Snakes: a technique for line smoothing and displacement in map generalisation

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Abstract: Line smoothing in map generalisation is commonly seen as a comparatively simple operation. In this paper, however, the snakes model will be examined as a basic technique with respect to combined operations of displacement and smoothing. For line smoothing, two different snake models and their parametrisation with curvature controlled and constant parameters are examined. Further the snakes model for smoothing will be combined with the already known displacement model. Two combined methods will be proposed, the alternation of algorithms and a linear combination of the snakes equation systems, of which the former shows more flexibility. Selected test examples are illustrated and discussed, followed by an assessment of the models.

1 Introduction

Active contours or energy minimizing splines - so called *snakes* - are used as filtering and smoothing functions in signal processing and computer visualisation. Hence it is not a big step from use in graphic modelling to generalisation of line and polygonal objects in maps. So far active splines algorithms in automated map generalisation have been developed for feature displacement by Burghardt and Meier [1997], Burghardt [2001] and Bader [2001]. Bearing in mind the snakes application in the graphics domain, however, it is obvious to develop algorithms for line smoothing as well and further to combine the operations smoothing and displacement. The analysis of ”well” generalised topographic maps provides evidence that in local conflicts booth operations are performed.

Line smoothing is a well known theme in automated generalisation. An overview is given, for instance, by Weibel [1997]. First research on line smoothing using snakes has been carried out by Burghardt [2002] based on the conventional snakes model introduced by Kass et al. [1987]. He segments a line using the ”curvness” (behavior of curvature) and smooths the elements separately with constant parameters.
Although the results are satisfactory from a (carto-)graphic point of view, modifications or alternative approaches for a snakes smoothing algorithm are possible. We intended to improve the existing snakes model by examining the following aspects:

- Comparison of the conventional snakes model by Kass et al. [1987] and *Tangent Angle FUnction Snakes* (TAFUS) - a model developed specifically for cartographical purposes (Borkowski et al. [1999]).
- Comparison of smoothing with constant and locally driven parameters (e.g. by curvature) and estimation of initial values.
- Smoothing with and without line segmentation to preserve constraint points.

- Conception and testing of combined algorithms for line smoothing and displacement.

A detailed evaluation of the proposed modifications on cartographic quality, speed and ease of use concerning parametrisation has been done by Steiniger [2003]. The algorithms has been implemented using MATLAB, a math oriented script language for research in signal processing. Thus, concerning the cartographic displacement, only geometric and not topologic constraints of lines are realised.
The formulae of the snakes techniques - partly modified - can be found in the Tables 1 and 2. The matrices A_T and A_P (Tab. 2) of the snakes iteration process are not given explicitly, since they can be looked up in the articles cited.

2 The basic snakes model for line smoothing

In the following section snakes will be established as filter techniques for smoothing of curves which are represented in maps as discrete lines.

2.1 The snakes model

In signal analysis splines are used to decompose a complex signal using simple base functions. The result is an approximation respectively smoothed version of the original signal. For line smoothing the signal g is a vector consisting of the vertex coordinates of a line. If the vertex position is described by two coordinates then the two directions are handled separately.

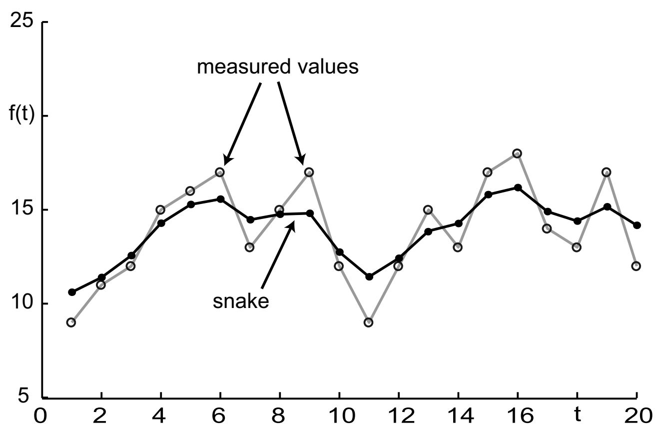

Figure 1: 1-dim signal smoothed by a snake function.

