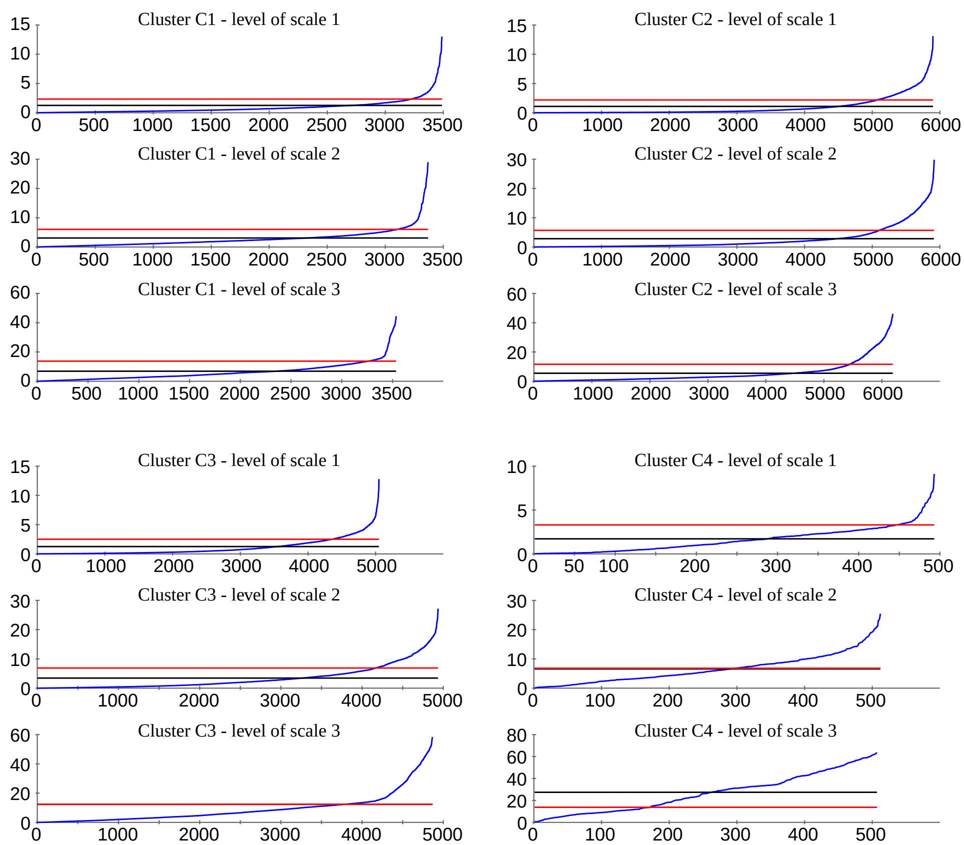


Figure 8.22: Locally extreme wavelet coefficient moduli sorted in ascending order (blue). The black lines depict the mean of the locally extreme wavelet coefficient moduli, the red lines the thresholds applied.

Level of scale	***Threshold***	*Mean (of locally extreme wavelet coeff. moduli)*	*Number of coefficients retained*	*Percentage of coefficients retained*
1	**1.88**	0.94	932	14.15
2	**5.12**	2.56	870	13.21
3	**11.00**	5.48	776	11.78

Table 8.4: Summary statistics for wavelet thresholding in cluster C2.

Level of scale	***Threshold***	*Mean (of locally extreme wavelet coeff. moduli)*	*Number of coefficients retained*	*Percentage of coefficients retained*
1	**2.2**	1.10	963	17.78
2	**6.88**	3.44	803	14.84
3	**9.24**	9.24	1757	32.46

Table 8.5: Summary statistics for wavelet thresholding in cluster C3.

Level of scale	***Threshold***	*Mean (of locally extreme wavelet coeff. moduli)*	*Number of coefficients retained*	*Percentage of coefficients retained*
1	**3.20**	1.61	65	11.27
2	**6.50**	6.47	224	38.82
3	**13.00**	26.62	347	60.14

Table 8.6: Summary statistics for wavelet thresholding in cluster C4.

using a (dilated) Gaussian $f_g = \exp\left(-\frac{|\boldsymbol{x}|^2}{4}\right)$ in order to avoid sharp edges in the reconstructed signal. For each scale, the 'window' of the weighting function f_g was scaled to the corresponding dilation of the wavelet.

Filtering Results and Uncertainties Introduced

Filtering Results The results of the approach are visualised in figures 8.23 to 8.25. When looking at the results produced, it becomes evident that the rugged terrain is retained with much more detail than the smoother areas, as a result of the less decimating thresholds applied to the 'high energy' clusters (see tables 8.5 and 8.6). Application of more decimating thresholds to the clusters C3 and C4 instantaneously leads to more simplified results, as can be seen in figures 8.26 and 8.27 (where thresholds of 1.8 times the mean were applied instead of using the mean or even half the mean; cf. tables 8.5 and 8.6).

A second effect clearly visible is that the lake surface still gets somehow perturbated despite retaining wavelet coefficients considered significant also at scale levels finer than the threshold scale. This is, to a large degree, due to the small spatial extent of the lake. Particularly at scale level 3, the chance is considerable that a wavelet hits not only the lake but also parts of its surrounding, thus causing a significant non-zero coefficient (cf. also the profile in figure 8.2). The lake is, then, perceived as part of a concave

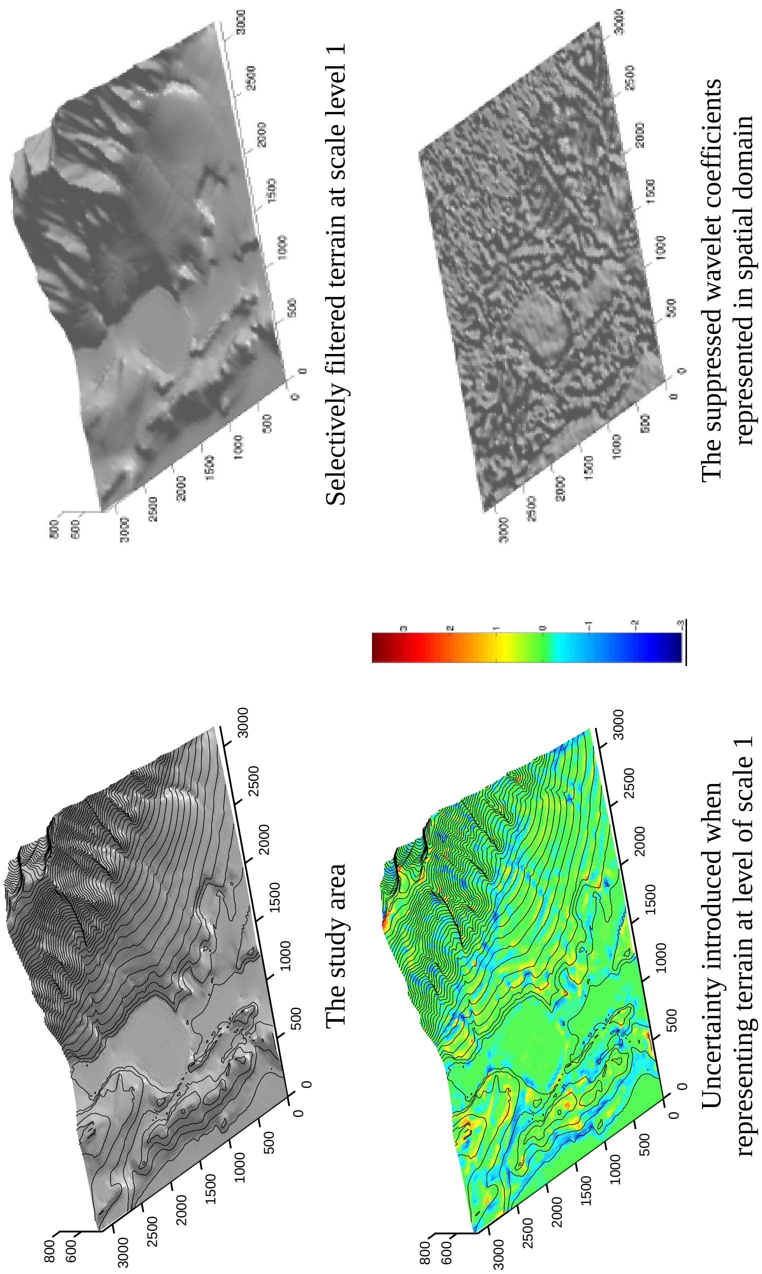


Figure 8.23: Terrain representation at scale level 1 resulting from applying selective filtering to the original data. Bottom line: Uncertainty introduced when representing the terrain at scale level 1, visualised as colorplot (bottom left) or by directly accessing the suppressed wavelet coefficients in the spatial domain (bottom right; all units in m; DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

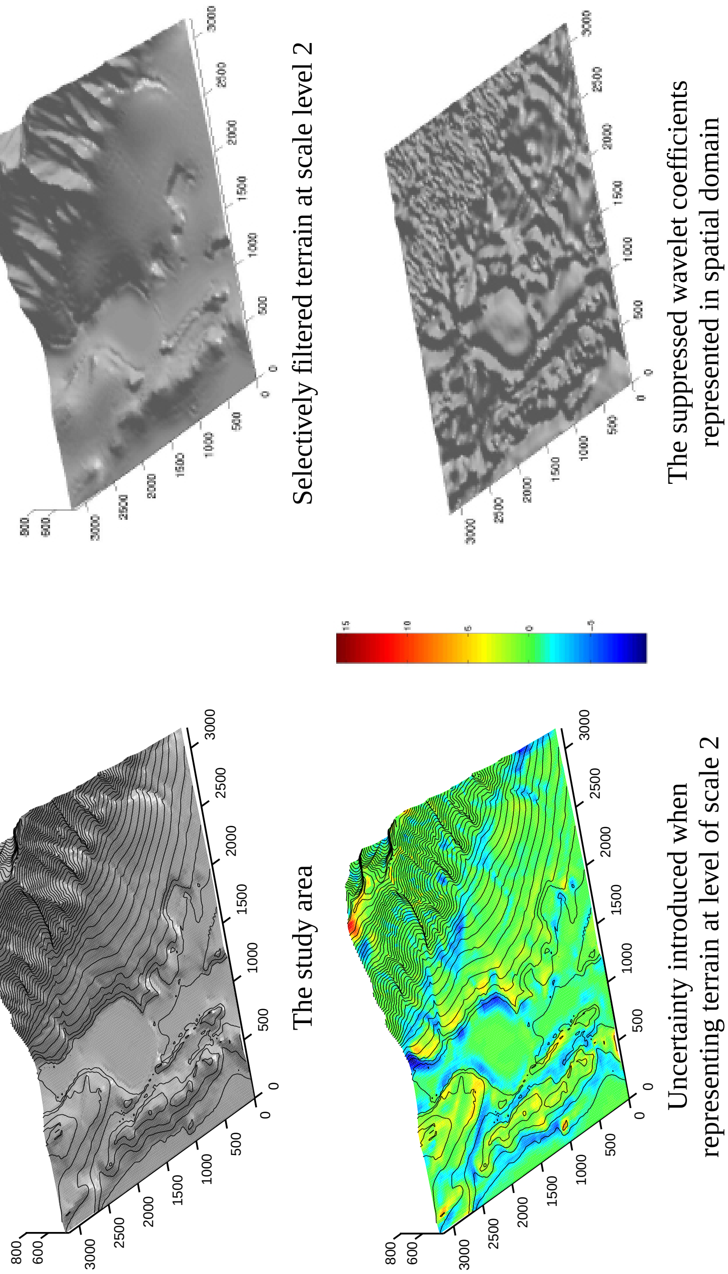


Figure 8.24: Terrain representation at scale level 2 resulting from applying selective filtering to the original data. Bottom line: Uncertainty introduced when representing the terrain at scale level 2, visualised as colorplot (bottom left) or by directly accessing the suppressed wavelet coefficients in the spatial domain (bottom right; all units in m; DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

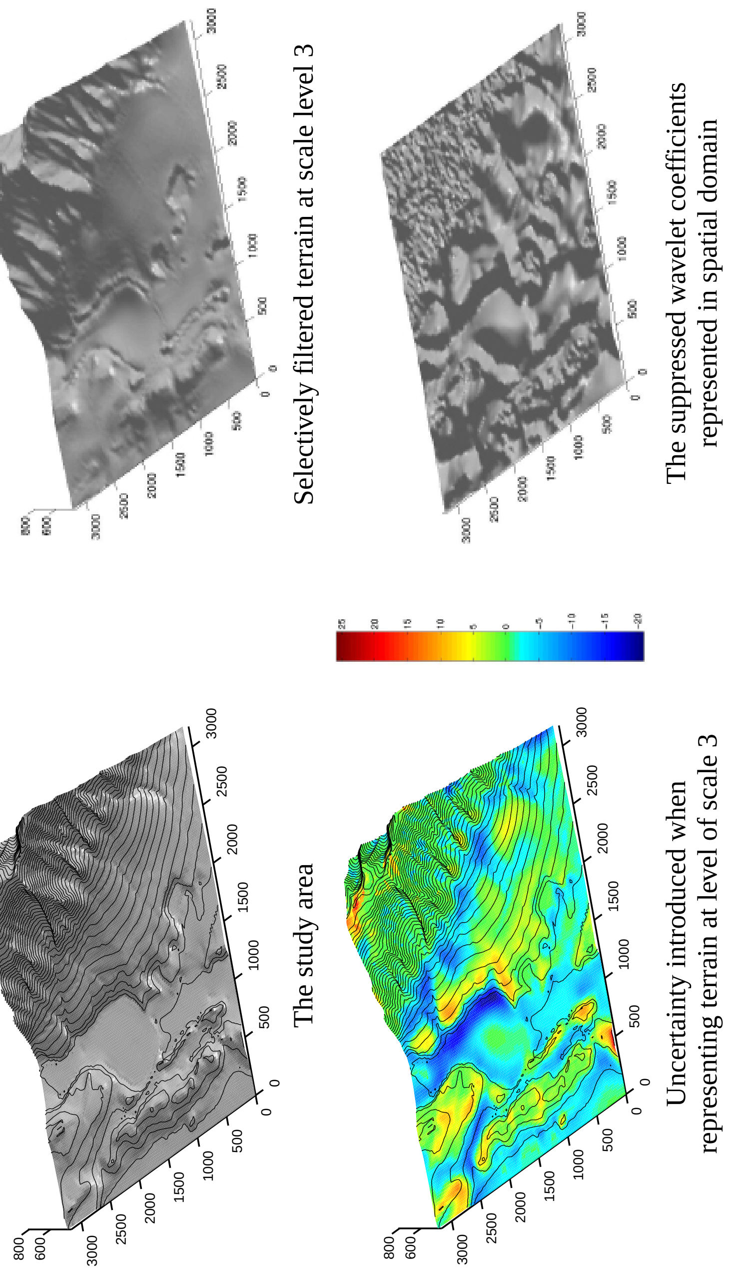


Figure 8.25: Terrain representation at scale level 3 resulting from applying selective filtering to the original data. Bottom line: Uncertainty introduced when representing the terrain at scale level 3, visualised as colorplot (bottom left) or by directly accessing the suppressed wavelet coefficients in the spatial domain (bottom right; all units in m; DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

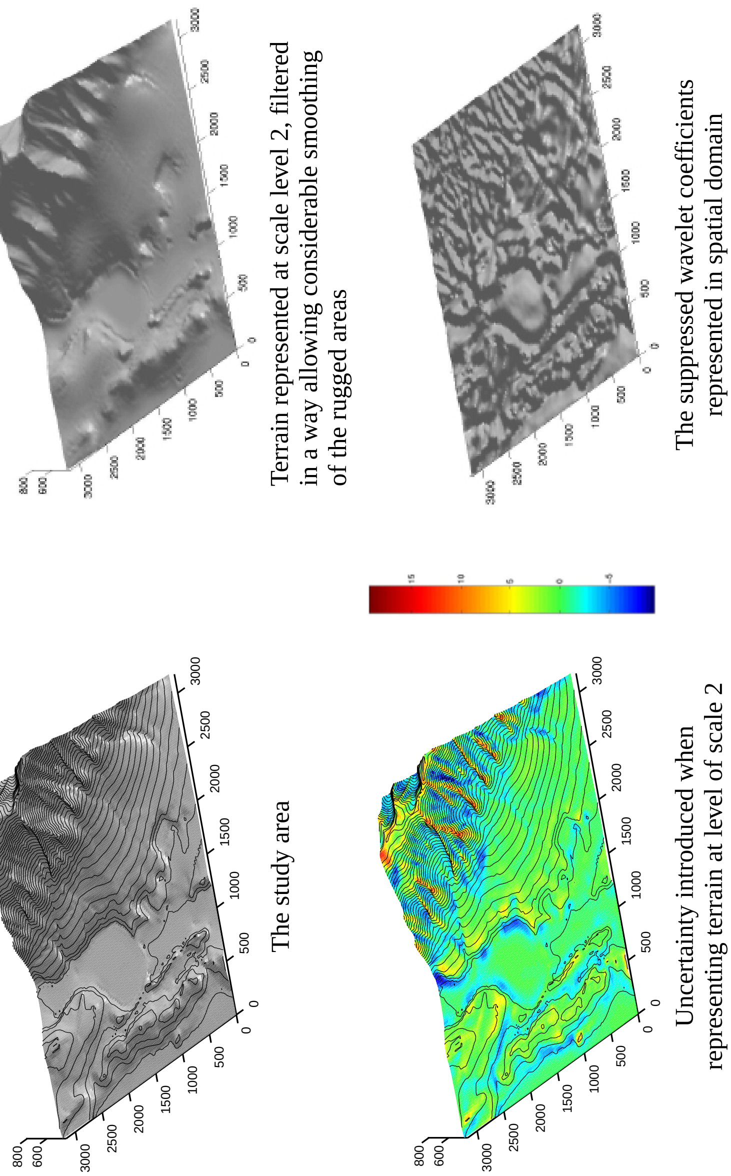


Figure 8.26: Terrain representation at scale level 2 resulting from applying selective filtering to the original data allowing more smoothing in the rugged areas. Bottom line: Uncertainty introduced when representing terrain at scale level 2 by thresholding the 'high energy' areas in a rather decimating way, visualised as colorplot (bottom left) or by directly accessing the suppressed wavelet coefficients in spatial domain (bottom right; all units in m; DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

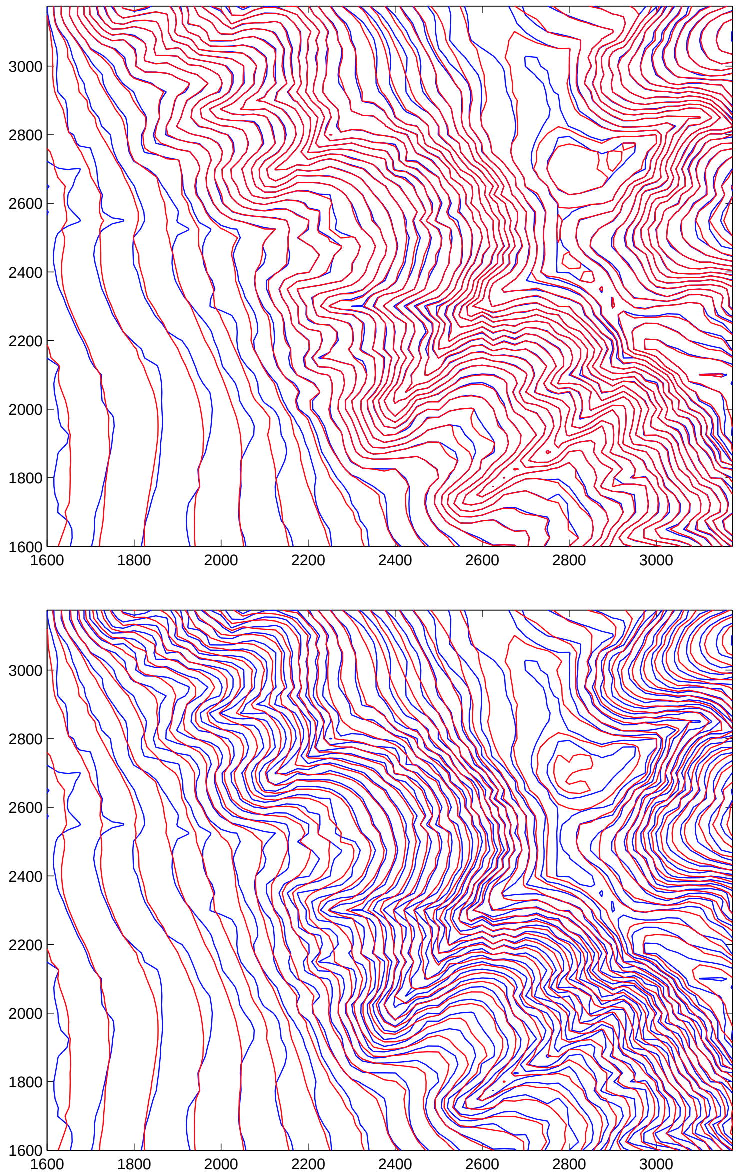


Figure 8.27: Contours (at 10 m contour interval) for the rugged part of the study area. Blue: Contours for the original data. Red: Contours for terrain represented at scale level 2; above: Retaining a high degree of detail in the rugged area (cf. figure 8.24), bottom: Allowing more smoothing in the rugged area (cf. figure 8.26). All units are expressed in m (DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

structure, indicated by negative wavelet coefficients. However, as may be intuitively clear, no significant structures are likely to be detected *on* the lake at finer scale levels. The corresponding coefficients are, therefore, suppressed and so they are missing to level the lake surface again. This means that the effect is not directly an artifact of the basic weighting procedure proposed and, therefore, can not be handled by adjusting the thresholds deciding the weighting or rejecting wavelet coefficients.

The poor reconstruction of the rolling hill at the eastern side of the study area also deserves some consideration, particularly at scale level 3. To analyse what happened there, it is worth looking at the fraction of the terrain being concentrated at scale levels 1 to 3 (in the spatial domain). From figure 8.28 it becomes clear that the rolling hill is, to a great extent, concentrated at scale level 3. Discarding this scale level for reconstruction purposes, thus, leads to 'erosion' of the rolling hill (see also fig 8.17). Retaining the significant wavelet extrema, as suggested in this approach, however retains only the ridge of the hill, letting foray the hill slope.

Uncertainty Introduced The locally resulting deviations from the original data can be estimated by means of the formula (8.7) and are visualised in figures 8.23 to 8.25, bottom lines; table 8.7 provides some summary statistics. The results depicted in figures 8.23 to 8.25 indicate that convex regions are 'eroded' by the filtering process while accumulation occurs across concave areas. This fact highlights the nature of wavelets as an *averaging technique*. Another indication to the averaging nature of wavelets may be found in table 8.7, which shows that the means of the signed deviations are concentrated around zero in all clusters and for all levels of scale with only very few exceptions.

Table 8.7 also indicates that both the entire study area as well as the individual clusters C1 to C3 tend to be 'accumulated', that is, super elevated, by the filtering applied. Only cluster C4 shows a tendency towards 'erosion'. However, this finding matches the results discussed in section 8.3.2, where the study area was characterised to a great extent as concave (see also figure 8.9). Another way to visualise this finding is provided in figure 8.29, where the signed deviations resulting from the applied filtering are plotted against their percental frequency (for each cluster and for all levels of scale). The intuitive interpretation of the pictures in figure 8.29 is that the higher the total percentage of small or no uncertainties caused, the less the smoothing occurred, and vice versa. As indicated by the figure, the uncertainties introduced by the applied selective filtering approach are far from being normally distributed. Interpretation of the standard deviations given in table 8.7 must, therefore, be handled with care. However, when attempting to draw conclusions from figure 8.29 it must be taken into consideration that, while the spatial extent of the clusters C1 to C3 is in the

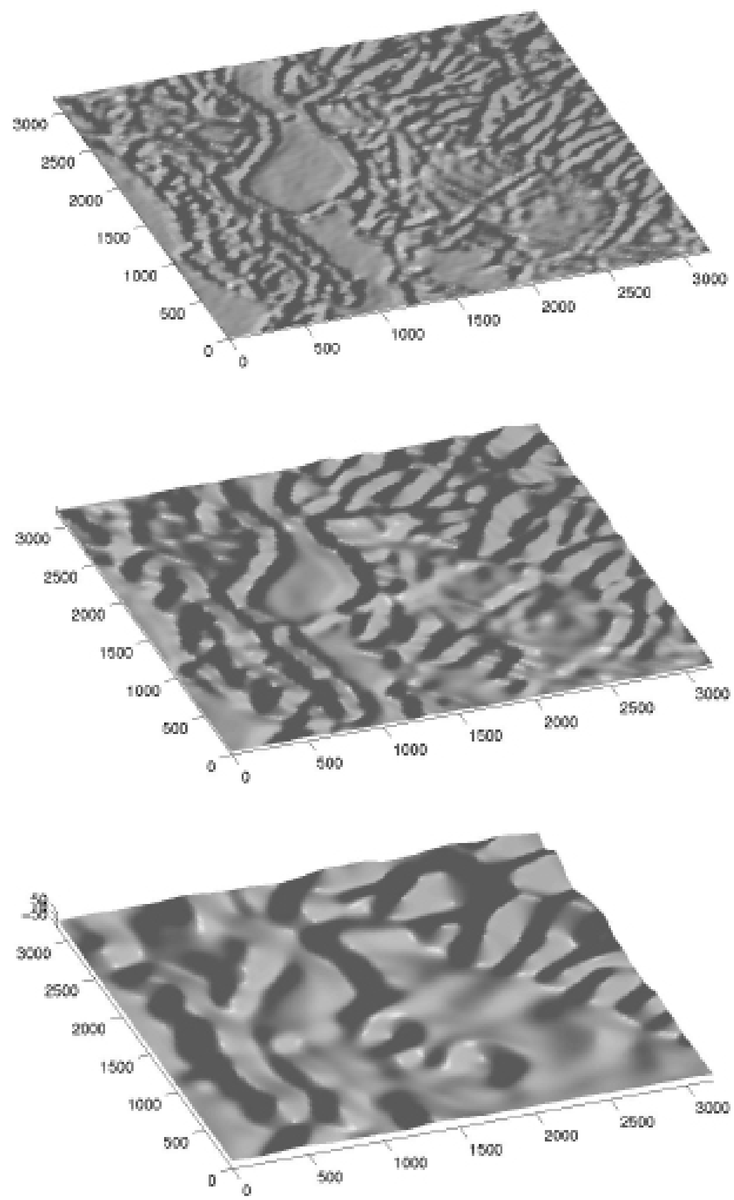


Figure 8.28: The fraction of the study area being concentrated at scale levels 1 (top), 2 (middle) and 3 (bottom; all units in m; DTM-Data: DHM25, reproduced by permission of the Swiss Federal Office of Topography (BAO13927)).

	Scale	*Min. deviation*	*Max. deviation*	*Abs. mean*	*SD (absolute)*	*Signed mean*	*SD (signed)*	*No. pos. deviations*	*Perc. pos. deviation of tot. deviation*	*No. neg. deviations*	*Perc. neg. deviations of tot. deviation*
Entire study area	1	-3.01	3.69	0.21	0.29	-0.03	0.36	5233	41.88	7964	58.12
	2	-9.59	15.81	1.12	1.25	-0.25	1.66	5736	39.01	9011	60.99
	3	-20.96	25.80	3.17	3.13	-1.16	4.30	5651	31.64	9680	68.36
C1	1	-1.77	2.32	0.29	0.33	-0.02	0.43	1416	46.24	1712	53.76
	2	-5.95	8.00	1.44	1.17	0.03	1.86	1892	50.97	1709	49.03
	3	-12.55	19.40	3.12	2.74	0.52	4.27	2146	57.89	1543	42.19
C2	1	-1.98	2.03	0.18	0.25	-0.04	0.31	1936	38.41	3595	61.59
	2	-5.16	5.50	0.80	0.88	-0.25	1.16	1979	34.43	3935	65.57
	3	-15.09	10.73	2.26	2.04	-1.05	2.86	1792	26.81	4263	73.19
C3	1	-3.0	2.17	0.19	0.26	-0.06	0.32	1639	35.55	2470	64.45
	2	-9.59	4.89	1.07	1.19	-0.54	1.51	1564	25.03	3146	74.97
	3	-20.96	14.56	3.98	3.92	-2.67	4.91	1309	16.46	3663	83.54
C4	1	-2.15	3.69	0.33	0.50	0.15	0.58	242	72.44	187	27.56
	2	-9.22	15.81	3.04	2.80	0.70	4.07	301	61.75	221	38.43
	3	-11.75	25.80	4.94	4.17	0.55	6.44	304	55.58	221	44.42

Table 8.7: Summary statistics for the generated uncertainty

same order of magnitude, C4 is smaller by a factor of 10. Therefore, the individual deviations occurring in C4 preponderate more when compared to the other clusters. For instance, the 'peak' in the northern part of cluster C4 levelled down by the applied filtering (cf. figures 8.23 to 8.25), causes the entire cluster to statistically display a down levelling tendency and entails considerable mean uncertainty values.

Figures 8.23 to 8.25 and 8.26 nicely visualise both the amount of uncertainty introduced and its spatial structure, however at the expense of not providing any statistical statements. While table 8.7, on the contrary, exposes uncertainty extrema together with some main statistical quantities, it does not allow for any interpretation of the occurring uncertainty in terms of its spatial structure. Moreover, the significance of the provided statistics is heavily affected due to the analysed deviations not being normally distributed. Interpretation of figure 8.29, finally, may be biased when not appropriately considering topographical occurrences. So, for instance, the lake situated in the study area mainly falls into clusters C2 and C3. Satisfying filtering results require a careful reconstruction of the lake, while the two clusters may otherwise be considerably smoothed. This requirement results in a high percental frequency of zero or small uncertainties, thus seemingly pretending a moderate smoothing of the entire cluster if the spatial structure of the induced derivations is not considered.

Discussion of the Selective Filtering Approach

The proposed approach to selective filtering in the wavelet domain is deemed a promising tool for controlled surface simplification in digital terrain modelling. This positive judgement is, particularly, due to the *localisation property* of the wavelet transform, which enables a *selective* and, to some degree, *intentional* procedure, thus opening a door to the 'traditional' concepts of cartographic generalisation. The localisation property also allows easy integration of *a priori* knowledge, if available. Finally, its ability to provide a *local estimate of the uncertainty introduced* when changing representation scale makes the method very attractive.

However, a range of problems have yet to be overcome. While the localisation property, at first suggests a lot of possibilities in terms of selective and, to some degree, intentional filter procedures, in the end, being honest, everything again comes down to *statistical analysis*. Further research is definitely needed to investigate relations between the amplitude of the wavelet coefficients and useful thresholds and/or thresholding techniques.

Another bulk of open questions is related to *boundary problems*. In the example provided in this section boundary problems were avoided by excluding the boundary regions from analysis. However, for successful application of the method, boundary issues call for thorough investigation and

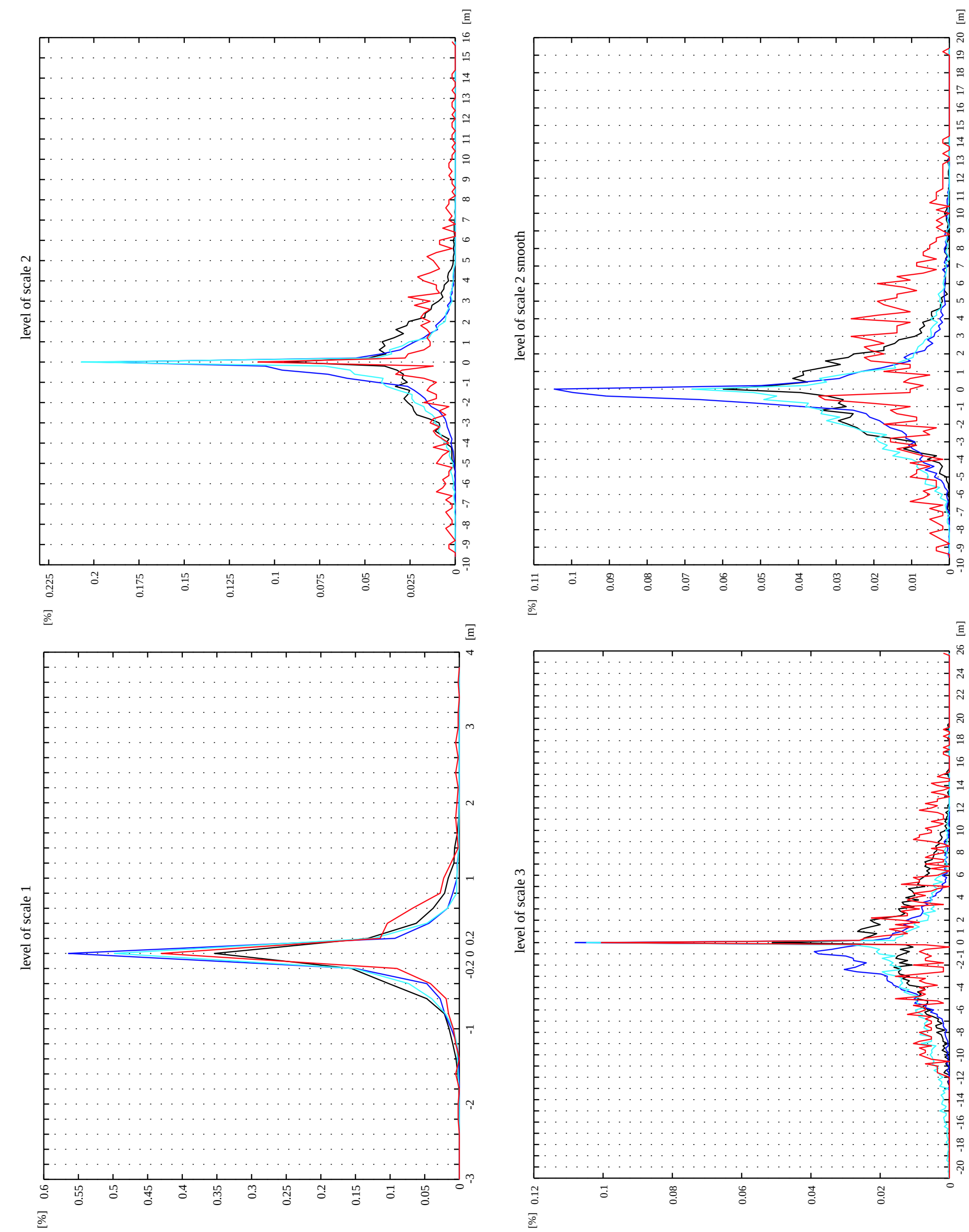


Figure 8.29: Percental frequency of the introduced uncertainty values. The colours denote the 4 clusters: Cluster C1 (black), C2 (blue), C3 (cyan), C4 (red).

proper treatment. In a similar way, the 'lake problem' discussed in the previous section may be interpreted as an 'internal boundary problem', for it eventually arises from the wavelet transform not properly interpolating the boundary of an existing, more or less well-defined geographical object.

8.5 Review

Recognising terrain as a function of, among other things, scale, means that 'scale-aware' techniques for terrain representation should be provided. This section presented the (discrete) wavelet transform as a promising tool for the analysis of scale dependencies in topography. It helps identify features at a range of scales, thus allowing simplification of the terrain surface, and multiscale terrain representation. Finally it allows for the investigation of the effect of scale on the terrain surface without introducing artifacts due to coarsening of the original DTM resolution.

Chapter 9

Conclusions and Outlook

9.1 Achievements

At the outset of this thesis the overall objective of the research was to *elaborate approaches* to *handling quality issues in digital terrain modelling.* It was pointed out that in order to gain practical relevance, approaches to TI quality handling must embrace both *producer* and *user perspectives*, and be equally applicable to both the production and application domains.

The achievements of the thesis are discussed on the basis of the research questions listed in section 1.5. These were:

Definition of what the concept of quality may encompass Of the five conceptions of quality discussed in section 1.3, *fitness for use* was considered best suited to provide an overall paradigm for quality handling in digital terrain modelling, because it:

- provides a common baseline for TI quality handling across the heterogeneous communities using such information; and
- apportions and emphasises the responsibilities between producers and users of TI.

However, the nature of digital terrain modelling is that of a 'process' rather than a 'product'. Therefore, it was recognised that the attempt to manage the various factors affecting the given TI quality generated and propagated throughout this process, requires integration of *process-related quality notions* within the fitness for use paradigm.

Based on such considerations, the *definition of quality* provided by the ISO Standard 19113 (Geographic Information - Quality Principles) was adopted, where quality is formalised *without reference to truth*, and is not restricted to notions of accuracy.

Development of a model of the terrain data life-cycle A *workflow framework for reliable application of TI in spatial modelling applications* was proposed in section 4.3.3, tracing the *life-cycle of terrain data* from its very conceptualisation to its actual storage in a virtual dataset (VDS), possibly accessible for re-use. The workflow arranges the influence domains of TI users and producers on the terrain data life-cycle, in that it:

- assigns to the *TI users* the responsibility to elaborate the *abstraction* to be modelled as well as the admissible simplifications and idealisations. That is, to formulate and specify the *rules* initiating the life-cycle; and

- apportions the *TI producer* the responsibility to create and maintain the dictionaries by which translation is provided from the real world to the mathematical and digital world and back.

Identification of the sources and roles of factors affecting TI quality within the life-cycle of terrain data Investigation of the life-cycle of terrain data revealed *three major factors affecting TI quality*, namely (chapter 5):

- *Uncertainty*, which arises from deficiencies in the individual steps of the DTM generation and the TI derivation process. Uncertainty, hence, combines aspects of both a *fitness for use* and a *product-related* understanding of quality (section 1.3), and thus contributes to the integration of the latter quality concept into the overall discussion.

- *Consistency*, which is a measure of the integrity of the digital terrain modelling workflow. Particularly, consistency issues concerning the actual workflow through the modelling stages pay attention to the *process-related* quality concept and hence may be interpreted as aspects of *quality assurance*.

- *Validity*, which is a measure of the genericness of given TI. Validity is concerned with the re-usability of data and models supplied for applications other than the ones envisioned originally, and thus provides information compatible with the idea of *fitness for use*.

Besides these aspects stemming from the workflow perspective taken in chapter 5, a few additional factors influencing TI quality were identified (chapter 6), including *history* of models and data (reviewing the data's life-cycle, for instance), *technical issues*, arising from digital implementation, and other *miscellaneous questions* such as contact information or information about access.

Identification of a set of metadata components appropriate to TI quality description Chapter 6 suggested three primary *metadata components* by which to organise knowledge about factors affecting TI quality:

- *Modifiers* make explicit the mappings involved in the digital terrain modelling workflow, and thus provide part of the *quality norm*.

- *Descriptors* provide *informative* data explaining the supplied TI by, for instance, recounting its life-cycle or describing details about its technical implementation or its availability.

- *Quality elements* measure the actual performance of the supplied TI.

These components were substantiated by naming *metadata elements* necessary for comprehensive TI quality documentation, further developed in appendix C. The components were accompanied by basic *principles for TI quality reporting*, including mechanisms for reporting quality on virtual data and for supporting quality assurance.

The concepts presented in chapter 6 basically extend the concepts provided by the ISO Standard 19113 (Geographic Information - Quality Principles), providing improvements in four respects:

- Aspects of quality *missing* in the Standard are added (such as measures of a dataset's information content, descriptors for recording technical quality or a structure to support the concept of validity). The processes of abstraction and specification essentially influence the *semantics* of the resulting data. Therefore, their description is inevitable for comprehensive TI quality documentation.

- The concept of *modifiers* is introduced to enable integration of the abstraction and the product specifications into the actual dataset.

- Concepts, structures and reporting principles for recording metadata describing not only supporting data but also *virtual data* are provided.

- A tool for *quality assurance* is added by allowing reporting of metainformation on metainformation.

While the perspective to reporting metainformation adopted in chapter 6 was rather a producer-oriented one, chapter 7 provided a *conceptual model for storing of metadata* documenting TI quality that:

- structures the metadata elements according to the kind of information they provide to a TI user, and

- incorporates a mechanism for *quality assurance*.

Development of methods for assessing and coping with factors affecting TI quality Within the scope of this research, this question could not be comprehensively addressed. Instead, one aspect of special importance was focused on, namely the issue of *scale*. The scale dependence of landforms is widely recognised, yet scale and its effects are rarely considered a factor affecting TI quality, and consequently it is not generally considered in discussions of TI quality. Wavelet techniques were presented in chapter 8 as a promising tool for:

- *Local estimation* of the uncertainty introduced when representing a terrain at different levels of scale by allowing local access to the fraction of the terrain surface concentrated at a specific level of scale. Therefore, to achieve terrain representation at different levels of scale, a *selective filtering procedure in the wavelet domain* was presented.

- Enabling *'scale aware' modelling techniques*, particularly by easily integrating methods for multiscale modelling.

- Allowing changes in the representation scale without necessarily changing the model's resolution, thus enabling decoupling of the level of detail displayed in the DTM from the DTM mesh size (that is, from its discretisation unit).

9.2 Insights

Section 1.4 defined digital terrain modelling as the numerical modelling and representation of the terrain surface with digital computers in order to allow derivation of TI required in spatial modelling applications. Its core task was equated to providing a mapping between the terrain surface and a suitable mathematical representation enabling the use of mathematical inference rules to derive statements about the topographic surface being investigated. Based on this view, the main insights provided by the thesis are briefly discussed.

Reliable DTMs After section 4.1, *reliable DTMs* are required to supply TI in a *consistent* and *replicable* way. This condition may not be met unless

- an *explicit* and *suitable* reconstruction of the topographic surface is provided (section 4.1), and

- *transparent access* to both the supporting data and the derived TI as proposed by the Virtual Data Set (VDS) concept is supported (section 4.3, chapters 6 and 7, appendix C).

It was argued in section 4.3 that the specification of a well-behaved, finite digital terrain reconstruction necessarily requires incorporation of additional *knowledge* and *assumptions* derived from *application-specific rules* into the modelling and representation process. 'Topographic reality' was identified to be best represented by a 'set of sets' $\{\mathcal{M}; (T_R; \mathcal{D}_T)\}$ conforming to the notion of a VDS, where

- $\mathcal{M}$ stands for a *set of metainformation* capturing, among other things, the rules driving the modelling process. $\mathcal{M}$ provides the metainformation about the model required for appraising the quality of the supplied TI;

- T_R and $\mathcal{D}_T$ provide the mathematical representation, in that T_R denotes a *terrain reconstruction* specified subject to the prescribed rules and *parameterised* by $\mathcal{D}_T$.

The Relationship between the 'Real' Terrain and its Formal Representation Any terrain representation always portrays an *abstraction* of the true terrain - never the true terrain itself. Due to the magnitude of possibilities in which to plausibly abstract a terrain, *maintainance of the linkage* between the 'real world', the abstraction, and the terrain representation is required

- to ensure correct and unambiguous interpretation of the *semantics* of a DTM, where semantics, according to section 4.3, are understood as the relationship between a real terrain and its digital representation; and

- to provide a *frame of reference* for DTM quality assessment.

TI Quality Documentation Investigation of common geospatial data quality standard's (such as the ISO Standard family on geographic information) fitness for TI quality description revealed several problems:

- Aspects of quality essential to comprehensive TI quality documentation were *missing* in the standards. Missing quality aspects include, for instance, measures of a dataset's information content, quality elements considering the influence of scale, or descriptors for recording technical quality.

- The processes of *abstraction* and *specification* essentially influence the semantics of the computed TI. Yet, structures to capture and make them explicit are missing.

- Concepts, structures and reporting principles for recording metadata describing not only the supporting data but also *virtual* data are not provided.

- *Quality assurance* (throughout the entire modelling process) is usually not supported.

To address these problems, *three primary metadata components* were suggested (section 6.1) by which to organise knowledge about the factors affecting TI quality in a way which basically extends the concepts provided by the ISO Standard 19113 (Geographic Information - Quality Principles). The three metadata components proposed are:

- *Modifiers*, which provide *normative* data to make explicit the mappings involved in the digital terrain modelling workflow, and thus provide part of the *quality norm*.

- *Descriptors*, which provide *informative* data explaining the supplied TI (by, for instance, reviewing its life-cycle or describing details about its technical realisation or its availability).

- *Quality elements*, which *measure* the actual performance of the supplied TI.

TI Scale-dependence The *scale-dependence* of landforms was recognised as an important yet rarely considered factor affecting TI quality. Scale is manifested by the degree of detail displayed and the degree of detail neglected, the latter always being a cause of *uncertainty*. Scale, therefore, critically affects both TI quality itself (section 5.1.2) as well as TI quality evaluation (chapter 8). The latter is affected in the sense that, in view of a specific application, TI accuracy needs to be assessed with respect to a terrain representation at the requested scale rather than by comparison with some 'true' values.

To address the issue of TI scale-dependence, in chapter 8 *wavelet techniques* were presented contributing to:

- assigning *scale ranges* to terrain surface features,

- enabling a *controlled* terrain surface simplification (that is, a controlled information reduction), and

- locally estimating the *uncertainty* introduced through representing terrain at a specific scale.

'Controlled', in this context, means that in view of a specific application, topographic structures assigned a scale finer than the envisioned modelling scale shall be preserved provided they are considered relevant by the application. A problem encountered when dealing with controlled information reduction not yet properly handled by geometric approaches like wavelet techniques is the sound preservation and representation of what was called 'inner boundaries' that result from more or less well-defined geographical objects (such as water bodies; cf. section 9.4.2).

9.3 Integration of Achievements and Insights

The *Pluggable Terrain Module* (PTM), whose basic concept was presented in chapter 7, serves as the vehicle for the integration of the achievements of this thesis. The PTM on the one hand, is designed as a *virtual dataset* (VDS) enabling the realisation of the concepts for reliable digital terrain modelling discussed in chapter 4 by:

- simulating a *continuous terrain surface* with the help of supporting data and terrain reconstruction methods driven by application-specific rules (both hidden within the dataset); and
- embodying *derivable TI* based on stored and thus persistent functions (hence encompassing more than terrain surface description itself). By embedding the functionality needed for terrain analysis it provides an application with proper tools that prevent it from choosing inappropriate approaches to handle TI.

On the other hand, the PTM concept is basically a *conceptual model for storage of metadata* documenting TI quality using the metadata components and principles proposed in chapter 6. This storage model provides a mechanism for structuring the metadata elements according to the kind of information they provide to a TI user. In this sense, the metadata storage model conforms to section 4.3, where a DTM was defined as $DTM := \{\mathcal{M}; (T_R; \mathcal{D}_T)\}$, in that

- the set of metadata $\mathcal{M}$ comprises the parts recording the PTM's *quality performance*, its *specification*, and its *explanation*, which together provide the structured context required for appraising the quality of the TI supplied;
- $\mathcal{D}_T$ is represented by the PTM's *sampled* and *supporting geospatial properties*; and
- the terrain reconstruction T_R is hidden in the specification schema in the form of stored functions; however, it manifests itself in the PTM's *virtual geospatial properties*.

9.4 Outlook

The title of the thesis features two main keywords: *quality handling* and *digital terrain modelling*. Chapter 7 presented the *Pluggable Terrain Module* (PTM) as a means to integrate these two foci. However, the concepts and approaches shown in this thesis can and need to be extended in both respects in order to improve their workability and be successfully applied in the PTM context.

9.4.1 The Quality Handling Perspective

The thesis focused on content, structuring, and representation issues of metadata appropriate for TI quality documentation. *Evaluation* and *communication* of the proposed body of metadata have not been addressed. Based on the contributed catalogue of metadata elements (chapter 6 and appendix C) and the proposed PTM storage model (chapter 7), ways of *gathering, evaluating, transferring*, and *communicating* metainformation via standardised PTM interfaces need to be developed.

Gathering and Evaluating Metainformation Enabling standardised access to metainformation documenting TI quality must prerequisite provision of such metainformation. This means that the supporting data must be accompanied by a comprehensive set of metadata. It is the duty of the TI producer to provide such metadata - in collaboration with the ordering application, if the data are produced on demand.