The smoothing spline function accomplishes two constraints: The first order and the second order derivative of a 1-d function should be minimised. For a 1-d signal and a line represented by tangent angle function the first order derivative is the tangent direction and the second order derivative is the curvature. This two constraints formalised in

$$E_{int} = \frac{1}{2}\alpha \cdot \left| \frac{dg}{ds} \right|^2 + \frac{1}{2}\beta \cdot \left| \frac{d^2 g}{ds^2} \right|^2$$

are called *internal energy* E_{int} and describing the characteristics of the curve. The Parameters α and β are used to weight the derivation terms against each other.
The first snakes model was introduced by Kass et al. [1987] for the use in the image analysis domain.

The model itself can be seen as enhanced spline model which has not only the property to smooth a signal it is also able to react against the smoothing. Therefore an external energy E_{ext} is added.

$$E_{Snake} = \int_0^1 \left(E_{ext} + \frac{\alpha}{2}\left|\frac{dg}{ds}\right|^2 + \frac{\beta}{2}\left|\frac{d^2g}{ds^2}\right|^2 \right) ds \rightarrow Min.$$

The external energy drives the deformation of the shape and will be defined by the purpose (solving proximity conflicts, reducing line details, contour detection in image recognition, etc.).
After defining the energies with respect to arc length s the minimisation of the energy sum is strived to receive a smoothed line. This is done by applying the *Calculus of Variation* and deriving the equivalent *Eulerian Equations* (Tab. 1). These equations will be discretised using *Finite Differences* and solved by an iteration process.

2.2 Two models of active splines

The original snakes model is based on direct manipulation of two dimensional x, y coordinates. Since this model has some disadvantages (condition of matrix, slow convergence of the algorithm and so forth) a new simpler model was developed by Borkowski et al. [1999] for cartographic purposes. The so called - *Tangent Angle FUnction Snakes* (TAFUS) - are based on line representation using the tangent angle function:

$$\varphi(s) := \arctan \frac{\dot{y}(s)}{\dot{x}(s)}$$

with the first order derivatives of the coordinate directions and with respect to arc length s.
If this model is used small changes of angles are obtained. To derive the new x, y coordinates from an unique backward transformation an additional constraint for the movements of points is necessary. Using angle representation can have disadvantages for planimetric positional accuracy. This problem is discussed for TAFUS in the article by Borkowski and Meier [2001].
The most important equations and relations for both line smoothing models can be found in Table 1.

2.3 Internal and external energy

As mentioned before the internal energy preserves the shape of the line. It can be seen in the upper part of Table 1 that the characteristics of E_{int} are driven by the parameters α and β in both snakes models. In the conventional model the parameter α for the first term is controlling the *elasticity* against the second term, the *stiffness* weighted by β. For the TAFUS model the parameter β controls a line property which is independent of the coordinate system - the curvature $\dot{\varphi}$ or variation of line direction with respect to arc length. If we take a look on the *Eulerian Equations* the parameter α controls the change of direction and β the change of line curvature. Equal parameter values for both models show different effects, since the relations among polar and orthogonal coordinates are not linear.

The external Energy will be defined in dependence of the purpose, e.g. smoothing or displacement. For the smoothing of a line E_{ext} is set by Burghardt [2002] to a constant value for the conventional snakes. This implies a vanishing external energy. If the filter characteristics of the snakes model will be calculated it can be shown that smoothing snakes realise a lowpass filter (Fig. 2) with domination of the parameter β on higher frequencies. Hence we set $\beta = 100, \alpha = 1$ we gain a more strongly smoothed line than set $\beta = 1, \alpha = 1$.

For the TAFUS model the external energy is set to the curvature of the line. We approximate the curvature by calculating the discrete differences of the line direction in a vertex. Therefore we obtain

Table 1: Snakes models used for line smoothing

SNAKES	TAFUS
internal (shape-) energy	
$E_{int} = \frac{1}{2}[\alpha \lvert \underline{\dot{v}} \rvert^2 + \beta \lvert \underline{\ddot{v}} \rvert^2]$	$E_{int} = \frac{1}{2}[\alpha \varphi^2 + \beta \dot{\varphi}^2]$
terms: elasticity $\dot{v}$ and stiffness $\ddot{v}$	terms: direction φ and curvature $\dot{\varphi}$
$\underline{\dot{v}} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}, \underline{\ddot{v}} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix}$	$\varphi = \arctan(\dot{y}/\dot{x})$ $\dot{\varphi} = \dot{x}\ddot{y} - \dot{y}\ddot{x}$
Eulerian-Equations	
$0 = \frac{\partial E_{ext}}{\partial x} - \alpha \ddot{x} + \beta x^{IV} + 1[-\dot{\alpha}\dot{x} + \ddot{\beta}\ddot{x} + 2\dot{\beta}\dddot{x}]$	$0 = \frac{\partial E_{ext}}{\partial \varphi} + \alpha \varphi - \beta \ddot{\varphi} - \dot{\beta}\dot{\varphi}$
(analogous for y; the term in brackets vanishes for constant α, β)	+ restriction: a snake point moves orthogonal towards the tangent
external energy for line smoothing	
$E_{ext} = const \quad \frac{\partial E_{ext}}{\partial x} = \frac{\partial E_{ext}}{\partial y} = 0$	$E_{ext} = (\varphi_0 - \varphi)\dot{\varphi}_0 \quad \frac{\partial E_{ext}}{\partial \varphi} = -\dot{\varphi}_0$