Documentation of TI quality permits TI producers to define how well their TI meet their product specification and assists TI users to assess how well a DTM meets their requirements. For this purpose, the quality of TI needs to be evaluated using consistent methods. To achieve this, a *framework for quality evaluation procedures* for TI must be provided, in accordance with the quality principles and guidance on reporting quality information described in chapters 6 and 7. As TI producers and users may view TI quality from different perspectives, procedures for TI quality evaluation may be expressed in two models, one for the TI producers and one for the TI users. The *TI producer model* is for testing and reporting conformance to specifications and the *TI user model* is for evaluating how well the quality of the DTM meets his or her requirements. In other words, the TI user model is intended for users who want to base a determination of fitness for use on an interpretation of the quality information provided. However, when a user requires more metainformation than provided, he or she may follow the TI producer model to get the additional information, in which case user requirements are treated as a product specification for purposes of using the TI producer model.

As far as the *TI producer model* is concerned, the metainformation given as part of the supporting data must be *propagated* so that the derived TI can be assigned appropriate quality documentation. For this purpose, methods to model the quality of both the supporting data and the derived TI, have to be found. Provided a continuous terrain surface is specified, derivation of basic TI can be accomplished by means of continuous functions. Hence, the uncertainty of the derived TI is likely to be estimated with the help of, for instance, stochastic simulation, Taylor methods, or interval arithmetic (cf. section 3.2.2). It must be investigated, whether these techniques are easily extendible to geometric constructs like paths of steepest descent, drainage areas, or visible regions. However, for aspects of *non-metric attribute accuracy*, other approaches to representation and propagation have to be developed.

Metadata Storage and Exchange In order to unambiguously store and exchange TI, a new XML-based Markup Language is currently being developed (Wirz 2001): the Terrain Markup Language (TML). To unambiguously *store* and *exchange* the appendant *metainformation*, TML must be provided with a metadata extension.

Metainformation Communication Finally, the metainformation must be made available to the PTM user. This task requires the disposability of tools to formulate queries and to provide, view, and comprehend corresponding answers - that is, tools to inspect a PTM. *Formalisation of possible queries* may be accomplished with the development of a 'query language' that is composed of elements that allow for:

- Specification of the TI for which a specific metainformation is requested,
- specification of the metadata element requested,
- specification of the location, the spatial extent, and the level of scale for which the metainformation is to be returned, and
- logical combination of such single queries (*and, or*, etc.).

To ensure successful perception of the communicated metainformation, it must be visualised in an understandable way. Such a powerful *visualisation* may be achieved through a combination of text, hypertext, tables, static and animated graphics, as well as interactive three-dimensional scenes. The user may be allowed to choose between different forms of display and to freely combine interactive scenes, pictures, tables, etc. However, it will be a future task to determine which combinations of displays best serve the intended communication of metainformation. Finally, extending the proposed concept of *metainformation on metainformation* (chapter 6), the user must always have access to information about the chosen display techniques. Such information is mandatory to the user to clearly understand the relation between the original and the displayed information, and hence to reduce the risk of misunderstanding and misinterpretation.

9.4.2 The Digital Terrain Modelling Perspective

Specification of Continuous Surfaces *Reliable* terrain models were characterised in section 4.1 as supplying TI in a *consistent* and *replicable* way. These conditions may not be met unless an explicit and suitable terrain reconstruction is provided from which the supplied TI may be derived. There are three major requirements such a terrain reconstruction has to meet. First, all given information must be appropriately considered. That is, all supporting data must be taken into account with all their semantic and geometric information. Second, the generated surface must bear all the characteristics and requirements demanded by the rules. Third, the reconstruction must allow estimation of the accuracy of the derived TI. This estimate must take into account, on the one hand, the numerical error propagated from the supporting data, and, on the other, the uncertainty introduced through the reconstruction schema itself.

The first task asks for a versatile reconstruction approach able to consider not only numerical, but also semantic information, and for an implementation enabling the reconstruction to include and react to this information. Higher-order Bézier splines, B-splines, and so called Coons-patches may be well suited in this regard. Reproduction of specific requirements calls for adaptive reconstruction techniques with a high degree of freedom that may be systematically and practically controlled. Again, higher-order parametric surfaces may be well suited. Finally, the reconstruction must propagate given uncertainty and estimate uncertainty introduced through the reconstruction process itself. One of the major difficulties remains to merge the three requirements in one reconstruction scheme.

TI Derivation from Continuous Terrain Surfaces Whereas the derivation from continuous digital surfaces is simple for elevation, slope, aspect, and curvature, the construction of, for instance, paths of steepest descent, valley lines, or drainage areas is more demanding. However, it is the ultimate goal of the PTM to provide all TI that will be needed in spatial models. To this end, the task of TI derivation from continuous terrains must be investigated on a conceptual level by analysing the TI to be supplied itself. Such an investigation should provide *concepts* and *definitions of TI*, as well as a comprehensive and structured *inventory*. To derive actual implementations, the provided definitions, then, may be translated into measurable units and values. This necessitates careful consideration of scale and discretisation issues. The concepts, then, must be mapped on different digital terrain representations and to the specified reconstruction schemes. It will have to be clarified whether all concepts will find a feasible realisation for each data model and each terrain reconstruction scheme. Finally, methods for uncertainty propagation for the extraction of TI must be studied.

TI Scale-dependence Finally, the thesis suggested that due to the *scale-dependence* inherent to any terrain representation, topographic surfaces may be best portrayed by means of multiscale representations. However, to develop a *reliable, multiscale TI representation tool*, the wavelet approach discussed in chapter 8 has yet to be developed further. For instance, even in the relatively small selective filtering example provided (section 8.4.5), a *multitude of parameters* had to be set. A user who may be unfamiliar with wavelet techniques, therefore, needs to be provided useful guidance in order to use the available methodology in an effective way. To this end, further research is needed to investigate the relations between the amplitude of the wavelet coefficients and useful thresholds and/or thresholding techniques.

Another set of open questions is related to *boundary* problems. For successful application of wavelet techniques, boundary problems call for thorough investigation and sound treatment. Particularly, the problem of wavelets not properly handling the 'inner boundaries' of more or less well-defined geographical objects (such as water bodies) within a dataset requires further attention. To address this family of problems, shift-invariant or so-called endpoint-interpolating wavelets may be tested.

The wavelet-based selective filtering procedure discussed in chapter 8 allows changes in the representation scale through controlled terrain surface simplification. In view of providing a *multiscale TI representation tool*, the algorithm may be extended to allow building a multiscale terrain model on the basis of:

- a user-defined sequence of envisioned levels of scale, and
- an application-defined list of relevant topographic structures to be accurately preserved across the scales.

Suitable algorithms then may be developed that enable reliable TI extraction from the multiscale terrain model at any desired level of scale.

Appendix A

Working through the Workflow for Reliable TI Application: The Example of Snowmelt Modelling

In this appendix, a snowmelt model for high alpine permafrost regions is outlined as an instance to exemplify in detail the individual steps of the workflow framework for reliable TI application discussed theoretically in section 4.3.3 (see also figure 4.11). The discussion in section 4.3.3 was based on the corresponding paper of Martinoni and Bernhard (1998); the example *spatial modelling application* outlined here is based on the master's thesis of Bernhard (1996). In his thesis, Bernhard presents a model to predict the temporal and spatial variation of snow cover during the melting process in a high alpine permafrost region. For a more thorough and detailed description of the *snowmelt model* developed, please be referred to Bernhard (1996), or Bernhard and Weibel (1999).

A.1 Problem Specification and Application Model Generation

The objective of the research was to estimate the temporal and spatial variation of the snow cover during the melting period in a high alpine permafrost region[1]. The generated *snowmelt model* is based on an *energy balance* approach. The solar radiation is the driving force for snowmelt. The snow

[1] Specifically, the study area was delimited to the rock glacier Murtèl, located to the southwest of the middle station of the Mount Corvatsch cable-car in the Swiss Alps. The rock glacier is approximately 400 m long and roughly 200 m wide and lies between 2620 m and 2850 m a. s. l. The Murtèl rock glacier exhibits a very rugged surface relief

cover is modelled as a function of the initial snow depth s_0, the absorbed radiance R_a, and physical snow characteristics χ_{snow} (such as the specific melting heat L_f or the snow density ρ_{snow}):

$$s_{0-n} = f(s_0, R_a, \chi_{snow}), \tag{A.1}$$

where n denotes the number of days since the initial snow depth measuring (i.e., since s_0)[2]. Essentially, the *snowmelt model* (A.1) is composed of two *submodels*: A snow cover submodel and a radiation submodel.

A.1.1 Snow Cover Submodel

The snow cover submodel provides snow depth values for all locations of the study area. This is achieved by extrapolating sampled snow depth measurements with the help of a *multiple regression model.* Such a regression model allows the establishment of a relationship between the *snow depths measured* at sample points and TI characterising the terrain surface, namely *slope, aspect* and *curvature.* The initial snow cover s_0 was modelled as a function of the initial snow fall[3] (sf), and of the TI slope (s), aspect (a), and curvature (c):

$$s_0 = f(sf, s, a, c). \tag{A.2}$$

A.1.2 Radiation Submodel

The radiation submodel assesses the energy balance on the Earth's surface. Accurate modelling of radiation fluxes, particularly of short-wave radiation fluxes (< 4 μm), is important for developing snowmelt models, especially in regions with rugged topography (such as high alpine permafrost regions), where the radiation rates vary substantially. In the actual case, only short-wave direct radiation was considered, resulting in a *radiation model* of the form:

$$R_{a_i}(\boldsymbol{x};\diamond) = I_m(\boldsymbol{x};\diamond)(1 - \alpha_i - c_i), \tag{A.3}$$

---

with ogive-like transverse ridges and furrows and is covered with coarse blocks. It is active and completely bare of vegetation.

[2]The *snowmelt model* is written as follows:

$$s_{0-n}(\boldsymbol{x}) = s_0(\boldsymbol{x}) - \frac{\sum_{i=1}^{n} R_{a_i}(\boldsymbol{x};\diamond)}{L_f \cdot \rho_{snow}},$$

where $s_0(\boldsymbol{x})$ is the initial snow depth at location $\boldsymbol{x}$, n is the number of days after determining the initial snow depth, R_{a_i} is the radiance absorbed at day i (per unit area $\diamond$), L_f is the specific melting heat, and ρ_{snow} is the mean snow density.

[3]Which, for the sake of simplicity, is assumed to be constant across the entire study area.

where $R_{a_i}(\boldsymbol{x};\diamond)$ denotes the radiance absorbed at day i (per unit area $\diamond$ "at" location $\boldsymbol{x}$), $I_m(\boldsymbol{x};\diamond)$ is the mean incident radiance (per unit area $\diamond$), α_i is the albedo value for day i[4], and c_i is a coefficient of cloud cover at day i[5].

The potential *incident direct radiance* $I(\boldsymbol{x})$ at a point $\boldsymbol{x}$, basically, depends on the elevation $z(\boldsymbol{x})$, the normal $\vec{n}_{\boldsymbol{x}}$ to the surface tangent plane in $\boldsymbol{x}$, the local horizon $hor(\boldsymbol{x})$ and the sun angle $\vec{S}(\boldsymbol{x})$ for location $\boldsymbol{x}$:

$$I = f(z, \vec{n}, hor, \vec{S}).$$

In more detail, the *mean incident direct radiance* $I_m(\boldsymbol{x};\diamond)$ per unit area $\diamond$ "at" location $\boldsymbol{x}$ may be computed according to the following, very simplified, formula:

$$I_m(\boldsymbol{x};\diamond) = \int_{t_r(\boldsymbol{x})}^{t_s(\boldsymbol{x})} I_0(\boldsymbol{x};\diamond)\cos(\vec{n}_\diamond, \vec{S}(t))\,dt, \tag{A.4}$$

where $t_r(\boldsymbol{x})$ and $t_s(\boldsymbol{x})$ denote the time of sunrise and sunsetting for location $\boldsymbol{x}$, and $I_0(\boldsymbol{x};\diamond)$ is the incident direct solar radiance referred to a horizontal unit plane "in" $\boldsymbol{x}$. The term $\cos(\vec{n}_\diamond, \vec{S}(t))$ provides the geometrical correction required to compute potential incident direct radiance referring to an area of arbitrary slope. Local horizon enters the model in that it is required to compute $t_r(\cdot)$ and $t_s(\cdot)$. Local horizon essentially affects the period of daylight, that is, the duration of potential solar irradiation. Elevation impacts on $I_0(\boldsymbol{x},\diamond)$ in that it influences the optical density of the atmosphere and the water content of the air.

Although *topography* is not a direct component of the snowmelt model (i.e. the application model), the application is influenced by topographic factors acting as inputs to the various submodels (namely the snow cover and the radiation (sub)model; figure A.1). As illustrated in figure A.1, the *UoD* implied for the (sub)submodel terrain that is implicitly part of the application model contains, at least, *elevation, slope, aspect, normal to the surface tangent plane, curvature,* and *local horizon.*

[4]Where *albedo* is parameterised as follows:

$$\alpha_i = \begin{cases} 0.4 + 0.5^{-0.05 n_d}, & \text{if } T_{i,mean} < 0°\text{C}, \\ 0.4 + 0.44^{-(0.252+0.167\ln(T^+))}, & \text{otherwise}, \end{cases}$$

where $T_{i,mean}$ stands for the mean air temperature at day i, n_d denotes the number of days since the last snow fall, and T^+ stands for the sum of positive daily temperature means (i.e., $T^+ := \sum_{i=1}^{n}(T_{i,mean} > 0°\text{C})$).

[5]Bernhard derived the coefficient c_i of cloud cover at day i by linear interpolation from available meteorological data and assumed it to be constant across the entire study area.

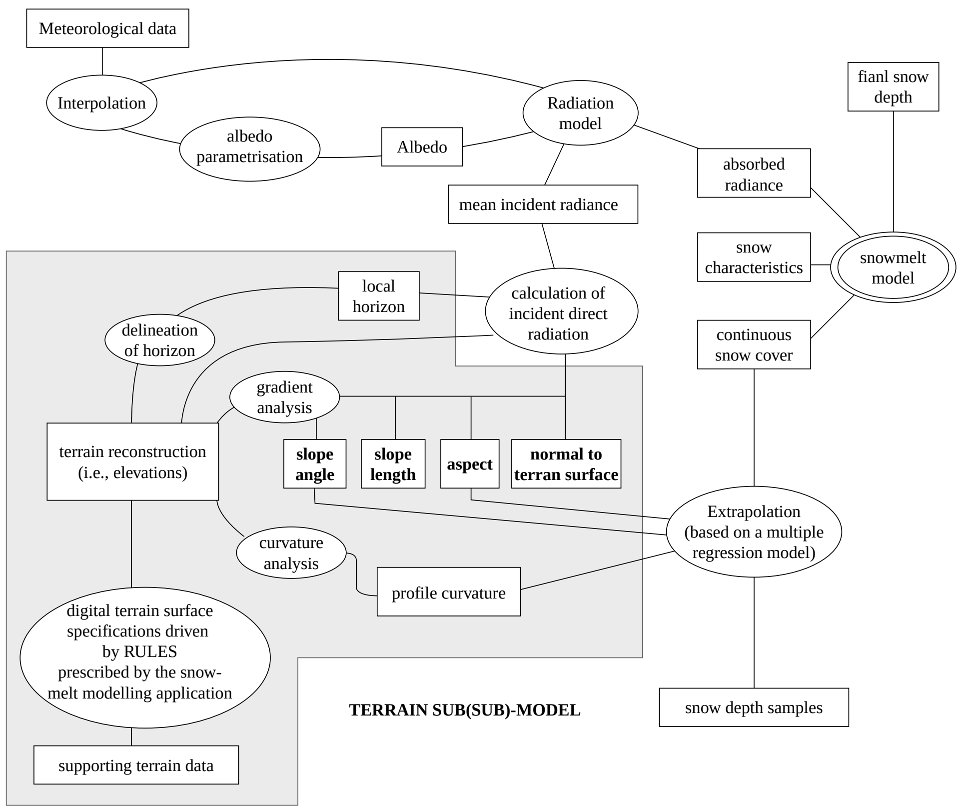


Figure A.1: A schematic overview on the snowmelt model.

A.2 Requirement Specification

From the snowmelt model outlined in the last section it follows that the topography is only implicitly part of the application model. The requirements with respect to TI are, therefore, also "indirect" in that they result from the submodel's requirements.

Required TI

- *Snow cover submodel:*
  - Slope,
  - aspect,
  - profile curvature.

  In the snow cover submodel, TI is required for two different purposes. First, to compute the correlation coefficients of the multiple regression. Here, TI is required with respect to the locations where snow depth is sampled (scattered points). Second, TI is required for extrapolation of snow depth values for arbitrary locations across the entire study area. Due to the radiation governing snowmelt being computed per unit area, the snow cover also is averaged by area. In this case, the TI is needed in terms of the application model's discretisation units.

- *Radiation submodel:*
  - Elevation,
  - normal to the surface tangent plane,
  - local horizon.

  As the incident direct radiance is computed per unit area, this TI is required with respect to the application models' discretisation units.

Desired Terrain Surface Properties Drainage continuity is a concept related to terrains shaped by fluvial processes and does not hold for terrains shaped, for instance, by processes of glacial erosion. The surface of the rock glacier shall, therefore, be modelled *without enforcing drainage continuity.*

"Micro"-TI[6] contained in the supporting data shall be retained in the surface generated.

[6]Such as coarse blocks covering the rock glacier. Coarse blocks impact on the model by acting as wind breaks and thus affecting snow cover erosion by wind transport. Coarse blocks also cast shadows on their next surroundings thus impacting on the snowmelt rate.

Modelling Scale The spatial variation of the snow cover is clearly correlated with the surface relief. While the snow pack, for instance, excesses the average in furrows, the snow depth on ridges is lower than the average snow cover. As mentioned in footnote[1], the Murtél rock glacier exhibits a very rugged surface relief with ogive-like transverse ridges and furrows, whose average diameter is approximately 10 m to 20 m. To accurately capture the surface relief, the modelling scale therefore needs to be in the range of a few metres (cf. also footnote[6] on the impact of coarse blocks on the model).

Accuracy and Resolution Demands

- *Slope, aspect, and profile curvature:* In order to accomplish a differentiated analysis of the model error, Bernhard (1996) classified the individual sample points on the rock glacier according to topographic parameters. Absolute and relative differences between computed and measured snow cover were evaluated for eight equidistant classes of aspect, and five equidistant classes of slopes (for slope values ranging from 0° to 50°). Within these classes, the differences showed a variation of up to 30%. These results suggest that the snow cover submodel requires input TI values of higher resolution and corresponding accuracy than the above mentioned class sizes (for instance, to allow a regression model of useful sensitivity, slope and aspect may be portrayed with a *resolution* of a single degree, and an *accuracy* of ±5°).

- *Normal to surface tangent plane:* Normals to the surface tangent plane enter the model through the correction term $\cos(\vec{n}_\diamond, \vec{S}(t))$ required in the calculation of incident direct radiance to correct for the slope of the area referred to. As the time dependence of the sun angle $\vec{S}(t, \boldsymbol{x})$ for location $\boldsymbol{x}$ is much more difficult to model than the normal to the surface tangent plane, the latter's accuracy should not exceed the accuracy bounds determined by the modelled temporal sun angle resolution.

- *Local horizon:* Local horizon is worth accurate and sophisticated modelling since it essentially affects the duration of daylight and therefore the total amount of daily potential solar irradiation (however, it is unfortunately not clear how the accuracy of computed local horizon may be quantified or assessed).

- *Elevation:*

  - *Accuracy:* Elevation impacts on the radiation model by affecting the optical density of the atmosphere and the water content of the air. However, the respective formulae are rather approximative, and therefore not imposing very rigorous accuracy demands on the requested elevation values. Hence, absolute (vertical) accuracy is not critical; the emphasis lies rather on the requirements made to *relative (vertical) accuracy* (cf. accuracy demands for slope, aspect, and curvature).

– *Resolution:* Again, the required elevation resolution is determined rather by accuracy and resolution demands of the TI to be derived (namely, slope, aspect, and curvature) than by radiation model demands. Taking into account the resolution requirements for slope, aspect and profile curvature as well as the large modelling scale, elevation may be represented with a resolution not falling below a single meter (for a detailed investigation of the correlation between the resolution of the elevations portrayed and that of derived slope and aspect values, see Carter (1992)).

Generally, the TI provided shall be supplied together with respective uncertainty estimates (supplied by way of intervals (cf. section 3.2.2) or standard deviations) to allow propagation of the expected uncertainties into the snowmelt model.

A.2.1 Reference Model

Categories Defined

- *Elevation*;
- *slope*;
- *aspect*, meaning azimuth direction of steepest slope;
- *surface normal* (i.e., normal to the tangent plane at a specific location);
- *(profile) curvature*, that is the change in slope angle along the path of steepest descent;
- *local horizon*, which provides the open hemisphere thus determining the duration and hence the total potential amount of daily potential solar irradiation.

Geometric Descriptions

- Elevation: *Continuous scalar field*;
- slope: *Continuous scalar field*;
- aspect: *Continuous directional field*;

- surface normal: *Continuous vector field*;
- profile curvature: *Continuous scalar field*.
- Local horizon: *Closed polygon in 3-dimensional space*, associated to a specific location (of specific extent, cf. above).

The *spatial concept* underlying these categories is one of surface layers. The above categories are conceptualised as continuous fields, that is, as existing everywhere across the entire study area. Note, however, that, since the absorbed radiation is computed per unit area, for elevation, slope, aspect, normals to surface tangent plane, and profile curvature, averaging over the application model's discretisation units will be required (cf. section A.2).

Establishment of Relationships

- *Functional/logical relationships:*
  - Slope, aspect, and surface normal are *derivatives* of the elevation field (i.e., they can be expressed as functions of the gradient of the elevation field).
  - Profile curvature is a *second derivative* of the elevation field, or, a derivation of the elevation's gradient field, respectively.
  - Local horizon results as a combination of slope angle and elevation values.
- *(Topo)logical relationships:*
  - Local horizon may be represented as a closed polygon in 3-dimensional space around a specific location or region, respectively.

A.2.2 Rules

TI to Be Represented *Elevation, slope, aspect, normals to surface tangent planes, profile curvature*, and *local horizon* must be represented, that is, derivable upon request (cf. section A.2). To allow reliable derivation of profile curvature, the terrain surface generated must be $\mathcal{C}^2$-continuous (where not explicitly specified otherwise). The terrain surface is conceptualised as a surface layer (i.e., interpreted as a continuous phenomenon).

Supporting Data Portrayal of coarse blocks in the supporting data would be very desirable. Coarse blocks impact on the model by acting as wind breaks and thus affecting snow cover evolution by wind transport. Coarse blocks also cast shadows on their next surroundings thus impacting on the snowmelt rate. If such "micro"-TI is portrayed in the supporting data, this information shall be retained in the generated terrain surface.

Support and Geometry of the Requested TI

- TI requested at *point locations*: Slope, aspect, and profile curvature (required at point locations to optimally fitting the regression model).
- *Averages* over extended areas: Elevation, slope, aspect, surface normal, profile curvature. Warnings may be released if essential information may be suppressed or hidden due to averaging (for instance, if averaging occurs over an area containing an extremum or a non-differentiable breakline).
- TI *related to an extended area:* Local horizon.

Required Surface Properties Since rock glaciers are a surface form shaped by periglacial processes, drainage continuity shall *not* be imposed on the terrain reconstruction (the filling of pits would, for instance, bias the results of snow depth extrapolations). "Micro"-TI contained in the supporting data shall be retained in the surface generated (possibly introducing breaklines, that is, non-differentiability). As mentioned in the first paragraph of this section, reliable derivation of profile curvature necessitates the terrain surface specified to be $\mathcal{C}^2$-continuous (where not indicated otherwise).

Modelling Scale For discussion of the required modelling scale, please refer to section A.2.

Accuracy and Resolution Demands For a detailed discussion of the accuracy and resolution demands, see section A.2. In summary, the emphasis is definitely on *relative* rather than on *absolute accuracy.* Uncertainty estimates by way of *intervals* or *standard deviations* shall be provided together with the TI supplied. Warnings indicating important suppression or distortion of information caused by the averaging operation would be very desirable.

A.3 Terrain Model Design

A.3.1 Mathematical Model

Definition of Variables

- *Defined model variables:*
  - Elevation $z:\ (\boldsymbol{x},\diamond) \mapsto z(\boldsymbol{x},\diamond)$;
  - slope $s:\ (\boldsymbol{x},\diamond) \mapsto s(\boldsymbol{x},\diamond)$;

  - aspect $a: \ (\boldsymbol{x}, \diamond) \mapsto a(\boldsymbol{x}, \diamond)$;
  - surface normal $\vec{n}_\diamond: \ (\boldsymbol{x}, \diamond) \mapsto \vec{n}_\diamond(\boldsymbol{x}, \diamond)$;
  - profile curvature $c_{prof}: \ (\boldsymbol{x}, \diamond) \mapsto c_{prof}(\boldsymbol{x}, \diamond)$;
  - (local) horizon $hor: \ (\boldsymbol{x}, \diamond) \mapsto hor(\boldsymbol{x}, \diamond)$.

- *Auxiliary variables* (incomplete list):
  - Area $A: \diamond \mapsto A(\diamond)$;
  - vector $\boldsymbol{x}: (x, y) \mapsto \boldsymbol{x} := (x, y)$;

Geometry Definitions

- *Defined model variables:*
  - Elevation z: Continuous scalar field, such that $\lim_{A(\diamond)\to 0} z(\cdot, \diamond) \in \mathcal{C}^2(\mathbb{D}; \mathbb{R})$;
  - slope s: Continuous scalar field;
  - aspect a: Continuous directional field;
  - surface normal $\vec{n}_\diamond$: Continuous vector field;
  - profile curvature c_{prof}: Continuous scalar field;
  - local horizon hor: Closed polygon in 3-dimensional space.

- *Auxiliary variables* (incomplete list):
  - Area A: Scalar value;
  - vector: Vector value;

Formalisation of Relationships Because the DTM is designed to match the application's modelling scale, the required TI can legitimately be formalised in its mathematical sense (at least for the limes $A(\diamond) \to 0$).

- *Elevation*: $z(\boldsymbol{x}, \diamond) = \begin{cases} z(\boldsymbol{x}), & \text{if } A = 0, \\ \frac{1}{A} \iint_\diamond z(x, y)\, dx\, dy, & \text{otherwise.} \end{cases}$.

- Gradient: $\vec{\nabla} z(\boldsymbol{x}, \diamond) =: (z_x, z_y)$, where:

$$
\begin{aligned}
z_x &= \begin{cases} \left.\frac{\partial z}{\partial x}\right|_{\boldsymbol{x}}, & \text{if } A = 0, \\ \frac{1}{A} \iint_\diamond \left.\frac{\partial z}{\partial x}\right|_{(x,y)} dx\, dy, & \text{otherwise,} \end{cases} \\
z_y &= \begin{cases} \left.\frac{\partial z}{\partial y}\right|_{\boldsymbol{x}}, & \text{if } A = 0, \\ \frac{1}{A} \iint_\diamond \left.\frac{\partial z}{\partial y}\right|_{(x,y)} dx\, dy, & \text{otherwise.} \end{cases}
\end{aligned}
$$

  With the components of the gradient vector computed as above, it follows:

- *Slope:* $s(\boldsymbol{x},\diamond) = \arctan\sqrt{z_x^2 + z_y^2}$;
- *aspect:* $a(\boldsymbol{x},\diamond) = \arctan\frac{z_y}{z_x}$;
- *surface normal:* $\vec{n}_\diamond(\boldsymbol{x},\diamond) = \begin{bmatrix} z_x \\ z_y \\ -1 \end{bmatrix}$.

- Second derivatives: $\left[\frac{\partial(z_x,z_y)}{\partial(x,y)}\right]_{\boldsymbol{x}} = \begin{bmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{bmatrix}_{\boldsymbol{x}}$, where:

$$\begin{aligned} z_{xx} &= \left.\frac{\partial^2 z}{\partial x^2}\right|_{\boldsymbol{x}}, \\ z_{yy} &= \left.\frac{\partial^2 z}{\partial y^2}\right|_{\boldsymbol{x}}, \\ z_{xy} &= \left.\frac{\partial^2 z}{\partial x \partial y}\right|_{\boldsymbol{x}}. \end{aligned}$$

It follows from the above for *profile curvature* c_{prof}:

$$c_{prof}(\boldsymbol{x},\diamond) = \begin{cases} \left.\frac{-(z_{xx}z_x^2 + 2z_{xy}z_xz_y + z_{yy}z_y^2)}{z_x^2+z_y^2}\right|_{\boldsymbol{x}}, & \text{if } A = 0, \\ \frac{1}{A}\iint_\diamond \left.\frac{-(z_{xx}z_x^2 + 2z_{xy}z_xz_y + z_{yy}z_y^2)}{z_x^2+z_y^2}\right|_{(x,y)} dx\,dy, & \text{otherwise.} \end{cases}$$

- *Local horizon:* $hor(\boldsymbol{x},\diamond) = \bigcap_{\boldsymbol{x}\in\diamond} hor(\boldsymbol{x})$.

Uncertainty Representation and Propagation

- DTM *range* $\mathbb{V}$: $\mathbb{V} = \mathbb{R}$.
- *Uncertainty representation:* Intervals or standard deviations.
- *Uncertainty propagation:* Uncertainty estimates may be derived by using interval mathematics or stochastic simulation methods (for a discussion of these techniques, please refer to section 3.2.2). As interval mathematics tend to overestimate the actual uncertainty, uncertainty propagation by means of stochastic simulation methods may be preferred despite the heavy computational load involved.

A.3.2 Spatial Data Model

It follows from the defined variables and the established relationships that all the TI to be represented can be written as a function of the elevation field (even local horizon, as its delineation basically results as a combination of viewshed computation and slope analysis; see, for instance, Dozier et al. (1981)). Consequently, the primary quantity to be sampled is *elevation*. However, as the emphasis in accuracy issues lies rather on gradient accuracy (i.e., on relative vertical accuracy) than on absolute vertical accuracy, additional sampling of surface gradients, aspects or similar information could substantially improve the fitness of the DTM generated for the application discussed.

Elevation was conceptualised as a continuous field (cf. section A.2.1). The *spatial data model* chosen, therefore, must support representation of continuous fields. From the two kinds of data models serving this purpose presented in section 2.3.2, only a *sampled model* is possible[7].

The rules mention that portrayal of coarse blocks in the supporting data would be very desirable. If such big blocks shall be recorded, an irregular data model must be chosen[8]. If blocks do not need to be represented in the supporting data (i.e., in case the data captured do not invariably imply a specific sampling scheme), the question which data model may be the most favourable also depends on the available data sources[9]. For instance, the expense caused and the uncertainty introduced depend on the amount and kind of preprocessing required[10].

A.4 DTM Implementation

A.4.1 Computer Representation

Following definition (4.2), the DTM is implemented as a VDS embodying both supporting data (by way of the supporting data set) and virtual data, that is, the TI derivable upon request.

[7]For two reasons, *Piecewise models* are not appropriate for the application at hand. First, terrain by its nature does not lend itself to be piecewisely sampled. The second and more cogent reason is that, unless the terrain surface is reconstructed by linear interpolation, averaged elevation and averaged first- and second-order derivatives cannot be associated with a common representative point (meaning that, for instance, slope or curvature computed for the point having exactly mean elevation do not correspond to mean slope or mean curvature). Therefore, to reliably provide the averaged TI values required by the application (cf. sections A.2.1 and A.2.2) using a piecewise spatial data model, at least 3 data layers (namely, elevation, gradient, and curvature) instead of only one for a sampled model would be necessary.

[8]Else, both scattered or regular sampling schemes could serve the purpose. Because of the large modelling scale implying a high data resolution and because of the very rugged topography being modelled, even a regular sampling scheme does not necessarily introduce an immense amount of redundancy.

[9]Possible data sources could be, for instance, on-site measurements from a field campaign, remote sensing technologies (laser-scanning, air- or space-borne sensors such as InSAR or SAR, digital photogrammetry, etc.), or contour maps.

[10]Bernhard in his thesis (1996) derived a RSG of 2 m resolution from photogrammetric data through the intermediate step of contouring the data (at 2 m contour interval).

Reconstruction of the Terrain Surface If the supporting data consist of *scattered elevation data* (or even *contours*), domain subdivision is most favourably specified as a constrained Delaunay triangulation honouring additional information that may be provided by an enhanced information base such polylines for contours or coarse blocks causing breaklines.

According to the specified requirements and rules (sections A.2 and A.2.2), the reconstructed rock glacier surface is allowed to display pits, and it needs to be $\mathcal{C}^2$-continuous except at breaklines and points. The glacier surface hence may be reconstructed using Akima's bivariate-quintic interpolation scheme (Akima 1978). Two major problems remain with this approach, however. First, bivariate-quintic interpolants tend to overshoot; and second, the method does not enable consideration of breaklines and points. While cubic Bézier-splines designed according to the Clough-Tocher method (Clough and Tocher 1965, Schneider 1998) address the first of the above problems, they do still not support consideration of breaklines and points. Another solution may be to reconstruct the terrain surface with the help of finite element methods, as these allow integration of additional knowledge such as gradient samples or breaklines and points into the reconstruction scheme (see, for instance, Ebner and Reiss (1978)).

In case of *regularly spaced supporting data*, domain subdivision into regular polygons is chosen. In this case, the terrain surface may be reconstructed by cubic or birational spline interpolation (Späth 1986).

Virtual TI

- *Elevation* at arbitrary location: see previous paragraph.

- *Slope, aspect, normal to surface tangent plane, profile curvature:* A known surface reconstruction scheme allows analytical parametrisation of the respective formalisations provided by the mathematical model (section A.3.1).

- *Local horizon:* Analytical parametrisation of the local horizon of an extended area is not straightforward. The solution chosen for Bernhard's investigation was to represent the extended area by its centre of gravity and to compute the local horizon for this centre of gravity[11]. Local horizon was implemented following an algorithm proposed by Dozier et al. (1981). Thereby, local horizon is given through the vertical angle to the horizon and the topographic horizon in all directions. That is, the local horizon of a location $\boldsymbol{x}_i$ is given as a sequence of points $[\boldsymbol{x}_1, ..., \boldsymbol{x}_k]$, where $\boldsymbol{x}_{k'}$ denotes the horizon of $\boldsymbol{x}_i$ in direction

[11]Depending on the extent of a discretisation unit, this approach bears the risk of heavily over or under estimating the open sky.

$\frac{k' \cdot 2\pi}{k}$, $(1 \leq k' \leq k)$, and k thus determines the discretisation of the unit circle.

This section gave a quick outline of some tasks of computer representation. Not discussed were very essential issues of the encoding step such as selection of data types (integer, double, etc.) or numerical methods to be used (for geometric computations, for solving equation systems, etc.).

A.5 Evaluation

In case there is no need to generate a new terrain representation, that is, in case an existing one can be re-used, the available DTMs must be assessed in light of the requirements specified (section A.2) and the rules prescribed (section A.2.2). If a supporting dataset rather than an actual DTM is at disposal, its suitability to parameterise a computer representation similar to the one sketched in the above section, that is, a computer representation meeting the rules prescribed in section A.2.2, may be assessed.

Appendix B

Surfaces Implied by the Common Methods for Gradient Calculation from Regular Grids

In section 5.1.3 it was remarked that TI extraction always in one form or another implicitly integrates surface reconstruction. As is shown in this appendix, gradient computation at a grid point from a regular square grid (RSG) using a second-order finite differences method[1], for instance, renders the gradient of a biquadratic surface through the 3 by 3 neighbourhood of the point (cf. slope calculation following Zevenbergen and Thorne (1987)). Gradient calculation by third-order finite differences, on the other hand, returns the gradient of a plane fitted by least squares to a 3 by 3 neighbourhood of the raster point.

B.1 Methods for Gradient Calculation

(i) *Second-order finite differences method* (Dozier and Strahler 1983):

$$\begin{aligned} \left.\frac{\partial z}{\partial x}\right|_{z_5} &= \frac{z_6 - z_4}{2\Delta x}, \\ \left.\frac{\partial z}{\partial y}\right|_{z_5} &= \frac{z_8 - z_2}{2\Delta y}, \end{aligned} \tag{B.1}$$

where the indices i of the elevation values $z_i,\ i = 1, ..., 9$, follow the scheme depicted in figure B.1; Δx and Δy denote the grid spacing in horizontal and vertical direction, respectively.

[1] A method also known as numerical differentiation (Dozier and Strahler 1983).

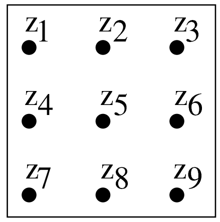


Figure B.1: Scheme for numbering the elevation values in a 3 by 3 DTM window.

(ii) *Third-order finite differences method*:

- Without specially weighting the non-diagonally adjacent neighbours (Sharpnack and Akin 1969):

$$\begin{aligned}\left.\frac{\partial z}{\partial x}\right|_{z_5} &= \frac{(z_3 - z_1) + (z_6 - z_4) + (z_9 - z_7)}{6\Delta x},\\ \left.\frac{\partial z}{\partial y}\right|_{z_5} &= \frac{(z_7 - z_1) + (z_8 - z_2) + (z_9 - z_3)}{6\Delta y}.\end{aligned} \qquad \text{(B.2)}$$

- Weighting the non-diagonally adjacent neighbours with $\sqrt{2}$, as implicitly proposed by Horn (1981):

$$\begin{aligned}\left.\frac{\partial z}{\partial x}\right|_{z_5} &= \frac{(z_3 - z_1) + 2(z_6 - z_4) + (z_9 - z_7)}{8\Delta x},\\ \left.\frac{\partial z}{\partial y}\right|_{z_5} &= \frac{(z_7 - z_1) + 2(z_8 - z_2) + (z_9 - z_3)}{8\Delta y}.\end{aligned} \qquad \text{(B.3)}$$

B.2 Interpolation of a Biquadratic Polynomial in a 3 by 3 DTM Window

Equation (B.4) writes elevation z as a complete *biquadratic polynomial*:

$$z = f(x, y) = Ax^2y^2 + Bx^2y + Cxy^2 + Dx^2 + Ey^2 + Fxy + Gx + Hy + I. \qquad \text{(B.4)}$$

The two components of the *gradient vector*, hence, result to be:

$$\begin{aligned}\frac{\partial z}{\partial x} &= 2Axy^2 + 2Bxy + Cy^2 + 2Dx + Fy + G,\\ \frac{\partial z}{\partial y} &= 2Ax^2y + Bx^2 + 2Cxy + 2Ey + Fx + H.\end{aligned} \qquad \text{(B.5)}$$

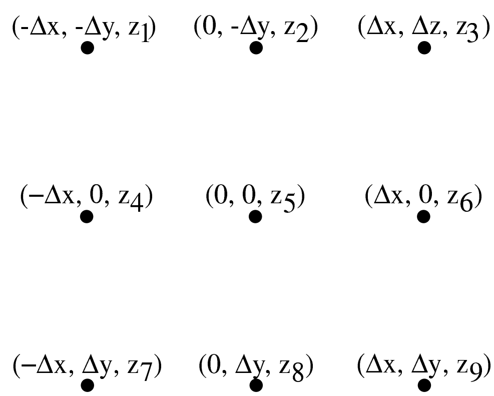


Figure B.2: Local coordinate system for a 3 by 3 DTM window.

The biquadratic polynomial given by equation (B.4) may be exactly interpolated to a 3 by 3 DTM window as the one depicted in figure B.1. Without loss of generality, to ease computations, consider the definition of a *local coordinate system* as illustrated in figure B.2. Interpolation of the polynomial (B.4) to a 3 by 3 DTM window, then, means the solution of the following linear equation system (LES) for the 9 coefficients $A, ..., I$:

$$P\boldsymbol{c} = \boldsymbol{z}, \tag{B.6}$$

where:

$$P := \begin{bmatrix} (\Delta x)^2(\Delta y)^2 & -(\Delta x)^2\Delta y & -\Delta x(\Delta y)^2 & (\Delta x)^2 & (\Delta y)^2 & \Delta x\Delta y & -\Delta x & -\Delta y & 1 \\ 0 & 0 & 0 & 0 & (\Delta y)^2 & 0 & 0 & -\Delta y & 1 \\ (\Delta x)^2(\Delta y)^2 & -(\Delta x)^2\Delta y & \Delta x(\Delta y)^2 & (\Delta x)^2 & (\Delta y)^2 & -\Delta x\Delta y & \Delta x & -\Delta y & 1 \\ 0 & 0 & 0 & (\Delta x)^2 & 0 & 0 & -\Delta x & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & (\Delta x)^2 & 0 & 0 & \Delta x & 0 & 1 \\ (\Delta x)^2(\Delta y)^2 & (\Delta x)^2\Delta y & -\Delta x(\Delta y)^2 & (\Delta x)^2 & (\Delta y)^2 & -\Delta x\Delta y & -\Delta x & \Delta y & 1 \\ 0 & 0 & 0 & 0 & (\Delta y)^2 & 0 & 0 & \Delta y & 1 \\ (\Delta x)^2(\Delta y)^2 & (\Delta x)^2\Delta y & \Delta x(\Delta y)^2 & (\Delta x)^2 & (\Delta y)^2 & \Delta x\Delta y & \Delta x & \Delta y & 1 \end{bmatrix},$$

$$\boldsymbol{c} := \begin{bmatrix} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \\ I \end{bmatrix} \quad \text{and} \quad \boldsymbol{z} := \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \\ z_7 \\ z_8 \\ z_9 \end{bmatrix}.$$

Solving the LES (B.6) yields:

$$\begin{aligned}
A &= \frac{z_1 + z_3 + z_7 + z_9 - 2(z_2 + z_4 + z_6 + z_8) + 4z_5}{4(\Delta x)^2(\Delta y)^2}, \\
B &= \frac{(z_7 - z_1) - 2(z_8 - z_2) + (z_9 - z_3)}{4(\Delta x)^2 \Delta y}, \\
C &= \frac{(z_3 - z_1) - 2(z_6 - z_4) + (z_9 - z_7)}{4\Delta x(\Delta y)^2}, \\
D &= \frac{z_4 - 2z_5 + z_6}{2(\Delta x)^2}, \\
E &= \frac{z_2 - 2z_5 + z_8}{2(\Delta y)^2}, \\
F &= \frac{z_1 - z_3 - z_7 + z_9}{4\Delta x \Delta y}, \\
G &= \frac{z_6 - z_4}{2\Delta x}, \\
H &= \frac{z_8 - z_2}{2\Delta y}, \\
I &= z_5.
\end{aligned} \tag{B.7}$$

Substitution of the coefficients (B.7) into the gradient components given by (B.5) yields, for the grid point $(0, 0, z_5)$:

$$\begin{aligned}
\left.\frac{\partial z}{\partial x}\right|_{z_5} &= \frac{z_6 - z_4}{2\Delta x}, \\
\left.\frac{\partial z}{\partial y}\right|_{z_5} &= \frac{z_8 - z_2}{2\Delta y},
\end{aligned} \tag{B.8}$$

which corresponds exactly to the second-order finite differences method described by equation (B.1).

B.3 Fitting a Plane to a 3 by 3 DTM Window by Least Squares

Consider the *plane equation* $Ax + By + Cz = D$, rewritten as:

$$z = A'x + B'y - D', \tag{B.9}$$

where $A' := \frac{-A}{C}$, $B' := \frac{-B}{C}$, $D' := \frac{-D}{C}$.

The two components of the *gradient vector*, hence, result to be:

$$\frac{\partial z}{\partial x} = A', \quad \frac{\partial z}{\partial y} = B'. \tag{B.10}$$

Without loss of generality, to ease computations, again a *local coordinate system* as the one depicted in figure B.2 is introduced.

B.3.1 Plane Fitting by Least Squares without Particular Weighting of the Non-Diagonally Adjacent Neighbours

Using a local coordinate system as described above, fitting a plane to a 3 by 3 DTM window results in the following *error equations*:

$$\begin{aligned}
-A'\Delta x - B'\Delta y - D' &= z_1 - r_1 \\
-B'\Delta y - D' &= z_2 - r_2 \\
A'\Delta x - B'\Delta y - D' &= z_3 - r_3 \\
-A'\Delta x - D' &= z_4 - r_4 \\
-D' &= z_5 - r_5 \\
A'\Delta x - D' &= z_6 - r_6 \\
-A'\Delta x + B'\Delta y - D' &= z_7 - r_7 \\
B'\Delta y - D' &= z_8 - r_8 \\
-A'\Delta x + B'\Delta y - D' &= z_9 - r_9,
\end{aligned}$$

which may be rewritten in matrix form as:

$$\underbrace{\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \\ z_7 \\ z_8 \\ z_9 \end{bmatrix}}_{=\boldsymbol{y}} - \underbrace{\begin{bmatrix} -\Delta x & -\Delta y & -1 \\ 0 & -\Delta y & -1 \\ \Delta x & -\Delta y & -1 \\ -\Delta x & 0 & -1 \\ 0 & 0 & -1 \\ \Delta x & 0 & -1 \\ -\Delta x & \Delta y & -1 \\ 0 & \Delta y & -1 \\ \Delta x & \Delta y & -1 \end{bmatrix}}_{=M} \underbrace{\begin{bmatrix} A' \\ B' \\ D' \end{bmatrix}}_{=\boldsymbol{x}} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \\ r_9 \end{bmatrix}. \tag{B.11}$$

From the *normal equation* $M^T M\boldsymbol{x} = M^T\boldsymbol{y}$, it follows that:

$$\boldsymbol{x} = (M^T M)^{-1} M^T \boldsymbol{y}. \tag{B.12}$$

Solution of (B.12) yields:

$$\begin{aligned}
A' &= \frac{(z_3 - z_1) + (z_6 - z_4) + (z_9 - z_7)}{6\Delta x}, \\
B' &= \frac{(z_7 - z_1) + (z_8 - z_2) + (z_9 - z_3)}{6\Delta y}, \\
D' &= -\frac{1}{9}(z_1 + z_2 + z_3 + z_4 + z_5 + z_6 + z_7 + z_8 + z_9).
\end{aligned} \tag{B.13}$$

Substitution of the coefficients (B.13) into the gradient components given by equation (B.10) shows the exact correspondence with the third-order finite differences method described in equation (B.2).

B.3.2 Least Squares Plane Fitting by Weighting the Non-Diagonally Adjacent Neighbours

Least squares plane fitting to a 3 by 3 DTM window particularly weighting non-diagonally adjacent neighbours with $\sqrt{2}$ (as implicitly suggested by Horn (1981)) leads to the following *error equations*:

$$\underbrace{\begin{bmatrix} z_1 \\ \sqrt{2}z_2 \\ z_3 \\ \sqrt{2}z_4 \\ z_5 \\ \sqrt{2}z_6 \\ z_7 \\ \sqrt{2}z_8 \\ z_9 \end{bmatrix}}_{=\boldsymbol{y}} - \underbrace{\begin{bmatrix} -\Delta x & -\Delta y & -1 \\ 0 & -\sqrt{2}\Delta y & -\sqrt{2} \\ \Delta x & -\Delta y & -1 \\ -\sqrt{2}\Delta x & 0 & -\sqrt{2} \\ 0 & 0 & -1 \\ \sqrt{2}\Delta x & 0 & -\sqrt{2} \\ -\Delta x & \Delta y & -1 \\ 0 & \sqrt{2}\Delta y & -\sqrt{2} \\ \Delta x & \Delta y & -1 \end{bmatrix}}_{=M} \underbrace{\begin{bmatrix} A' \\ B' \\ D' \end{bmatrix}}_{=\boldsymbol{x}} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \\ r_9 \end{bmatrix}. \qquad \text{(B.14)}$$

Solution of the *normal equation* (cf. equation (B.12)), in this case, yields:

$$\begin{aligned} A' &= \frac{(z_3 - z_1) + 2(z_6 - z_4) + (z_9 - z_7)}{8\Delta x}, \\ B' &= \frac{(z_7 - z_1) + 2(z_8 - z_2) + (z_9 - z_3)}{8\Delta y}, \\ D' &= -\frac{1}{13}(z_1 + z_2 + z_3 + z_4 + z_5 + z_6 + z_7 + z_8 + z_9). \end{aligned} \qquad \text{(B.15)}$$

Substitution of the coefficients (B.15) into the gradient components given by (B.10) shows the correspondence with the weighted third-order finite differences method described by equation (B.3).