All terms are functions with respect to arc length s, dotted for partial differentiation with respect to s.

a locally driven smoothing: a high curvature results in a strong external force and hence in a strong smoothing. The calculated filter characteristics for this TAFUS model shows a combined lowpass and highpass filter resulting in a bandpass (see Figure 2). Therefore this model is only recommendable for limited smoothing in the high frequency domain and using convenient stopping criterion for the iterative smoothing process.

2.4 Solution via iteration process

Obtaining a specific degree of smoothness is possible in two ways:

- **one step**: We can set the parameters α, β to fixed values and obtain the desired smoothness immediately. But this is hard to do, because it requires a lot of experience by the user. Besides, this is not advisable for use in automated generalisation environments.

- **iteration**: Another approach is to set the parameters to constant values (e.g. $\alpha = 1, \beta = 1$) and start an iterative smoothing process. Therefore it is necessary to define a stopping criterion.

Tests have been shown that the curvature - a line feature that is independent of the coordinate system - can be used as such a criterion. However more research is needed on generalised maps to evaluate a scale dependet maximum of curvature where legibility of a map is still ensured.

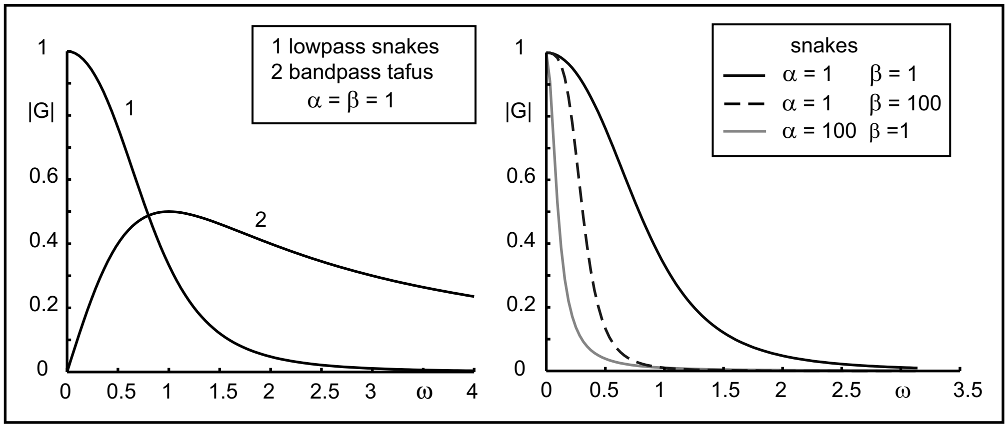

Figure 2: Filter characteristics of the smoothing snakes model with respect to angular frequency ω.

3 Results and improvement of the smoothing algorithm

The evaluation of the tests for the basic smoothing algorithms revealed some weaknesses. The side effects, their handling and improvements are discussed below.

Parametrisation An inappropriate setting of values can lead to a distorted line. This is due to numerical problems caused by ill conditioned filter matrices. A solution is given either by well known *regularisation* or by *scaling* of the filter matrix elements to preserve continuity in scale (c.f. Steiniger [2003]). Figure 3 shows on the left side deformations of line shapes caused by extreme parameter values. To avoid such and other effects we recommend to fix the parameter $\alpha = 1$ for TAFUS and conventional snakes.

Movement of boundary points For generalisation it is necessary to preserve connectivity as well as keep the location of navigation points (e.g. crossroads). This requires a fixed position for boundary points of a line. Strong point translations are observable for the 5 outer points only, if the matrices are *scaled*. To prevent translations it is recommended to *rescale* the outer matrix coefficients and to mirror the outer points on the end of the line before calculating the smoothing. Here, *rescaling* means to set the values of the two outer diagonal matrix elements to the value of the third diagonal coefficient and after it to ensure that the sum for the elements of a matrix row is equal to one.