Appendix C

Metadata Elements

C.1 Modifiers

Modifiers are used to *enforce behaviour*, that is, to ensure that certain properties of the real terrain persist in the conceptualisation and so also in the DTM and the TI derived. Consequently, modifiers are clearly *normative* (as pointed out in section 6.1.1).

Table C.1 summarises the modifiers, their modifier elements and associated modifier attributes identified in section 6.1.1 as essential to exhaustively expose the workflow process involved in reliable TI generation (cf. table 6.1).

For these modifiers, this appendix provides:

- a short description of the *modifiers* and of their associated *modifier elements* (sections C.1.1 and C.1.2),
- selected parts composing the modifier elements (tables C.2 through C.6), including:
  - the types of *reporting scopes* to which they may be applied,
  - a short description of their respective *modifier attributes*, and
  - the *metadata* attributable to them (in view of *quality assurance*).

C.1.1 Abstraction Modifier

The **abstraction modifier** is defined as a modifier that supports or informs the linkage between the real world and the abstraction, thus exposing a DTMs UoD, reference model, rules and spatial concepts[1]. Links which might otherwise have been lost in the process of abstraction.

[1]In this sense, the idea of an **abstraction modifier** is very similar to the concept of "abstractive uncertainty" introduced by Duckham (1999, 2001).

Modifier	*Modifier element*	*Modifier attribute*
Abstraction modifier	Abstractive information	Abstraction Information base Dictionary
Specification modifier	Supporting property type specification	Mathematical specification Supporting type representation Geometry specification Sampled attribute specification
	Virtual property type specification	Mathematical specification Virtual type representation Geometry specification Virtual attribute specification Stored function

Table C.1: Modifiers, modifier elements and their associated modifier attributes.

An **abstraction modifier** comprises the modifier element **abstractive information** which, basically, is a collection of modifier attributes recording a model's semantic distance to perceived reality resulting from the process of abstraction (see tables C.1 and C.2). **Abstractive information**, hence, addresses *abstraction uncertainty* (discussed in sections 5.1.2 and 5.1.4). As abstraction uncertainty is, crucially, effective on a model level, the *reporting scope* of **abstractive information** is set to the entire, possibly virtual, dataset, thus ensuring that the entire dataset is based on the same abstraction.

The modifier element **abstractive information** comprises the three modifier attributes **abstraction**, **information base**, and **dictionary** (see tables C.1 and C.2). The attribute **abstraction**, basically, records the modelled TI's distance to perceived reality resulting from the process of abstraction[2] (table C.3). It basically exposes the *reference model*, combined with elements of the *rules* (for a detailed discussion of the concept of rules, see section 4.3). The **information base** provides an overview of the body of information embraced by the DTM (table C.3) and therefore may be interpreted as a subset of the *rules* (section 4.3). The modifier attribute **dictionary**, finally, defines all the terms used in the abstraction and specification steps. It must provide sufficiently rich definitions and examples so that there will be no ambiguity concerning the meaning of the terms used.

[2]Therefore, the modifier attribute **abstraction** lends from the notion of an *"abstraction modifier"* as proposed by CEN/TC287 (1996).

Modifier element	*Reporting scope*	*Cardinality*	*Modifier attributes*[3] → *obligation*	*Associated metadata*[3] → *obligation*
Abstractive information	Dataset (sampled/ supporting or virtual)	1	• **Abstraction** → m cf. table C.3 • **Information base** → m cf. table C.3 • **Dictionary** → m cf. table C.3	• **Lineage** → o, particularly **source**, **domain characteristics** and **update** (cf. section C.2.1) • **Purpose descriptor** → o (cf. section C.2.2) • **Validity** → o (cf. section C.2.4) • **Conformance** → o, particularly **completeness** (cf. section C.3.2) • **Structural integrity** → o (cf. section C.3.2)

Table C.2: Overview of the **abstraction modifier**.

[3] The arrows indicate the obligation (m: mandatory; a: alternative (in the sense of mutually exclusive); o: optional).

Modifier attribute	*Modifier attribute entries*
Abstraction → m	• The *modelled TI* in terms of its properties, its conceptualisation (spatial, functional, etc.), and of its relationships. • The modelling *scale*.
Information base → m	• The *body of knowledge* input to the terrain reconstruction, as well as the *assumptions* it is based upon. • The *thematic resolution* (expressed by the 'amount' of TI modelled, that is, the number of TI structures conceptualised) and hence the model's (semantic) expressiveness. • The *degree and magnitude of detail* displayed (as implied by the modelling scale).
Dictionary → m	• Definition of all the terms used in the abstraction and specification steps.

Table C.3: Overview of the modifier attributes of the **abstraction modifier** element **abstractive information**.

C.1.2 Specification Modifier

The **specification modifier** is a modifier designed to document the mappings involved in proceeding from conceptual models (such as the reference model and spatial concepts) to highly formalised representations (such as the mathematical model or computer representation). A **specification modifier**, therefore, on the one hand may *enforce behaviour* by ensuring that properties stressed in the abstraction process persist through the process of specification and thus are carried into the computer representation. On the other hand, it may provide estimates of the *specification uncertainty* (cf. sections 5.1.2 and 5.1.4) introduced. Hence, a **specification modifier** is understood as a collection of textual descriptions of the distortions resulting from mathematical formalisation and digital encoding of the TI modelled. As specification uncertainty was identified to be effective at a variable or property type[4] level (cf. section 5.1.4), the *reporting scope* of modifier elements associated to a **specification modifier** is set to property types, thus ensuring that all data items (or properties, supporting or virtual) of a specific property type are based on the same specifications.

A **specification modifier** comprises the modifier elements **supporting property type specification** and **virtual property type specification** (see tables C.1 and C.4). The element **supporting property type specification** serves to specify property types that must be sampled. The element **virtual property type specification** is designed to specify property types that need not to be sampled but that are derived from supporting property

[4]Remember, property types denote variables mapped to a digital representation (see section 2.3.3).

Modifier element	*Reporting scope*	*Cardinality*	*Modifier attributes → obligation*	*Associated metadata → obligation*
Supporting property type specification	Sampled or supporting property type	1	• **Mathematical specification** → m (table C.5) • **Supporting type representation** → m (table C.5) • **Geometry specification** → m (table C.5) • **Sampled attribute specification** → m (table C.5)	• **Lineage** → o, particularly **source**, **domain characteristics** and **update** (cf. section C.2.1) • **Purpose descriptor** → o (cf. section C.2.2) • **Validity** → o (cf. section C.2.4) • **Technical quality** → o (cf. section C.2.6) • **Conformance** → o, particularly **completeness** (cf. section C.3.2) • **Structural integrity** → o (cf. section C.3.2)
Virtual property type specification	Virtual property type	1	• **Mathematical specification** → m (table C.5) • **Virtual type representation** → m (table C.6) • **Geometry specification** → m (table C.5) • **Virtual attribute specification** → m (table C.6) • **Stored function** → m (table C.6)	Same as for **supporting property type specification** (see above row)

Table C.4: Overview of the **specification modifier**.

types. That is, the modifier element **virtual property type specification**, basically, specifies *persistently stored methods* (or *stored functions*).

Supporting Property Type Specification

This modifier element comprises the modifier attributes **mathematical specification**[5], **supporting type representation**[6], **geometry specification**, and **sampled attribute specification** (tables C.1, C.4 and C.5). While, in terms of the modelling sequence discussed in section 2.3, the first two of these modifier attributes help to render explicit the *mathematical model* and the chosen *spatial data model*, the last two provide the actual *computer representation*.

To the attribute **geometry specification**, always a **stored function** (described in the next paragraph) of kind 'approximation' is associated (table C.5; see also figure 7.8). This association is required due to the distinction between sampled and supporting data made in chapters 6 and 7. Sampled data is meant to exclusively consist of sampled elements such as spot heights. Supporting data may comprise a sampled and a reconstructed component such as contours built from a sampled point sequence by interpolation. Hence, the **stored function** is required to provide an exact specification for the numerical realisation of the geometry defined by the modifier attribute **mathematical specification**.

Virtual Property Type Specification

This modifier element comprises the modifier attributes **mathematical specification**[7], **virtual type representation**, **geometry specification**, **virtual attribute specification**, and **stored function** (tables C.1, C.4 and C.6). While, in terms of the modelling sequence discussed in section 2.3, the first two of these modifier attributes help to render explicit the *mathematical model* and the chosen *spatial data model*, the last three provide the actual *computer representation*.

To the modifier attributes **geometry specification** and **virtual attribute specification**, always a **stored function** is associated (table C.6; see also figure 7.9). In the **geometry specification** case, this association is required to provide an exact specification for the numerical realisation of the geometry prescribed by the modifier attribute **mathematical specification**. In the **virtual attribute specification** case, the **stored function**

[5]As indicated in table C.5, the **mathematical specification** may be directly derived from the *mathematical model*.

[6]As indicated in table C.5, the **supporting type representation** may be directly derived from the *rules* and the chosen *spatial data model*.

[7]As indicated in table C.5, the **mathematical specification** may be directly derived from the *mathematical model*.

Modifier attribute	*Modifier attribute entries*	*Associated metadata*
Mathematical specification → m	• The *model 'type'* (i.e., deterministic, stochastic, etc.). • The *impact domain* (or area) of the variable/property type: local, zonal, global. • The *geometry* definition. • The *mathematical formalisation* of the conceptualised relationships (by definitions in closed form or by application-dependent parameters and thresholds). • The *requirements* and *prerequisites* of the data.	
Supporting type representation → m	• The property type's *data model* (including its 'localisation qualities', cf. section 5.1.2). • The *sampling scheme* (including a description of the sampling pattern). • The *sampling resolution* or *scale*.	
Geometry specification → m	• The *geometry type* (scalar, vector, directional, etc.). • The *dimension*. • The *coordinate system* (polar, Cartesian, etc.). • The *coordinate type* (or value type, such as int, double, etc.). • The *spatial reference system*.	**Stored function** (table C.6).
Sampled attribute specification → m	• The *property name/value type* pair defining the specified attribute. • The *precision* to be reported. • The *frame of reference*.	

Table C.5: Overview of the modifier attributes of the **specification modifier** element **supporting property type specification**.

Modifier attribute	*Modifier attribute entries*	*Associated metadata*
Mathematical specification → m	Same as in the **supporting property type specification** case (see table C.5).	
Virtual type representation → m	• The property type's *data model* (including its 'localisation properties', cf. section 4.3.1).	
Geometry specification → m	Same as in the **supporting property type specification** case (see table C.5).	**Stored function**
Virtual attribute specification → m	• The *property name/value type* pair defining the specified attribute. • The *precision* to be reported. • The *frame of reference*.	**Stored function**
Stored function → m	• The *operation kind*, which is a single operation or any combination of the following: approximation (including interpolation as a special case), differentiation, integration, trigonometric operation, arithmetics, statistics, (partial) differential equation, etc. • The stored function *type* (deterministic, stochastic, etc.). • The function's *'domain'* (local, zonal, global). • Further specifications depending on the operation kind[8].	**Stored function descriptor** (section C.2.7)

Table C.6: Overview of the modifier attributes of the **specification modifier** element **virtual property type specification**.

provides an exact specification of how the virtual attribute may be derived from the supporting property types when requested. **Stored function**s, therefore, make up the persistent part of virtual data. To support the reporting principle of providing two-fold metainformation for virtual data (discussed in section 6.2.2), so-called **stored function descriptor**s are associated to **stored function**s (table C.6; see also figure 7.9). The **stored function descriptor**s (discussed in section C.2.7) are introduced to assess a **stored function**'s performance.

[8]In digital terrain modelling, the probably most frequently used stored functions are of kind *'approximation'* and *'differentiation'*. A **stored function** of kind *approximation*, for instance, needs to provide further specifications concerning: the *numerical method* applied (e.g., splines, kriging, finite element approaches); the *domain subdivision* the approxima-

Descriptor	*Descriptor element*
Lineage	Source Domain characteristics Survey Preprocessing Transformation Conversion Update
Purpose descriptor	Purpose
Usage descriptor	Usage
Validity	Semantic validity Validity across scales Temporal validity
Model expressiveness	Modelling scale Model explicitness Thematic resolution Domain consistency properties
Technical quality descriptor	Technical quality
Stored function descriptor	Technical performance Stability Artifacts control

Table C.7: Descriptors associated descriptor elements.

C.2 Descriptors

Descriptors provide *informative* data explaining the supplied TI with regard to its history, its validity and expressiveness, its original purpose and current usage, or its technical implementation.

Table C.7 summarises the descriptors and their associated descriptor elements identified in section 6.1.2 as necessary to provide comprehensive TI documentation on an *informative* level (cf. table 6.2).

tion is based upon (including description of the algorithm applied for constructing the domain subdivision); the *basis functions* used, including their *order* (that is, description of the *function space* to which the approximation belongs); the *prerequisites* of to the data used; the *coordinate system* referred to (including specification of possibly used local coordinates); the *tolerances* specified (e.g., for multi-valued logics); etc.

A **stored function** of kind *differentiation* needs to provide further specifications concerning: the *numerical method* applied (e.g., analytical differentiation, numerical differentiation, finite differences); the *increment* possibly applied; the *prerequisites* of the data used; the *tolerances* specified; etc.

Descriptor	*Descriptor element*	*Reporting scope*	*Cardinality*	*Descriptor attributes*	*Associated metadata*[9] $\rightarrow$ *obligation*
Lineage	**Source**	No restrictions apply	1	see table C.10	-
	Domain characteristics	No restrictions apply	1	see table C.10	-
	Survey	Sampled dataset	1	see table C.10	• **Lineage** → o, namely **source**
	Preprocessing	Sampled/supporting or virtual dataset, property type or individual data item	Many	see table C.10	• **Lineage** → o, namely **source** • **Structural integrity** → o, namely **logical integrity**, **structural integrity** (cf. table C.29) • **Formal correctness** → o (cf. table C.30) • **Conformance** → o, namely **completeness** (cf. table C.32)
	Transformation	Sampled/supporting or virtual dataset, property type or individual data item	Many	see table C.10	Same as **preprocessing**
	Conversion	Sampled/supporting or virtual dataset, property type or individual data item	Many	see table C.10	Same as **preprocessing**
	Update	Sampled/supporting or virtual dataset, property type or individual data item	Many	see table C.10	Same as **preprocessing**

Descriptor	*Descriptor element*	*Reporting scope*	*Cardinality*	*Descriptor attributes*	*Associated metadata*[9] → *obligation*
Usage descriptor	**Usage**	Sampled/supporting or virtual dataset, property type or individual data item	Many	see table C.12	• **Lineage** → o, namely **source** (cf. table C.10) • **Conformance** → o, namely **completeness** (cf. table C.32)
Purpose descriptor	**Purpose**	No restrictions apply	1	see table C.11	• **Lineage** → o, namely **source** (cf. table C.10)
Validity	**Semantic validity**	No restrictions apply	1	see table C.13	Same as **purpose**
	Validity across scales	No restrictions apply	1	see table C.13	Same as **purpose**
	Temporal validity	No restrictions apply	1	see table C.13	Same as **purpose**
Model expressiveness	**Modelling scale**	Sampled/supporting or virtual dataset, property type or individual data item	1	see table C.14	**Lineage** → o, namely **source** (cf. table C.10) **Conformance** → o, namely **completeness** (cf. table C.32)
	Model explicitness	Supporting or virtual dataset	1	see table C.14	Same as **modelling scale**
	Thematic resolution	Supporting or virtual dataset	1	see table C.14	Same as **modelling scale**
	Domain continuity properties	Supporting or virtual property type	1	see table C.14	Same as **modelling scale**
Technical quality descriptor	**Technical quality**	Supporting or virtual dataset; **supporting-** and **virtual property type specification**	1	see table C.15	• **Lineage** → o, namely **source** (cf. table C.10) • **Conformance** → o, namely **completeness** (cf. table C.32)

Table C.8: Overview of the proposed descriptors.

Descriptor	*Descriptor element*	*Reporting scope*	*Cardinality*	*Descriptor attributes*	*Associated metadata*[9] $\rightarrow$ *obligation*
Stored function descriptor	**Technical performance**	**Stored function**	1	see table C.16	• **Lineage** $\rightarrow$ o, namely **source** (cf. table C.10)
	Stability	**Stored function**	1	see table C.16	• **Lineage** $\rightarrow$ o, namely **source** (cf. table C.10)
	Artifacts control	**Stored function**	1	see table C.16	• **Lineage** $\rightarrow$ o, namely **source** (cf. table C.10)

Table C.9: Overview of the **stored function descriptor**.

[9]The arrows indicate the obligation (m: mandatory; a: alternative (in the sense of mutually exclusive); o: optional).

For these descriptors, this appendix provides:

- a short explanation of the *descriptors* and their associated *descriptor elements*,
- selected parts composing the descriptor elements, including:
  - the types of *reporting scopes* to which they may be applied (tables C.8 and C.9),
  - a short characterisation of their respective *descriptor attributes* (tables C.10 through C.16), and
  - the *metadata* attributable to them (in view of *quality assurance*; tables C.8 and C.9).

C.2.1 Lineage

Lineage is defined to comprehensively track the *history* of a DTM (including, in as much as known, the history of its abstraction and specification). On a data level, the source(s) of the data as well as its compilation, transformations and insertions are documented in chronological order. **Lineage** comprises the descriptor elements **source**, **domain characteristics**, and **survey**, as well as of a number of elements describing the (pre)processing that took place, namely **preprocessing**, **transformation**, **conversion**, and **update** (tables C.7 and C.8). A short characterisation of the descriptor elements constituting **lineage** is provided in tables C.8 and C.10.

C.2.2 Purpose Descriptor

A **purpose descriptor** consists of the single element **purpose** (tables C.7 and C.8), which shall record the TI producer's rationale for creating a DTM. **Purpose** may also contain information about a DTM's intended use (table C.11). The descriptor element **purpose**, in this sense, fulfils the tasks of the quality overview element *purpose* defined in the ISO Standard 19113 Geographic Information - Quality Principles (ISO/TC211 1999). Note that a DTM's intended use is not necessarily the same as its current use, which is documented by the **usage descriptor** (exposed in section C.2.3).

C.2.3 Usage Descriptor

A **usage descriptor** comprises the single descriptor element **usage** (tables C.7 and C.8), which documents the TI's current use (table C.12). The element **usage**, hence, corresponds to the quality overview element *usage* defined in the ISO Standard 19113 Geographic Information - Quality Principles (ISO/TC211 1999).

Descriptor element	*Descriptor attribute entries → obligation*
Source	• The *date* of TI generation. → m • The person(s) or organisation *initiating* the production of the TI. → o • The person(s) or organisation *owning* the terrain data. → o • The *purpose* for creating the TI. → o
Domain characteristics	• The *area* covered. → m • The TI's *scale range*. → m • The TI's *semantics*. → m • The *temporal characteristics* of the TI. → o
Survey	• The data *source* (field campaign, map, other digital data, etc.). → m • The *methods* used to gather the data. → o • The *instruments* used for data collection. → o • The *recording media* involved. → o • The *time period* of data collection. → o
Preprocessing	• The *type* of preprocessing that took place (correction, calibration, georeferencing, etc.). → m • The *algorithms* used. → m • The *assumptions* the preprocessing is based upon. → o • The *frames of reference* referred to. → m
Transformation	• The *type* of transformation that took place (coordinate transformation, aggregation, classification, cartographic displacement, generalisation, etc.). → m • The *algorithms* used for the transformation (e.g., map projection used). → m • The *assumptions* the transformation is based upon. → o • The *frames of reference* referred to. → m
Conversion	• The *type* of conversion that occurred (scanning, digitising, vectorisation, etc.). → m • The *instruments* used. → o • The *algorithms* used. → o
Update	• The *type* of update that occurred (creation, modification, deletion, insertion, etc.). → m

Table C.10: Overview of the descriptor attributes of the descriptor elements of **lineage**.

Descriptor element	*Descriptor attribute entries → obligation*
Purpose	• The *rationale for creation*. → o • The *intended use*. → o

Table C.11: Overview of the descriptor attributes of the descriptor elements of a **purpose descriptor**.

Descriptor element	*Descriptor attribute entries → obligation*	
Usage	• The applications for which the data (supporting or virtual) or computer representation have been used.	→ o
	• Information on *usage* and *access.*	→ o

Table C.12: Overview of the descriptor attributes of the descriptor elements of a **usage descriptor**.

Descriptor element	*Descriptor attribute entries → obligation*	
Semantic validity	• Textual description of whether the modelling is based on *intrinsic properties* of the portrayed terrain or on properties depending on the *purposes of a specific application.*	→ o
Validity across scales	• *Scale effects*, introduced through the abstraction process, the spatial data model used, characteristics of the measuring device, etc.	→ m
	• Possibly occurring *self-similarities.*	→ o
Temporal validity	• Textual description of the expected *sensitivity* to *temporal variation.*	→ o

Table C.13: Overview of the descriptor attributes of the descriptor elements of **validity**.

C.2.4 Validity

Validity is a measure of genericness. It may be used to qualify both, the TI (supporting or virtual) as well as its abstraction and, particularly, its specification. The definition of the proposed elements of validity is guided by the considerations set out in section 5.3. **Validity**, hence, comprises the descriptor elements **semantic validity**, **validity across scales** and **temporal validity** (tables C.7 and C.8). A short characterisation of the descriptor elements constituting **validity** is provided in tables C.8 and C.13.

C.2.5 Model Expressiveness

Model expressiveness is a collection of descriptor elements aiming at documenting properties of the provided TI which impact on a DTM's expressiveness. It comprises the descriptor elements **modelling scale**, **model explicitness**, **thematic resolution**, and **domain continuity properties** (table C.7 and C.8). A brief characterisation of the descriptor elements constituting **model expressiveness** is provided in tables C.8 and C.14.

Descriptor element	*Descriptor attribute entries → obligation*	
Modelling scale	• The *scale*.	→ m
	• The degree and details of *generalisation* (hence indicating the amount of detail represented).	→ m
Model explicitness	• Textual description of the dataset's *explicitness* (including, for instance, the ratio of explicitly represented to implicitly derived information).	→ o
Thematic resolution	• Textual description of the dataset's *thematic resolution*, that is, a recording of the variety of topographic structures explicitly modelled.	→ o
Domain continuity properties	• Textual description of the geometry for which TI of a property type is available.	→ o

Table C.14: Overview of the descriptor attributes of the descriptor elements of **model expressiveness**.

Descriptor element	*Descriptor attribute entries → obligation*	
Technical quality	• Characteristics of the present format (headers, etc.).	→ o
	• Storage needs.	→ o
	• Runtime efficiency.	→ o
	• Portability.	→ o
	• Flexibility.	→ o
	• Robustness.	→ o
	• Ease of use.	→ o

Table C.15: Overview of the descriptor attributes of the descriptor elements of a **technical quality descriptor**.

C.2.6 Technical Quality Descriptor

A **technical quality descriptor** comprises the single descriptor element **technical quality** (tables C.7 and C.8), which describes technical properties of the DTM implementation (as suggested by Brändli (1998); see tables C.8 and C.15).

C.2.7 Stored Function Descriptor

To conform to the reporting principle of providing two-fold metainformation for virtual data (discussed in section 6.2.2), a **stored function descriptor** is introduced, which assesses a **stored function**'s performance (cf. section C.1.2). A **stored function descriptor** comprises the three descriptor elements **technical performance**, **stability**, and **artifacts control** (tables C.7 and C.9) that are briefly characterised in table C.16.

Descriptor element	*Descriptor attribute entries → obligation*	
Technical performance	• Storage needs. • Runtime efficiency. • Computational demands (such as RAM required). • Complexity control.	→ o → o → o → o
Stability	• *Numerical stability.* • *Structural stability* (including convergence issues, sensitivity to perturbations, etc.) [10].	→ m → m
Artifacts control	• Possibly arising *boundary problems*, and the approach chosen to handle them. • Further identification of possibly or actually resulting *artifacts*, depending on the kind of operation assessed [11].	→ o → o

Table C.16: Overview of the descriptor attributes of the descriptor elements of a **stored function descriptor**.

C.3 Quality Aspects

Quality elements are metadata components which *measure* the actual performance of the provided TI. Table C.17 summarises the quality aspects, their quality elements and associated quality subelements identified in section 6.1.3 as necessary to comprehensively assess the performance of the supplied TI with respect to the criteria set forth in its specification (cf. table 6.3).

For these quality aspects, this appendix provides:

- a brief description of the *quality aspects*, their *quality elements* and associated *quality subelements*,
- selected parts composing the quality subelements, including:
  - the types of *reporting scopes* to which they may be applied,
  - their *metric scopes*,
  - possible quality *measures* (however, examples rather than an exhaustive listing are provided),
  - corresponding data quality *units* (again, examples rather than an exhaustive listing are provided), and
  - the *metadata* attributable to them (in view of *quality assurance*).

[10] A more detailed discussion of the notion of structural stability is provided in section 6.2.2.

[11] When, for instance, characterising an interpolating **stored function**, comments to over- or undershooting of the interpolation result or to the interpolant's ability for shape control may be provided.

Quality aspect		*Quality element*	*Quality subelement*
Representative quality	*Accuracy elements*	Metric attribute accuracy	Absolute accuracy Relative accuracy Polyhedral value accuracy ...
		Non-metric attribute accuracy	Value-typed attribute accuracy Classification accuracy ...
		Shape fidelity	Slope fidelity Curvature fidelity Drainage structure fidelity ...
	Information content	Supporting information content	Randomness Dimensionality Sampling resolution and pattern Redundancy Information elements provided
		Information Substance	Scale Reported precision Data persistence Support
Consistency		Structural integrity	Logical integrity Topological integrity Functional integrity Temporal integrity
		Formal correctness	Formal consistency Data structure consistency Domain consistency
		Conformance	Information persistence Operationalisation consistency Encoding consistency Specification consistency Format consistency Completeness

Table C.17: Quality aspects, quality elements and their associated quality subelements.

C.3.1 Representative Quality

Representative quality is designed to report on the implications of the *data uncertainty* inevitably introduced through parametrisation of the computer representation by the actual data (cf. the discussion in sections 5.1.2 and 5.1.4). Data uncertainty is, crucially, effective on the level of the individual data items (section 5.1.4). **Representative quality**, therefore, relates to the individual - sampled, supporting or virtual - data items. It comprises the *accuracy elements* **metric attribute accuracy**, **non-metric attribute accuracy**, and **shape fidelity**. However, due to the "systemness" of the phenomenon terrain (cf. section 2.2), **representative quality** also has a "systemness" component, expressed by means of the *measures of information content* **supporting information content** and **information substance** (table C.17).

Quality subelement	*Characterisation*	*Reporting scope*	*Metric scope*	*Cardinality*	*Quality attributes*	*Associated metadata → obligation*
Absolute accuracy	Accuracy of the stated value to the true value with respect to a specified datum	Sampled/supporting or virtual dataset, property type or single data item	Interval or ratio scale *vector* data	1	table C.19	• **Lineage** → o, namely **source** (cf. table C.10)
Relative accuracy	Accuracy within a dataset or to data items of another dataset	Sampled/supporting or virtual dataset, property type or single data item	Interval or ratio scale *vector* data	1	table C.19	Same as **absolute accuracy**
Polyhedral value accuracy	Accuracy of the values assigned to the faces of a polyhedral domain tesselation relative to ground truth	Sampled/supporting or virtual dataset, property type or single data item	Interval or ratio scale *raster* data	1	table C.19	Same as **absolute accuracy**
Temporal accuracy	Correctness of the temporal references of data (i.e., reports on the uncertainty in time measurement)	Sampled/supporting or virtual dataset, property type or single data item	Interval or ratio scale *temporal* data	1	table C.19	Same as **absolute accuracy**

Table C.18: Overview of the subelements of the **representative quality** element **metric attribute accuracy**.

Metric Attribute Accuracy

Metric attribute accuracy is an *accuracy element* of **representative quality** with subelements such as **absolute accuracy**, **relative accuracy**, **polyhedral value accuracy** or **temporal accuracy** (tables C.17, C.18 and C.19)[12].

Quality subelement	*Quality attribute*		
	Measure [12] → m	*Unit* → m	*Associated metadata* → *obligation*
Absolute accuracy	• Standard deviation, variance, • confidence level, • interval (cf. section 3.2.2), • ...	• Measurement unit (ground distance), • measurement unit, • measurement unit, • ...	• **Structural integrity** → o, namely **logical integrity** (table C.29). • **Formal correctness** → o, namely **formal consistency**, **domain consistency** (table C.30). • **Conformance** → o, namely **completeness** (table C.32).
Relative accuracy	• Variance/ covariance matrix	• -	Same as the attributes of **absolute accuracy**
Polyhed-ral value accuracy	• Existence, number or percentage of polyhed- ral faces dis-playing de-viations lar-ger than prescribed specification limits, • maximal deviation, • fuzzy set.	• Boolean variable, number or percentage, • measurement unit, • measurement unit.	Same as the attributes of **absolute accuracy**
Temporal accuracy	• Mean deviation, • standard deviation, variance, • interval.	• Time unit, • time unit, • time unit.	Same as the attributes of **absolute accuracy**

Table C.19: Overview of the quality attributes of the subelements of **metric attribute accuracy**.

[12]Note that examples rather than an exhaustive listing are provided here.

Quality subelement	*Characterisation*	*Reporting scope*	*Metric scope*	*Cardinality*	*Quality attributes*	*Associated metadata → obligation*
Value-typed attribute accuracy	Accuracy of an attribute defined by a value	Sampled/supporting or virtual dataset, property type or single data item	Nominal or ordinal data	1	table C.21	• **Lineage** → o, namely **source** (cf. table C.10)
Classification accuracy	Accuracy of the assignment of attributes to classes	Sampled/supporting or virtual dataset, property type or single data item	Nominal, ordinal, interval or ratio scale data	1	table C.21	Same as **value-typed attribute accuracy**

Table C.20: Overview of the subelements of the **representative quality** element **non-metric attribute accuracy**.

Non-metric Attribute Accuracy

Non-metric attribute accuracy is an *accuracy element* of **representative quality** with subelements such as **value-typed attribute accuracy** or **classification accuracy** (tables C.17, C.20 and C.21). Note that examples rather than an exhaustive listing are provided here.

Quality subelement	*Quality attribute*		
	Measure → m	*Unit* → m	*Associated metadata* → *obligation*
Value-typed attribute accuracy	• Existence, number or percentage of incorrect items or of items with a larger deviation than prescribed by specification limits, • standard deviation, variance, • confidence level, • ...	• Boolean variable, number, percentage, • measurement unit, • measurement unit • ...	• **Structural integrity** → o, namely **logical integrity** (cf. table C.29). • **Formal correctness** → o, namely **formal consistency**, **domain consistency** (cf. table C.30). • **Conformance** → o, namely **completeness** (table C.32).
Classification accuracy	• Percentage of misclassification, • misclassification matrix, • fuzzy set, • ...	• Percentage, • percentage matrix, • measurement unit, • ...	Same as the attributes of **value-typed attribute accuracy**

Table C.21: Overview of the quality attributes of the subelements of **non-metric attribute accuracy**.

Shape Fidelity

Shape fidelity is an *accuracy element* of **representative quality** reporting the degree to which the shape of the digital representation of linear or areal TI conforms to the phenomenon's true shape (with respect to a specified, established datum). **Shape fidelity** comprises quality subelements such as **slope fidelity**, **curvature fidelity**, or **drainage structure fidelity** (tables C.17, C.22 and C.23).

Quality subelement	*Characterisation*	*Reporting scope*	*Metric scope*	*Cardinality*	*Quality attributes*	*Associated metadata → obligation*
Slope fidelity	Assesses the similarity of the slope of some digitally represented linear or areal TI with a slope datum accepted as true	Sampled/supporting or virtual dataset, property type or single data item	Interval or ratio scale field data	1	table C.23	• **Lineage** → o, namely **source** (cf. table C.10)
Curvature fidelity	Assesses the similarity of the curvature of a di gitally represented linear or areal TI with a curvature datum accepted as true	Sampled/supporting or virtual dataset, property type or single data item	Interval or ratio scale field data	1	table C.23	Same as **slope fidelity**
Drainage structure fidelity	Assesses the similarity of the drainage structure of a DTM with a drainage structure accepted as true	Sampled/supporting or virtual dataset, property type or single data item	Interval or ratio scale data	1	table C.23	Same as **slope fidelity**

Table C.22: Overview of the subelements of the **representative quality** element **shape fidelity**.

Quality subelement	*Quality attribute*		
	Measure → m	*Unit* → m	*Associated metadata* → *obligation*
Slope fidelity	• Existence, number or percentage of deviating surface network nodes and edges, • conformance of surface network topology, • ...	• Boolean variable, number, percentage, • boolean variable, • ...	• **Structural integrity** → o, namely **logical integrity** (cf. table C.29). • **Formal correctness** → o, namely **formal consistency**, **domain consistency** (cf. table C.30). • **Conformance** → o, namely **completeness** (table C.32).
Curvature fidelity	• Existence, number or percentage of deviating surface network nodes and edges, • degree of similarity or conformance of concave and convex regions, • ...	• Boolean variable, number, percentage, • percentage, boolean variable, • ...	Same as the attributes of **slope fidelity**
Drainage structure fidelity	• Degree of similarity or conformance of drainage network topology, • degree of similarity or conformance of catchment areas, • differences in the number of peaks and pits, • ...	• Percentage, boolean variable, • percentage, boolean variable, • number, • ...	Same as the attributes of **slope fidelity**

Table C.23: Overview of the quality attributes of the subelements of **shape fidelity**.

Quality subelement	*Characterisation*	*Reporting scope*	*Metric scope*	*Cardinality*	*Quality attributes*	*Associated metadata → obligation*
Randomness	Assesses the randomness of the supporting data with respect to the modelled terrain surface	Sampled/supporting dataset, property type or single data item	No restrictions apply	1	table C.25	• **Lineage** → o, namely **source** (cf. table C.10)
Dimensionality	Evaluates the dimensionality of supporting data elements	Sampled/supporting dataset, property type or single data item	No restrictions apply	1	table C.25	Same as **randomness**
Sampling resolution and pattern	Evaluates the resolution (or density) and pattern of the supporting data	Sampled/supporting dataset, or property type	No restrictions apply	1	table C.25	Same as **randomness**
Redundancy	Measures the redundancy of the supporting data	Sampled/supporting dataset	No restrictions apply	1	table C.25	Same as **randomness**
Information elements provided	Counts the number of conditions contributed to terrain reconstruction	Sampled/supporting dataset, property type or single data item	No restrictions apply	1	table C.25	Same as **randomness**

Table C.24: Overview of the subelements of the **representative quality** element **supporting information content**.

Supporting Information Content

Supporting information content is an element of **representative quality** aiming at reporting the amount of knowledge and information for terrain surface reconstruction deducible from the *supporting data*. **Supporting information content**, hence, is critical to a TI user evaluating the reliability of a terrain model parametrisation.

Supporting information content comprises the subelements **randomness**, **dimensionality**, **sampling resolution and pattern**, **redundancy**, and **information elements provided** (tables C.17, C.24 and C.25). While, in terms of the *measures of information content* proposed in section 4.3.1, **randomness** and **dimensionality** are rather *individual or probability measures*, **sampling resolution and pattern**, **redundancy** and **information elements provided** conform more to the concept of *'systemness' measures.*

The subelement **dimensionality** evaluates the dimensionality of supporting data elements (table C.24). That is, it investigates whether a supporting data item is a point, a linear or an areal feature. **Dimensionality** is a measure of the degrees of freedom left to the modelling operations domain subdivision and reconstruction, that is, it indicates the number of modelling operations that are influenced by the supporting data (cf. sections 4.2 and 4.3.1).

Redundancy measures the redundancy of the supporting data (table C.24) and thus indicates the potential of the supporting dataset for further compression.

The subelement **information elements provided** counts the the number of conditions contributed to terrain reconstruction by a supporting data element (table C.24), either by itself or in combination with other supporting data (cf. section 4.3.1).

Quality subelement	*Quality attribute*		
	Measure → m	*Unit* → m	*Associated metadata* → *obligation*
Randomness	• Existence, number or percentage of of terrain specific (i.e., coordinate-random) supporting data items, • ...	• Boolean variable, number, percentage, • ...	• **Structural integrity** → o, namely **logical integrity** (cf. table C.29). • **Formal correctness** → o, namely **formal consistency**, **domain consistency** (cf. table C.30). • **Conformance** → o, namely **completeness** (table C.32).

Quality subelement	*Quality attribute*		
	Measure → m	*Unit* → m	*Associated metadata* → *obligation*
Dimensionality	• Dimension of a supporting data item, • ...	• Number • ...	Same as the attributes of **randomness**
Sampling resolution and pattern	• Magnitude of the smallest TI recognisable, • sampling density, • sampling pattern, • ...	• Lenght, width and height measure (ground distances), • number of data items per squared ground distances, • string(regular, scattered, clustered, etc.), • ...	Same as the attributes of **randomness**
Redundancy	• Number or percentage of supporting data items suppressible without altering the dataset's information content, • number of supporting data items required to represent the data in an alternative bases, • ...	• Number, percentage, • number • ...	Same as the attributes of **randomness**
Information elements provided	• Number of information elements provided by a supporting data item, • ...	• Number, • ...	Same as the attributes of **randomness**

Table C.25: Overview of the quality attributes of the subelements of **supporting information content**.

Quality subelement	*Characterisation*	*Reporting scope*	*Metric scope*	*Cardinality*	*Quality attributes*	*Associated metadata → obligation*
Scale	Documents the TI's scale range	Sampled/supporting or virtual dataset, property type or single data item	No restrictions apply	1	table C.27	• **Lineage** → o, namely **source** (cf. table C.10)
Reported precision	Documents the precision with which the TI is represented	Sampled/supporting or virtual dataset, property type or single data item	No restrictions apply	1	table C.27	Same as **scale**
Data persistence	Asks whether TI is sampled or de rived (whether it is of supporting or virtual type)	Sampled/supporting or virtual dataset, property type or single data item	No restrictions apply	1	table C.27	Same as **scale**
Support	Describes the support of the TI (in terms of support geometry and selection function)	Sampled/supporting dataset, property type or single data item	No restrictions apply	1	table C.27	Same as **scale**

Table C.26: Overview of the subelements of the **representative quality** element **information substance**.

Information Substance

While **supporting information content** focuses on the information content of the *supporting dataset*, **information substance** is a *measure of information content* that describes the performance of the entire dataset (i.e., of both, *supporting and virtual data*) in terms of the provided information. To this end, the quality subelements **scale**, **reported precision**, **data persistence**, and **support** are proposed (tables C.17, C.26 and C.27).

The subelement **support** describes the support of the supplied TI. It provides a TI user the support *geometry* together with the corresponding *selection function* (for discussion of the concept of selection functions, see section 2.1.2). If a supporting data element represents, for instance, a grid cell, the support *geometry* is a regular square whose extent must be documented in ground distances. The *selection function*, in this case, corresponds to the characteristic function of the grid cell.

Quality subelement	*Quality attribute*		
	Measure → m	*Unit* → m	*Associated metadata* → *obligation*
Scale	• TI scale range, • ...	• Ratio value (in ground distances), • ...	• **Structural integrity** → o, namely **logical integrity** (cf. table C.29). • **Formal correctness** → o, namely **formal consistency**, **domain consistency** (cf. table C.30). • **Conformance** → o, namely **completeness** (table C.32).
Reported precision	• Number of significant internal decimal places, • ...	• Number, • string (int, float, etc.), • ...	Same as the attributes of **scale**
Data persistence	• Type of TI, • ...	• String (sampled, supporting, virtual), • ...	Same as the attributes of **scale**
Support	• Support geometry and selection function, • ...	• String (referring to ground distances), • ...	Same as the attributes of **scale**

Table C.27: Overview of the quality attributes of the subelements of **information substance**.

C.3.2 Consistency

Consistency shall document the *integrity* of the *digital terrain modelling workflow*. The definition of the quality elements making up **consistency**, their quality subelements and associated quality attributes is guided by the considerations presented in section 5.2. **Consistency**, hence, comprises the quality elements **structural integrity**, **formal correctness** and **conformance** (table C.17).

Consistency is designed to be applicable to both the *data* and the *metadata* describing it (tables C.29, C.30 and C.32). The potentiality of its application to qualify other metadata elements, particularly the modifiers **abstraction modifier** and **specification modifier**, lets **consistency** become a useful tool of *quality assurance* (as discussed in section 6.2.3).

Structural Integrity

Structural Integrity is the *consistency* element concerned with the *logical consistency* of both the TI (supporting or virtual, i.e., made available through a stored function) and its corresponding conceptualisation and operationalisation. **Structural integrity** comprises the subelements **logical integrity**, **topological integrity**, **functional integrity**, and **temporal integrity** (tables C.17, C.29 and C.28).

Quality subelement	*Quality attribute*		
	Measure → m	*Unit* → m	*Associated metadata* → *obligation*
Logical integrity	• Occurrence of contradictions, • degree of consistency, • number of violations, • ...	• Boolean variable, • percentage, • number, • ...	• **Structural integrity** → o, namely **logical integrity**. • **Formal correctness** → o, namely **formal consistency**, **domain consistency** (cf. table C.30). • **Conformance** → o, namely **completeness** (table C.32).
Topological integrity	• Occurrence of topological inconsistencies, • degree of consistency of prescribed topologic relations, • number of topologic violations, • ...	• Boolean variable, • percentage, • number, • ...	Same as the attributes of **logical integrity**
Functional integrity	• Occurrence of functional inconsistencies, • degree of consistency of modelled functional relationships, • number of functional inconsistencies, • ...	• Boolean variable, • percentage, • number, • ...	Same as the attributes of the attributes of **logical integrity**
Temporal integrity	• Occurrence of erroneously ordered events or sequences, • existence, number or percentage of violations of prescribed evenet sequences, • ...	• Boolean variable, • boolean variable, number, percentage, • ...	Same as **logical integrity**

Table C.28: Overview of the quality attributes of the subelements of **structural integrity**.

Quality subelement	*Characterisation*	*Reporting scope*	*Metric scope*	*Cardinality*	*Quality attributes*	*Associated metadata → obligation*
Logical integrity	Tests the integrity of the logical structure imposed on the data (cf. section 5.2.1)	No restrictions apply	No restrictions apply	Many	table C.28	• **Lineage** → o, namely **source** (cf. table C.10)
Topological integrity	Assesses the integrity of the topological structure imposed, that is, the degree of adherence to characteristics of geometry remaining invariant under continuous mapping transformations (section 5.2.1)	*Spatial* data	No restrictions apply	Many	table C.28	Same as **logical integrity**
Functional integrity	Assesses the integrity of the functional relationships that are modelled (cf. section 5.2.1)	No restrictions apply	No restrictions apply	Many	table C.28	Same as **structural integrity**
Temporal integrity	Tests the correctness of ordered events or sequences	No restrictions apply	*Temporal* data	Many	table C.28	Same as **structural integrity**

Table C.29: Overview of the subelements of the **consistency** element **structural integrity**.

Quality subelement	*Characterisation*	*Reporting scope*	*Metric scope*	*Cardinality*	*Quality attributes*	*Associated metadata → obligation*
Formal consistency	Tests the syntactical and logical consistency of pieces of code or data (section 5.2.1)	Supporting or virtual dataset, property type or data item; **supporting-** and **virtual property type specification**	No restrictions apply	1	table C.31	• **Lineage** → o, namely **source** (cf. table C.10)
Data structure consistency	Assesses the integrity of spatial data structures and the built topology (section 5.2.1)	Supporting or virtual dataset, property type or data item; **supporting-** and **virtual property type specification**	No restrictions apply	1	table C.31	Same as **formal consistency**
Domain consistency	Assesses the adherence of data values to their specified value domains	Supporting or virtual dataset, property type or data item	No restrictions apply	1	table C.31	Same as **formal consistency**

Table C.30: Overview of the subelements of the **consistency** element **formal correctness**.

Formal Correctness

Formal correctness is a formal **consistency** element concerned with the *syntactical correctness* of the data and code. To report **formal correctness**, the quality subelements **formal consistency**, **data structure consistency**, and **domain consistency** are provided (tables C.17, C.30 and C.31).

Quality subelement	*Quality attribute*		
	Measure → m	*Unit* → m	*Associated metadata* → *obligation*
Formal consistency	• Occurrence of syntactical errors, • ...	• Boolean variable, • ...	• **Structural integrity** → o, namely **logical integrity** (cf. table C.29). • **Formal correctness** → o, namely **formal consistency**, **domain consistency**. • **Conformance** → o, namely **completeness** (cf. table C.32).
Data structure consistency	• Degree of adherence of the data structure to prescribed topologic relations, • ...	• Percentage, • ...	Same as the attributes of **formal consistency**
Domain consistency	• Existence, number or percentage of data values which do not adhere to their specified value domains, • ...	• Boolean variable, number, percentage, • ...	Same as the attributes of **formal consistency**

Table C.31: Overview of the quality attributes of the subelements of **formal correctness**.

Conformance

Conformance is concerned with the *workflow integrity* of the digital terrain modelling process. Paying attention to the *process-related quality concept*, **conformance** may be interpreted as a tool to enforce *quality assurance*. To attain this objective, the quality subelements **information persistence**,

Quality subelement	*Characterisation*	*Reporting scope*	*Metric scope*	*Cardinality*	*Quality attributes*	*Associated metadata → obligation*
Information persistence	Assesses whether the body of information and knowledge once introduced persists through the modelling chain and whether all the in formation needed is actually available (section 5.2.1)	No restrictions apply	No restrictions apply	1	table C.33	• **Lineage** → o, namely **source** (cf. table C.10)
Operationalisation consistency	Assesses the conformance of the TI operationalisation with its respective specification (section 5.2.1)	No restrictions apply	No restrictions apply	1	table C.33	Same as **information persistence**
Encoding consistency	Assesses the conformance of digital encoding and implementation with the corresponding specifica tion (section 5.2.1)	Supporting or virtual dataset, property type or data item; **supporting-** and **virtual property type specification**	No restrictions apply	1	table C.33	Same as **information persistence**
Specification consistency	Assesses the conformance of the supplied TI with its specification	Supporting or virtual dataset, property type or data item	No restrictions apply	1	table C.33	Same as **information persistence**

Quality subelement	*Characterisation*	*Reporting scope*	*Metric scope*	*Cardinality*	*Quality attributes*	*Associated metadata → obligation*
Format consistency	Assesses the degree to which data is stored (persistently or virtually) in accordance with the structure of the dataset	Supporting or virtual dataset, property type	No restrictions apply	1	table C.33	Same as **information persistence**
Completeness	Assesses the degree to which all features that have a conceptualisation (incl. their attributes and relationships) are preserved through the modelling chain	No restrictions apply	No restrictions apply	1	table C.33	Same as **information persistence**

Table C.32: Overview of the subelements of the **consistency** element **conformance**.

operationalisation consistency, **encoding consistency**, **specification consistency**, **format consistency**, and **completeness** are suggested (tables C.17, C.32 and C.33).

The element **completeness** is, basically, concerned with errors of *commission* (that is, of excess TI present in the dataset) and *omission* (that is, TI absent in the dataset; see also table C.33).

Quality subelement	*Quality attribute*		
	Measure → m	*Unit* → m	*Associated metadata* → *obligation*
Information persistence	• Degree to which informations persist through a modelling step, • number of information elements lost through a modelling step, • ...	• Percentage, • number, • ...	• **Structural integrity** → o, namely **logical integrity** (cf. table C.29). • **Formal correctness** → o, namely **formal consistency**, **domain consistency** (cf. table C.30). • **Conformance** → o, namely **completeness**.
Operationalisation consistency	• Degree of adherence of operationalised TI to its specification, • ...	• Percentage, • ...	Same as the attributes of **information persistence**
Encoding consistency	• Degree to which property types are encoded in accordance with the corresponding specifications, • ...	• Percentage, • ...	Same as the attributes of **information persistence**

Quality subelement	*Quality attribute*		
	Measure → m	*Unit* → m	*Associated metadata* → *obligation*
Specification consistency	• Degree to which data is stored in accordance with the corresponding specifications, • existence, number or percentage of TI not conforming to its specification, • ...	• Percentage, • boolean variable, number, percentage, • ...	Same as the attributes of **information persistence**
Format consistency	• Existence, number or percentage of data that are not in accordance with the structure of the dataset, • ...	• Boolean variable, number, percentage, • ...	Same as the attributes of **information persistence**
Completeness	• Existence, number or percentage of excess TI items, • existence, number or percentage of missing TI items, • ...	• Boolean variable, number, percentage, • boolean variable, number, percentage, • ...	Same as the attributes of **information persistence**

Table C.33: Overview of the quality attributes of the subelements of **conformance**.

Appendix D

An Introduction to Wavelets

This appendix provides a rough background to the wavelet transform. The first section exposes its concept. At first, the focus will be on qualitative understanding rather than on mathematical details. The more formal section D.2 will complete the theoretical understanding. The chapter concludes by presenting some basic methods for analysing data by wavelets.

For a thorough introduction to wavelets, readers with little mathematical knowledge are referred to Hubbard (1996). On an undergraduate level, Nievergelt (1999), Stollnitz et al. (1996), Triebfürst and Saurer (1999) and Frazier (1999) are a good base to start with. Detailed mathematical discussions are provided by Daubechies (1992), Mallat (1998), Blatter (1998) or Louis et al. (1998). An introduction to the application of wavelets in signal processing applications may be found in Strang and Nguyen (1996), Mallat (1998) or Bethge et al. (1997). To readers interested specifically in the application of wavelets in the geosciences, Klees and Haagmans (2000) or Gerstner et al. (2001) are suggested.

D.1 A Brief Introduction to Wavelets

Many physical processes, signals or other data are in fact 'objects' (such as functions) living on different *scales*. Associated with these scales may be *meshes* of different *sizes* (or resolutions). To mathematically describe such objects, an appropriate representation of them is required. In order to obtain such representations, most applications ask for (Gerstner et al. 2001):

(i) *Scalability*,

(ii) *spatial localisation.*

Put in other words, requirement (i) says that the representation needs to be able to capture features of an object on a coarse mesh as efficiently as on a fine mesh. That is, the description of the object should allow for a *multiscale representation.* Requirement (ii) means that *local* features should only be realised in a 'spatial neighbourhood'. (i) and (ii), together, would allow for an efficient representation of an object.

Basic Idea *Wavelets* are a concept, which, in principle, provides a methodology satisfying both (i) and (ii). The concept is perhaps most conveniently sketched in terms of the multiscale decomposition of a function $f: \mathbb{R} \to \mathbb{R}$. The basic idea is to split f into a sum of functions that are obtained by *scaling* and *shifting* one basis function ψ (see figure D.1). In the notation:

$$\psi_{j,k_j}(x) := 2^{-j/2}\, \psi(2^j x - k_j), \quad j, k_j \in \mathbb{Z}, \tag{D.1}$$

the index j determines the width of ψ_{j,k_j} and thus indicates the *level of resolution* or *scale*[1] while k_j is a *translation parameter* and refers to the *location* of the function (see figure D.1, where k_j is mapped on the abscissa). In these terms, the function f is represented by a *multiresolution decomposition* written in the *basis* given by (D.1) and corresponding expansion *coefficients* $d^j_{k_j}$:

$$f(x) = \sum_{j=-\infty}^{\infty} \sum_{k_j=-\infty}^{\infty} d^j_{k_j}\, \psi_{j,k_j}(x), \tag{D.2}$$

where the first sum is taken over all scales and the second over all locations. As can be seen from equation D.1, all functions ψ_{j,k_j} used for the representation are derived from the same function, the *mother wavelet* ψ. While the outer sum provides a decomposition by functions with varying oscillation speed, the inner sum gives a decomposition with different localisations thus allowing localisation of the variable x in time or space.

The mother wavelet should have properties that entail a meaningful interpretation of the decomposition. A proper choice makes it possible to split the signal recursively into a coarse overall approximation and a rough 'reminder' capturing the details (see figure D.2, and also section D.2.3). So, a decomposition is obtained ranging from very rough to very smooth components.

Compact Support The most important cases are the ones where the localisation effect can be most effective. Mathematically, this means, that ψ has *compact support*, that is, that ψ vanishes outside a fixed interval (see figure D.1). Then, each of the basis functions ψ_{j,k_j} in the decomposition D.2 has a compact support as well which is centred around $2^j k_j$ with width

[1] Resolution decides upon the magnitude of the details displayed and the magnitude of the details 'falling through the meshes' and thus introduces scale.

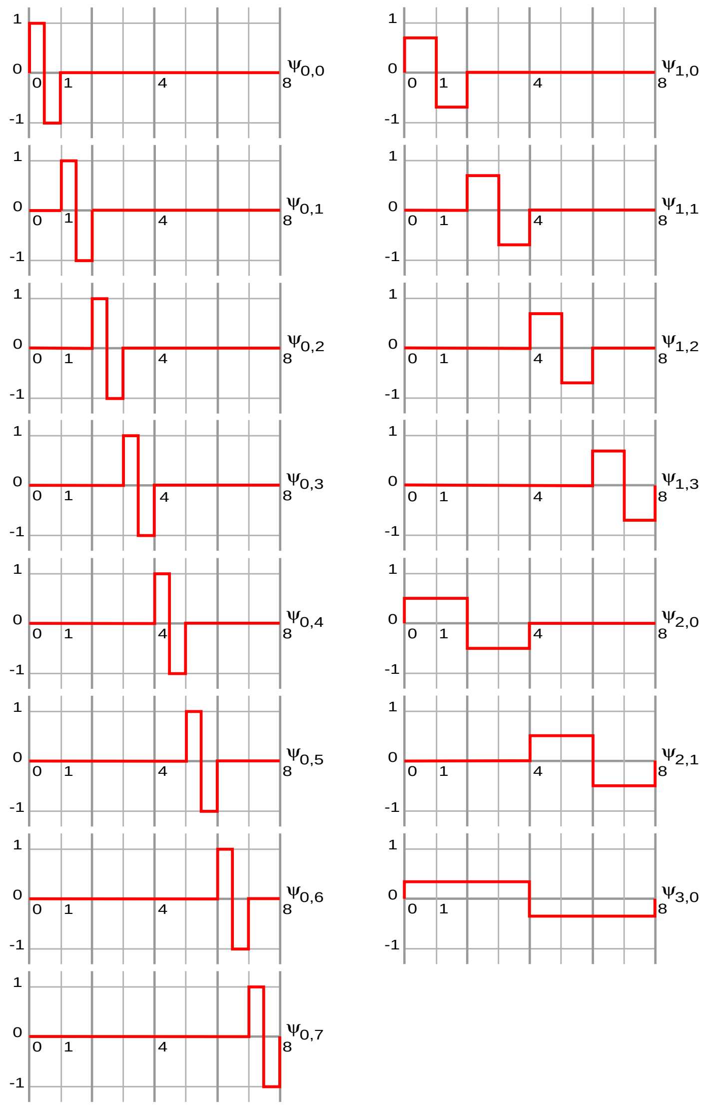


Figure D.1: Haar wavelet bases ψ_{j,k_j}^{Haar} for $j = 0, ..., 3$.

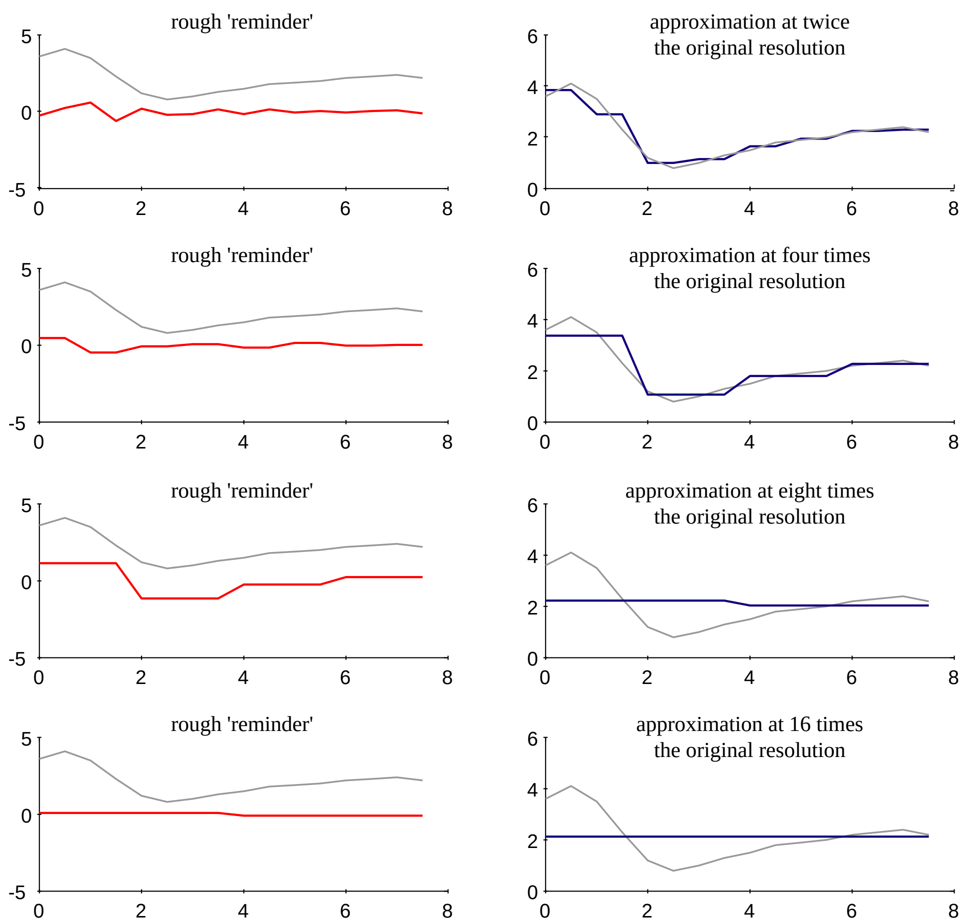


Figure D.2: Wavelet decomposition (using the Haar wavelet). A sequence of decreasing resolution approximations (portrayed in blue) to a function (gray), along with the detail signal (red) capturing the function's rough part.

proportional to 2^j. Hence, for instance, for small values of j the functions ψ_{j,k_j} represent the high-frequency parts of the function f (see figure D.1).

Energy Conservation The 'energy' of a finite energy function $f \in L^2(\mathbb{R})$[2] is defined as: $\|f\|^2_{L^2(\mathbb{R})} := \iint |f(x)|^2 \, dx$. An important property of wavelets that is used in different contexts is that:

$$\|f\|^2_{L^2(\mathbb{R})} \sim \sum_{j=-\infty}^{\infty} \sum_{k_j=-\infty}^{\infty} |d^j_{k_j}|^2 \tag{D.3}$$

holds independently of f. The relation $a \sim b$ says that a can be estimated from above and below by a constant multiple of b. In other words, equation D.3 means that any wavelet coefficient is a measure of the fraction of energy provided by its basis functions. The *stability estimate* also implies that *small disturbances of the function f result in small deviations in the expansion coefficients*, and vice versa.

Uncertainty Principle In the case of the Fourier transform, the *Heisenberg uncertainty principle* imposes a limit to any signal representation: it can not have a precise location and a precise frequency at the same time. Mathematically spoken the product of the uncertainty in space and the uncertainty of the frequency must exceed a minimal value. An analogue *uncertainty principle* also applies to the wavelet transform. It expresses the trade-off between localisation in time/space and frequency/scale. For an exact mathematical discussion of the issue please be referred to Dahlke and Maass (1995) or Murenzi and Antoine (1997).

D.2 More on the Theory of Wavelets

D.2.1 The Continuous Wavelet Transform

The Basic Formalism To describe the *continuous wavelet transform* (CWT) consider, for a parameter $a \neq 0, \; a \in \mathbb{R}$, the family of functions $\{\psi_a \mid a \neq 0, \; a \in \mathbb{R}\}$, which are generated by *dilation* of ψ as:

$$\psi_a(x) := \frac{1}{\sqrt{|a|}} \, \psi(\frac{x}{a}).$$

For $|a| > 1$, the function is expanded, for $|a| < 1$ it is 'compressed'. Hence, a can be viewed as a *scale*. Additionally, a second parameter b is introduced denoting the *location* by which ψ_a can be translated. Thus,

[2] L^2 denotes the space of square Lebesgue integrable functions, or energy space.

considered is the family of functions $\{\psi_{a,b} \mid a \neq 0,\ a, b \in \mathbb{R}\}$, where:

$$\psi_{a,b}(x) := \frac{1}{\sqrt{|a|}}\, \psi(\frac{x-b}{a}). \tag{D.4}$$

The factor $|a|^{-1/2}$ assures that all the functions making up the family (D.4) have the same norm. The CWT of a function f is an *integral transformation* of the form:

$$\mathcal{W}_\psi f(a,b) := \int_{\mathbb{R}} f(x)\psi_{a,b}(x)\, dx, \tag{D.5}$$

in terms of the *scale* a and *location* b. For $f \in L^2$ and appropriately chosen ψ, the CWT $\mathcal{W}_\psi f$ is *invertible*. The *inverse continuous wavelet transform* (ICWT) is given by:

$$f(x) = \frac{1}{c_\psi} \int_{\mathbb{R}} \int_{\mathbb{R}} \mathcal{W}_\psi f(a,b)\, \psi_{a,b}(x) \frac{da\, db}{a^2}\ , \tag{D.6}$$

where $c_\psi := 2\pi \int_{\mathbb{R}} \frac{|\hat{\psi}(\xi)|^2}{|\xi|}\, d\xi$ and $\hat{\psi}(\cdot)$ denotes the Fourier transform of $\psi(\cdot)$.

The terminology *continuous* means, that a and b are real numbers. Applying the CWT to a function f, different information on f is obtained for each pair of parameters a and b; namely, the information on the scale being represented by a and a set of values describing the function at different locations b at this scale. By the wavelet transform (D.5), the function f is *analysed*, whereas by the inverse wavelet transform (D.6), the function is *synthesised* from its wavelet transform.

The correspondence $f(x) \rightarrow \mathcal{W}_\psi f(a,b)$ represents a function of one variable by a function of two variables, into which a lot of correlations are built in. Basically, it is this *redundancy* of the representation which is exploited by the wavelet applications discussed in section D.3.

The Requirements on ψ Different choices of the mother wavelet ψ may well influence the outcome of an analysis. For the formalism to make sense, the analysing wavelet ψ needs to satisfy a few conditions:

(i) ψ should be square integrable (that is, $\psi \in L^2(\mathbb{R})$.

(ii) ψ must be *admissible*, that is, the integral

$$c_\psi := 2\pi \int_{\mathbb{R}} \frac{|\hat{\psi}(\xi)|^2}{|\xi|}\, d\xi < \infty$$

must converge. This condition implies (and, for ψ regular enough, is equivalent to) the so-called *zero mean* condition:

$$\int_{\mathbb{R}} \psi(x)\, dx = 0\, . \tag{D.7}$$

The zero mean condition implies that ψ must be oscillating.

(iii) In addition to (ii), ψ *may* be required to have a certain number of *vanishing moments*, that is:

$$\int_{\mathbb{R}} x^n \psi(x)\, dx = 0, \quad n = 0, 1, ..., N. \tag{D.8}$$

This property improves the efficiency of ψ at detecting singularities in the signal.

It may be pointed out that conditions (i) and (ii) are essential requirements for the CWT, while (iii) makes life easier but is not compulsory.

The CWT in two dimensions In two (and more) dimensions, additionally to dilations and translations *rotations* need to be introduced. Thus, considered is the family of functions $\{\psi_{a,\theta,\boldsymbol{b}} \mid a \in \mathbb{R}\backslash 0,\ \theta \in [0, 2\pi),\ \boldsymbol{b} \in \mathbb{R}^2\}$, where:

$$\psi_{a,\theta,\boldsymbol{b}}(\boldsymbol{x}) = \frac{1}{a}\,\psi(R_\theta^{-1}(\frac{\boldsymbol{x}-\boldsymbol{b}}{a})), \tag{D.9}$$

and R_θ is the matrix

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$

The CWT, then, becomes:

$$\mathcal{W}_\psi f(a, \theta, \boldsymbol{b}) := \int_{\mathbb{R}^2} f(\boldsymbol{x})\, \psi_{a,\theta,\boldsymbol{b}}(\boldsymbol{x})\, d\boldsymbol{x}, \tag{D.10}$$

and the corresponding *resolution of the identity* is:

$$f(\boldsymbol{x}) = \frac{1}{c_\psi} \int_{\mathbb{R}} \frac{da}{|a|^3} \int_{\mathbb{R}^2} d\boldsymbol{b} \int_0^{2\pi} d\theta\, \mathcal{W}_\psi f(a, \theta, \boldsymbol{b})\, \psi_{a,\theta,\boldsymbol{b}}(\boldsymbol{x}). \tag{D.11}$$

D.2.2 The Discrete Wavelet Transform

Discretising the CWT In the continuous case, considered was the family of functions (D.4). Discretising the CWT, then, means to restrict a and b to discrete values only. The discretisation of the *dilation parameter* seems natural: choose $a = a_0^j$, where $j \in \mathbb{Z}$, and the *dilation step* $a_0 \neq 1$ is fixed. For convenience, assume $a_0 > 1$ (although it does not matter as long as negative as well as positive powers of j are admitted). As already illustrated in figure D.1, different values of j correspond to wavelets of different widths.

It follows that the discretisation of the *translation parameter* b should depend on j: narrow (high frequency) wavelets are translated by small steps in order to cover the whole time or space range, while wider (lower frequency) wavelets are translated by larger steps. For $j = 0$, it seems natural to discretise b by taking only the integer (both positive and negative) multiples of one fixed b_0, where b_0 is appropriately chosen so that the $\psi(x - kb_0)$ 'cover' the whole time or space range. Again, for convenience, assume $b_0 > 0$. For different values of j, the width of $a_0^{-j/2}\psi(a_0^{-j}x)$ is a_0^j times the width of $\psi(x)$, so that the choice $b = kb_0a_0^j$ will ensure that the discretised wavelets at level j 'cover' the range in the same way that the $\psi(x - kb_0)$ do.

To sum up, convenient choices are $a = a_0^j$ and $b = kb_0a_0^j$, where j, k range over $\mathbb{Z}$, and $a_0 > 1,\ b_0 > 0$ are fixed (the appropriate choices for a_0 and b_0 depend on the wavelet ψ). The corresponding, discretely labelled wavelets are, then:

$$\begin{aligned} \psi_{j,k}(x) &:= a_0^{-j/2}\,\psi(\frac{x - kb_0a_0^j}{a_0^j}) \\ &= a_0^{-j/2}\,\psi(a_0^{-j}x - kb_0). \end{aligned} \tag{D.12}$$

For a given function f, this results in the *discrete wavelet transform* (DWT):

$$\begin{aligned} (\mathcal{W}_\psi f)_{j,k} &:= \int_{\mathbb{R}} f(x)\,\psi_{j,k}(x)\,dx \\ &= a_0^{-j/2}\int_{\mathbb{R}} f(x)\,\psi(a_0^{-j}x - kb_0)\,dx. \end{aligned} \tag{D.13}$$

Frames In the discrete case, there does not exist, in general, an inverse transform analogous to formula (D.6) for the continuous case. Reconstruction of f from $(\mathcal{W}_\psi f)$, if at all possible, must therefore be done by some other means. Questions arise, such as:

- Is it possible to completely characterise f by knowing $(\mathcal{W}_\psi f)$?

- Is it possible to reconstruct f in a numerically stable way from $(\mathcal{W}_\psi f)$?

The answer is that, yes it is, if the $\{\psi_{j,k} \mid j, k \in \mathbb{Z}\}$ constitute a so-called *frame*. However, an introduction to the concept of frames is beyond the scope of this review. For an excellent discussion of the topic, please be referred to Heil and Walnut (1989), Daubechies (1992), Blatter (1998), or Louis et al. (1998).

As in the continuous case, the DWT often provides a rather *redundant* description of the original function. This redundancy can be exploited, or eliminated to reduce the transform to its bare essentials, as discussed in the next section.

D.2.3 Orthonormal Wavelet Bases and Multiresolution Analysis

Orthonormal Wavelet Bases

For some very special choices of ψ and a_0, b_0, the $\psi_{j,k}$ constitute an *orthonormal basis* for $L^2(\mathbb{R})$. In particular, for the choice $a_0 = 2,\ b_0 = 1$, there exist wavelets ψ such that the

$$\psi_{j,k}(x) := 2^{-j/2}\psi(2^{-j}x - k) \tag{D.14}$$

constitute an orthonormal basis for $L^2(\mathbb{R})$. The simplest and oldest example of a function ψ for which the dilations and translations $\psi_{j,k}$ defined by (D.14) constitute an orthonormal basis for $L^2(\mathbb{R})$ is the Haar function (illustrated in figure D.1):

$$\psi(x) = \begin{cases} 1, & \text{if } 0 \leq x < \frac{1}{2}, \\ -1, & \text{if } \frac{1}{2} \leq x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Multiresolution Analysis

The Basic Idea The computational costs of a DWT are considerable. Therefore, it is desirable to reduce the number of computations. This can be done with a *multiresolution analysis* (MRA). Before giving an exact mathematical formulation of a MRA, a qualitative motivation is provided.

To show how a MRA works, think of a function f_k in a subspace V_k of $L^2(\mathbb{R})$. The idea, then, is to try to split up f_k into a high frequency and a low frequency component. The smooth (low frequency) components may be described by an *orthogonal projection* $P_{k+1}f_k$ of f_k onto a smaller space V_{k+1} containing the 'smooth functions' of V_k. By projecting f_k onto V_{k+1} one gets $P_{k+1}f_k$ as a blurred approximation of f_k. To answer the question of how the detailed (high frequency) information that is lost during the projection can be retained, let W_{k+1} denote a space complementing V_{k+1} in V_k:

$$V_k = V_{k+1} \oplus W_{k+1}.$$

Each function in V_k can be expressed in a unique way as the sum of functions in V_{k+1} and W_{k+1}. Let $Q_{k+1}f_k$ be the projection of f_k onto W_{k+1}. Then, the projection $Q_{k+1}f_k$ describes the detailed (high frequency) information that is lost when projecting f_k onto V_{k+1}, that is:

$$f_k = P_{k+1}f_k + Q_{k+1}f_k.$$

The procedure can then be continued with $P_{k+1}f_k$ in an analogous way by writing V_{k+1} as a direct sum of the spaces V_{k+2} (containing the 'smooth

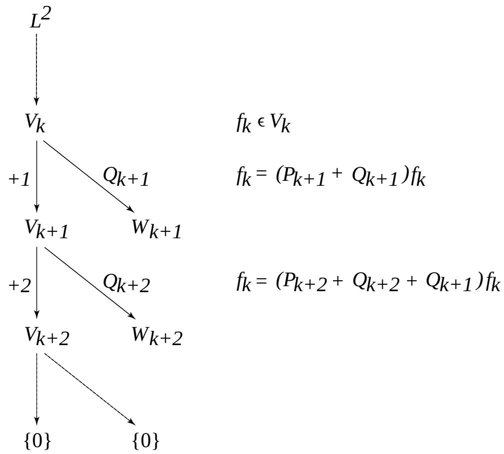


Figure D.3: Multiresolution decomposition of $L^2(\mathbb{R})$.

elements' of V_{k+1}) and W_{k+2} (complementing V_{k+2} in V_{k+1}). The corresponding projectors are P_{k+2} and Q_{k+2}. This leads to:

$$P_{k+1}f_k = P_{k+2}f_k + Q_{k+2}f_k,$$

and

$$f_k = P_{k+2}f_k + Q_{k+2}f_k + Q_{k+1}f_k, \tag{D.15}$$

respectively. Repeating this process recursively on the approximations gives the full decomposition, as indicated in figure D.3 (an example of a multiresolution decomposition is depicted in figure D.2, where the right column shows a sequence of approximations (blue) of decreasing frequency to an original function (gray). The right column captures the corresponding detailed (high frequency) information lost in the projection step).

In (D.15), $P_{k+2}f_k$ represents a 'smooth' (low(er) frequency) approximation of f_k, containing details about f_k starting from a specific magnitude (cf. section D.3.2). $Q_{k+2}f_k$ and $Q_{k+1}f_k$, respectively, contain the components of f_k to specific frequency bands, where $Q_{k+1}f_k$ corresponds to a higher frequency than $Q_{k+2}f_k$. Equation (D.15) may be understood as decomposition of the function f_k in *frequency bands* of high frequency and a *'mix' of low frequencies*. A MRA is an exact mathematical formulation of such a decomposition process.

The Exact Definition A sequence $\{V_j\}_{j\in\mathbb{Z}}$ of closed subspaces of $L^2(\mathbb{R})$ is a *multiresolution analysis* of $L^2(\mathbb{R})$ if the following conditions are satisfied

(Mallat 1998):

$$
\begin{aligned}
... \subset V_2 \subset V_1 \subset V_0 \subset V_{-1} \subset V_{-2} \subset ... \quad & \text{(nested subspaces)}, && \text{(D.16)}\\
f \in V_j \Longleftrightarrow f(2^j \cdot) \in V_0 \quad & \text{(dilation)}, && \text{(D.17)}\\
f \in V_0 \Longleftrightarrow f(\cdot - k) \in V_0,\ \forall k \in \mathbb{Z} \quad & \text{(translation)}, && \text{(D.18)}\\
\bigcap_{j \in \mathbb{Z}} V_j = \{0\}, \quad & && \text{(D.19)}\\
\overline{\bigcup_{j \in \mathbb{Z}} V_j} = L^2(\mathbb{R}), \quad & && \text{(D.20)}\\
\exists \phi \in V_0 \text{ so that the } \phi_{0,k}(x) := \phi(x-k) \quad & && \\
\text{constitute an } \textit{orthonormal basis} \text{ for } V_0 \quad & \text{(scaling function)}. && \text{(D.21)}
\end{aligned}
$$

Condition (D.19) means that when dilating f sufficiently, at infinite resolution, there is no detail information left. Condition (D.20) implies that any original function in $L^2(\mathbb{R})$ can be reconstructed exactly.

The Correspondence between MRA and Orthonormal Wavelet Bases The link between wavelet theory and MRA is that for each MRA there exists a wavelet ψ whose scaled and translated versions

$$\psi_{j,k}(x) := 2^{-j/2}\, \psi(2^{-j}x - k)$$

provide an *orthonormal basis* for the spaces W_j, for all $j \in \mathbb{Z}$ (cf. equation (D.14)). It turns out that there are many examples of such MRA's, corresponding to many examples of orthonormal wavelet bases. There exists an explicit recipe for the construction of ψ: since $\phi \in V_0 \subset V_{-1}$, and the $\phi_{-1,k}(x) = \sqrt{2}\,\phi(2x-k)$ constitute an orthonormal basis for V_{-1} (by (D.17) and (D.21) above), there exist $\alpha_k = \sqrt{2}\,\langle \phi, \phi_{-1,k}\rangle$ so that $\phi(x) = \sum_k \alpha_k\, \phi(2x-k)$. It then suffices to take $\psi(x) = \sum_k (-1)^k \alpha_{1-k}\, \phi(2x-k)$.

Two-dimensional Orthonormal Wavelet Bases and MRA's

Most two-dimensional formulations of MRA comprise a *tensor product extension* of their one-dimensional counterpart to derive the respective bases:

$$\Phi := \phi \otimes \phi, \quad \Psi^h := \phi \otimes \psi, \quad \Psi^v := \psi \otimes \phi, \quad \Psi^d := \psi \otimes \psi. \qquad \text{(D.22)}$$

For each level of resolution, a smooth(er) *approximation* to the original function and three *detail signals* are generated. Ψ^h captures horizontal changes in the function, Ψ^v vertical details and Ψ^d diagonal details, respectively.

For a two-dimensional wavelet transform two *translation parameters* k_1 and k_2 are required to identify the wavelet coefficient at location $(k_1, k_2) =:$

$\boldsymbol{k}$. For each level of scale j a matrix of horizontal, vertical and diagonal wavelet coefficients is computed, equivalent to equation (D.13) for the one-dimensional case:

$$\begin{aligned} d_{\boldsymbol{k}}^{j,h} := (\mathcal{W}_{\Psi^h} f)_{j,\boldsymbol{k}}^h &= \int_{\mathbb{R}^2} f(\boldsymbol{x}) \Psi_{j,\boldsymbol{k}}^h(\boldsymbol{x})\, d\boldsymbol{x}, \\ d_{\boldsymbol{k}}^{j,v} := (\mathcal{W}_{\Psi^v} f)_{j,\boldsymbol{k}}^v &= \int_{\mathbb{R}^2} f(\boldsymbol{x}) \Psi_{j,\boldsymbol{k}}^v(\boldsymbol{x})\, d\boldsymbol{x}, \\ d_{\boldsymbol{k}}^{j,d} := (\mathcal{W}_{\Psi^d} f)_{j,\boldsymbol{k}}^d &= \int_{\mathbb{R}^2} f(\boldsymbol{x}) \Psi_{j,\boldsymbol{k}}^d(\boldsymbol{x})\, d\boldsymbol{x}. \end{aligned} \tag{D.23}$$

Any finite energy function $f(\boldsymbol{x}) \in L^2(\mathbb{R}^2)$ can be approximated by the bases (D.22):

$$f(\boldsymbol{x}) = \sum_{k_1} \sum_{k_2} \left(d_{\boldsymbol{k}}^J \, \Phi_{J,\boldsymbol{k}} + \sum_{j=1}^{J} \left(d_{\boldsymbol{k}}^{j,h} \, \Psi_{j,\boldsymbol{k}}^h + d_{\boldsymbol{k}}^{j,v} \, \Psi_{j,\boldsymbol{k}}^v + d_{\boldsymbol{k}}^{j,d} \, \Psi_{j,\boldsymbol{k}}^d \right) \right), \tag{D.24}$$

where $\boldsymbol{k} := (k_1, k_2)$, J is the level of scale of the coarsest approximation, and the $d_{\boldsymbol{k}}^{\cdot}$ denote the respective coefficients resulting from the wavelet transform.

D.3 Methods for Analysing Data by Wavelets

In this section, discussed are some important properties of wavelets that are commonly exploited for analysing data by wavelets. The objective is to expose the basic ideas motivating the techniques used in chapter 8. The discussion is restricted to the one-dimensional case. Extension to two (or more) dimensions can be accomplished following the guidelines indicated in sections D.2.1 and D.2.3.

D.3.1 The Significance of the Wavelet Coefficients

The wavelet transform is the sum over the entire support of a function f multiplied by scaled and shifted versions of a wavelet ψ. This process produces *wavelet coefficients* d_k^j that are a function of scale j and location k. The coefficients d_k^j may be interpreted as a measure of how closely *correlated* the wavelet is with a respective section of the function f. The higher the modulus of d_k^j, the more the similarity (see figure D.4). Note however, that the results depend on the shape of the wavelet chosen. In other words, the wavelet coefficients constitute the results of a *regression* of the original function f performed on the wavelets.

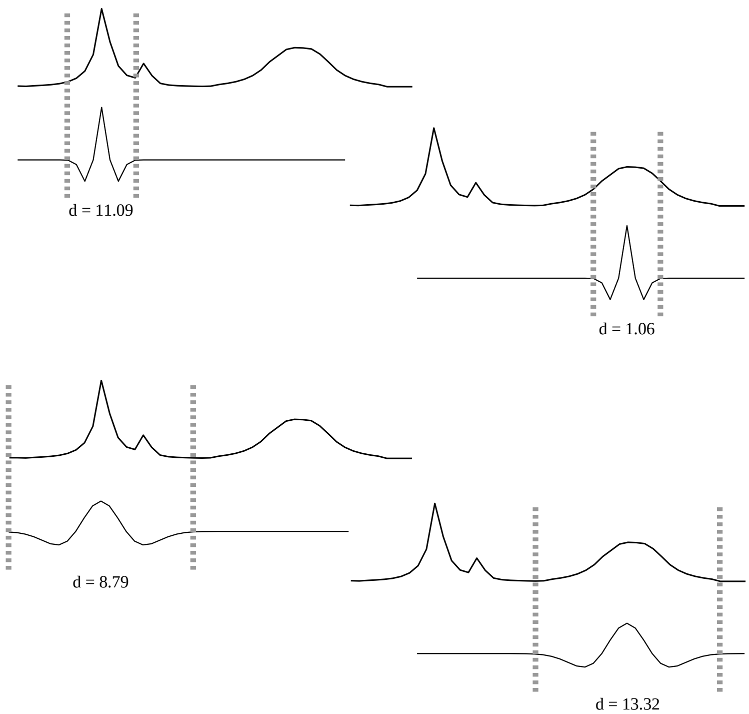


Figure D.4: Wavelet coefficients d computed for two levels of scale at two different locations.

D.3.2 Principles of Filtering in the Wavelet Domain

Filter Properties of the Wavelet Transform

Filters are deployed in many signal processing applications, for instance, to split up signals into high frequency and low frequency components, to emphasise specific frequency bands, or to attenuate the influence of data errors or noise. Important to such applications is the interpretation of Shannon's sampling theorem, which provides a relation between the magnitude of a detail of a function or signal f and the signal's Fourier transform. In other words, the sampling theorem provides a description of the splitting up of a function f into its frequency components. The most common filters are so-called *(linear) convolution filters* φ. Computed, then, is $f_\varphi := f * \varphi$.

Preliminary Considerations The material in this and the next paragraph is, basically, taken from Louis et al. (1998).

Assume, a function f is built from details of magnitude greater than L, that is, f is a linear combination of characteristic functions[3] χ_I:

$$f(x) := \sum_{j=1}^{N} \alpha_j \chi_{I_j}(x),$$

whose support satisfies $|\chi_{I_j}| > L,\ (j = 1, ..., N)$. The modulus of the Fourier transform of χ_I is:

$$|\hat{\chi}_I(\xi)| = \frac{1}{\sqrt{2\pi}}\,|\exp(-i(c+d)\xi/2)\,2\frac{\sin(\xi\frac{|I|}{2})}{\xi}| = \frac{|I|}{\sqrt{2\pi}}\,|\mathrm{sinc}(|I|\frac{\xi}{2})|,$$

where $I = [c, d],\ |I| := d - c \geq L$ and

$$\mathrm{sinc}(x) := \begin{cases} \frac{\sin(x)}{x}, & x \neq 0, \\ 1, & x = 0. \end{cases}$$

Figure D.5 depicts the graph of $\hat{\chi}_{[-1,1]}(\xi) = \mathrm{sinc}(\xi)$. To simplify things, consider the support of $\hat{\chi}_I(\cdot)$ as being essentially limited to the interval $[\frac{-2\pi}{|I|}, \frac{2\pi}{|I|}]$. This means that details of f of a magnitude $|I| \geq L$ are made out of frequencies $|\xi| \leq \frac{2\pi}{L}$. Formulated more trenchant, *corresponding to a detail of f of magnitude L is the frequency $\frac{2\pi}{L}$.*

The inverse interpretation, then, is: if $|\hat{f}(\xi_0)| \gg 0$, the function f is expected to feature details of magnitude $\frac{2\pi}{\xi_0}$.

[3] *Characteristic functions* are defined as:

$$\chi_I(x) := \begin{cases} 1, & \text{if } x \in I, \\ 0, & \text{otherwise.} \end{cases}$$

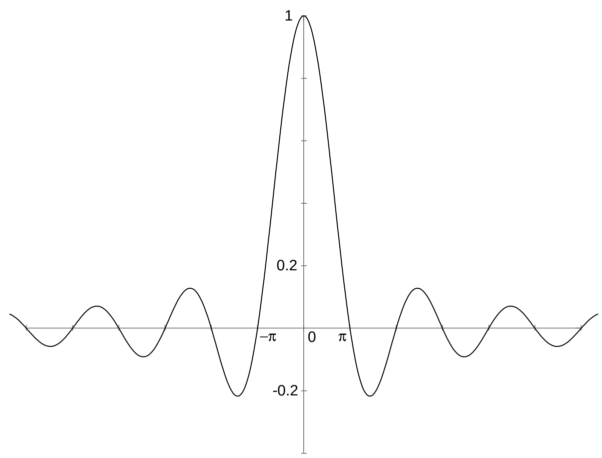


Figure D.5: The sinc function is the Fourier transform of the characteristic function $\chi_{[-1,1]}$.

Link to the Wavelet Transform Based on the above considerations, this paragraph investigates to what extent the wavelet transform may be interpreted as a filter. To this end, it is important to note, that, for each *fixed value of* a, the wavelet transform $\mathcal{W}_\psi f(a,b)$ may be interpreted as a *convolution*[4] of f with the scaled wavelet $\psi(\frac{\cdot}{a})$:

$$\mathcal{W}_\psi f(a,b) = \frac{1}{\sqrt{|a|}}\,(f * \psi(\frac{\cdot}{-a}))(b).$$

For a *fixed* value of a, the wavelet transform of f, therefore, corresponds to a *filtering* of f with $\psi(\frac{\cdot}{a})$. From the requirements on ψ discussed in section D.2.1, it follows that $\hat{\psi}(0) = 0$ (from (D.7)) and that $\lim_{\xi\to\infty} \hat{\psi}(\xi) = 0$ (from $\psi \in L^2(\mathbb{R})$). ψ, thus, is a *band filter*[5].

Using the inverse Fourier transform, $\mathcal{W}_\psi f(a,b)$ can be written as:

$$\mathcal{W}_\psi f(a,b) = \sqrt{|a|}\int_{\mathbb{R}} \hat{\psi}(a\xi)\hat{f}(\xi)^*\, e^{-ib\xi}\, d\xi, \tag{D.25}$$

where $\hat{f}(\cdot)^*$ denotes the complex conjugate of $\hat{f}(\cdot)$.

[4]For two functions f and g, the *convolution* $f * g$ is defined as:

$$f * g(x) := \int_{\mathbb{R}} f(x-t)g(t)\,dt, \quad x \in \mathbb{R}.$$

[5]*Band filters* are filters φ whose behaviour in the frequency domain is of the form: $\hat{\varphi} \sim \chi_{B_1 \leq |\xi| \leq B_2}$, that is, band filters analyse the frequency band in between B_1 and B_2.

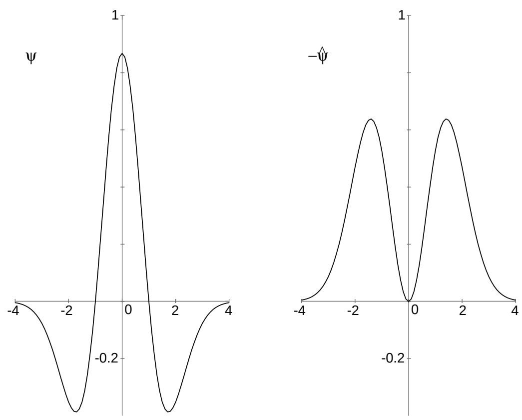


Figure D.6: The Mexican hat wavelet ψ_{Mh} (left) and its Fourier transform (right).

Consider, for instance, the one-dimensional counterpart of the *Mexican hat* wavelet used in chapter 8:

$$\psi_{Mh}(x) := \frac{2}{\sqrt{3}}\,\pi^{-1/4}(1-x^2)\,e^{-x^2/2}.$$

Its Fourier transform is $\hat{\psi}_{Mh}(\xi) = -\frac{2}{\sqrt{3}}\,\pi^{-1/4}\,\xi^2\,e^{-\xi^2/2}$ (see figure D.6). $|\hat{\psi}_{Mh}(\xi)|$ has global maxima at $\xi_0 = \pm\sqrt{2}$ with $|\hat{\psi}(\sqrt{2})| = \frac{4}{\sqrt{3}\,\pi^{1/4}\,e}$.

With $\hat{\psi}_{Mh}(\xi)$ being concentrated around ξ_0, $\hat{\psi}_{Mh}(a\xi_0)$ is centred at $\frac{\xi_0}{a}$. This means that, for *fixed* values of a, $\mathcal{W}_{\psi_{Mh}}f(a,\cdot)$ is determined by the frequencies of f around $\frac{\xi_0}{a}$. Or, looking in the inverse direction, $\mathcal{W}_{\psi_{Mh}}f(a,\cdot)$ contains only information about frequency components of f around $\frac{\xi_0}{a}$. By associating different frequencies with signal details of varying magnitudes (as discussed in the last paragraph), this means that, for *fixed* values of a, $\mathcal{W}_{\psi_{Mh}}f(a,\cdot)$ captures only information about details of f of magnitude $\frac{2\pi a}{\xi_0}$.

Filtering Principles

Filtering in the wavelet domain is performed according to the same principles as filtering in the frequency domain: the signal (or function) is mapped to the wavelet domain, where it is altered in a 'convenient way' and subsequently re-transformed to the original domain. In the wavelet domain, there are several possibilities to realise such a procedure (see, for instance, Strang and Nguyen (1996) or Mallat (1998)). However, the procedure is always based on the fact that the wavelet transform maps high frequency signal components to fine scales while low frequency signal components are mapped to coarser scales (Bethge et al. (1997); see also the last paragraph).

To exemplify this effect, look, for instance, at the wavelet transform of the signal $f(x) := A\sin(\xi_0 x)$, computed with the one-dimensional *Mexican Hat* wavelet $\psi(x) := \frac{2}{\sqrt{3}}\pi^{-1/4}(1-x^2)e^{-x^2/2}$. The corresponding Fourier transforms are:

$$\hat{f}(\xi) = -i\sqrt{\frac{\pi}{2}}A\,[\delta(\xi-\xi_0)-\delta(\xi+\xi_0)],$$

where $\delta(\cdot)$ denotes the Dirac function, and

$$\hat{\psi}(\xi) = \frac{-2}{\sqrt{3}}\,\pi^{-1/4}\,\xi^2 e^{-\xi^2/2},$$

respectively.

Using equation (D.25), it follows:

$$\mathcal{W}_\psi f(a,b) = |a|^{3/2}\,\pi^{1/4}\sqrt{\frac{8}{3}}A\,\xi_0^2\,e^{-a^2\xi_0^2/2}\sin(\xi_0 b) =: A_{\mathcal{W}}\cdot f(b), \qquad \text{(D.26)}$$

where $A_{\mathcal{W}} := |a|^{3/2}\,\pi^{1/4}\sqrt{\frac{8}{3}}\,\xi_0^2\,e^{-a^2\xi_0^2/2}$.

Equation (D.26) says that while $\mathcal{W}_\psi f(a,b)$ has the same period as $f(x)$, its amplitude increases to reach a maximum at the 'point'[6]:

$$(a_{max}(\xi_0), A_{\mathcal{W}}(a_{max},\xi_0)) = (\sqrt{\frac{3}{2}}\,\xi_0^{-1},\ (24\pi)^{1/4}\sqrt{\xi_0}\,e^{-3/4}),$$

and then decreases again. In other words, the wavelet transform maps high frequency oscillations to oscillations of the same frequency, whose amplitudes are concentrated at fine scales, while lower frequency oscillations are mapped to amplitudes centred at coarser scales (figure D.7). The exact amplitude value (at a scale a) depends on both, the amplitude A and the oscillation frequency ξ_0 of the original signal f, as indicated by (D.26) and $A_{\mathcal{W}}(a,\xi_0)$ (see figure D.7).

These relations may be exploited in varying ways to split up high frequency from low frequency signal components

- by neglecting information provided by the wavelet transform that refers to high frequency signal oscillations; or
- by suppressing wavelet coefficients which do not exceed a given threshold value.

[6]The amplitude as well as the 'point' at which the maximum is reached, depend, of course, on the properties of the wavelet chosen.

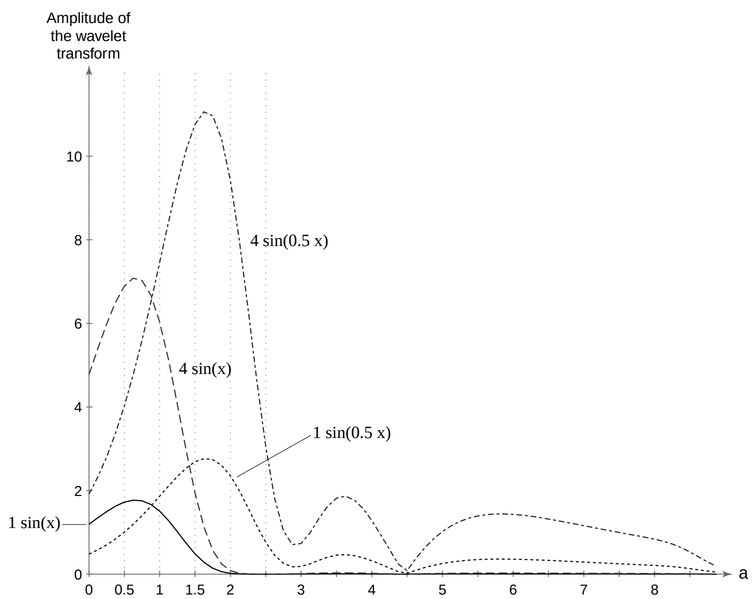


Figure D.7: Scale-dependent amplitudes of four sinusoidal oscillations in the wavelet domain.

D.3.3 Vanishing Moments and Multiscale Differential Operators

To measure the *local regularity* of a function or signal, it is not important to use a wavelet with a narrow frequency support, but *vanishing moments* are crucial. The next paragraph is based on Gerstner et al. (2001), while the considerations referring to *multiscale differential operators* stem from Mallat (1998).

Vanishing Moments A wavelet ψ is said to have N *vanishing moments*, if it satisfies:

$$\int_{\mathbb{R}} x^n \psi(x)\, dx = 0, \quad \text{for } 0 \le n < N. \tag{D.27}$$

Condition (D.27) implies that ψ (and also its scaled and shifted versions $\psi_{a,b}$) are *orthogonal* to polynomials of degree $N-1$. This, in turn, implies that for a sufficiently smooth function f, which is expanded in its Taylor series, the CWT only depends on the N-th reminder of the Taylor series. $\mathcal{W}_\psi f(a,b)$ gives a measure of the oscillatory behaviour of the function f in a neighbourhood of b whose size depends on a. Thus, for $a \to 0$, one has (up to a constant factor) $\mathcal{W}_\psi f(a,b) \to f^{(N)}(b)$.

Multiscale Differential Operator As proofed in Mallat (1998), a wavelet with N vanishing moments can be written as the N-th order derivative of a function θ. The resulting wavelet transform, then, is a *multiscale differential operator*. For the sake of completeness, Mallat's main result is portrayed here, however without proof.

Theorem D.1 (Mallat) *A wavelet ψ with a fast decay*[7] *has N vanishing moments if and only if there exists θ with a fast decay such that:*

$$\psi(x) = (-1)^N \frac{d^N \theta(x)}{dx^N}\,.$$

As a consequence,

$$\mathcal{W}_\psi f(a,b) = a^N \frac{d^N}{db^N}\,(f * \theta(\frac{\cdot}{-a}))(b). \tag{D.28}$$

Moreover, ψ has no more than N vanishing moments, if and only if

$$\int_{\mathbb{R}} \theta(x)\, dx \neq 0.$$

[7] A wavelet ψ is said to have *fast decay* if, for any decay exponent $m \in \mathbb{N}$, there exists a C_m such that: $\forall x \in \mathbb{R},\ \ |\psi(x)| \le \frac{C_m}{1+|x|^m}$.

If $K := \int_{\mathbb{R}} \theta(x)\, dx \neq 0$, then, the convolution $\frac{1}{\sqrt{|a|}}(f * \psi(\frac{\cdot}{-a}))(b)$ can be interpreted as a *weighted average* of f with a kernel dilated by a. So, equation (D.28) proves that $\mathcal{W}_\psi f(a,b)$ is a N*-th order derivative* of an averaging of f over a domain proportional to a.

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Education

1978 – 1984	Primary school in Zürich.
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1991 – 1997	Studies in geography at the University of Zurich. Minors in forest engineering, chemistry, geology and mathematics.
1997	Diploma in geography with a thesis on "Extraktion hydrologischer Strukturen aus triangulierten Geländemodellen" ("Extraction of Hydrological Networks from Triangulated Digital Terrain Models") advised by Prof. R. Weibel and dipl. phys. Martin Heller.
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1997 – 2001	Research assistant at the Department of Geography, University of Zurich, funded by the Swiss National Science Foundation under contract No. 20-47153.96. Dissertation with a thesis on "Models and Experiments for Quality Handling in Digital Terrain Modelling" advised by Prof. R. Weibel and Dr. B. Schneider.