Constraint points To improve legibility and cognition between the map and the reality it might be useful to fix some salient points of a line in position. The criterion for selecting a point as "landmark" is the change of tangent direction. As threshold for selection we used for continuously differentiable - means relatively smooth - lines the empirical value $\pi/3$ and for lines with breakpoints $\pi/2$ (see left picture of Figure 4). To fix such points the line has to be split up in the point and the resulting segments are smoothed independently. A smooth transition from segment to segment can be achieved by observing the previous modifications.

Local control for parameter To preserve shape characteristics of a line - for instance a meander of a river or serpentines of a pass road - the value of parameter β (with fixed $\alpha = 1$) can be locally controlled. A control by curvature is reasonable, since it represents the change of line direction. Figure 4 shows two examples for smoothing with constant and variable parameters.

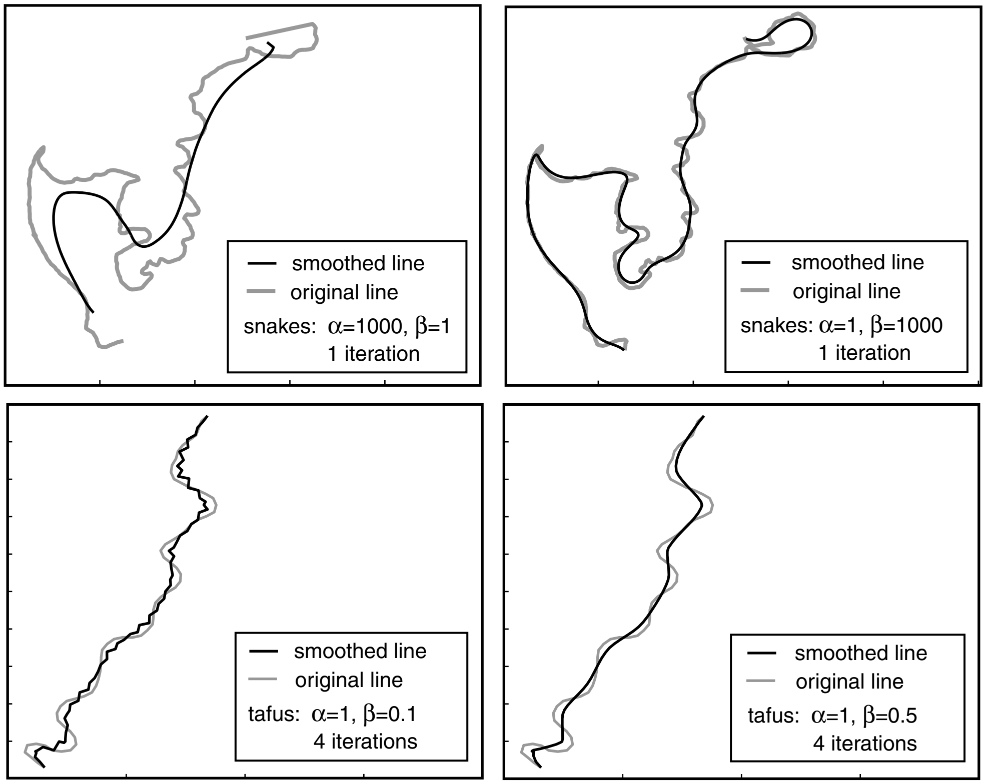

Figure 3: Examples for line smoothing using different parametrisation. Bad results caused by extreme parameter values on the left side.

The results for a few iteration steps showed better preserved line characteristics, but are converging to the ordinary solution if more than five iterations were computed. Due to the fact that more than five steps are usual and the calculation of a filter matrix with variable coefficients increases computational cost we can not recommend this approach without further reservations.
Besides this it is also possible to accelerate smoothing by calculating the parameter value using the curvature in the opposite way. Here a strong smoothing on segments with high curvature is obtained.

Stability of solution If raw data are used then points are usually not digitised at an equidistant spacing. Hence it might happen that very short distances with large tangent changes between points occur. Since the TAFUS model is based on the tangent angle function this affects shape distortions of the line. Therefore it is useful to set restrictions and filter out extreme values (e.g. for curvature), interpolate equidistant points (Borkowski and Meier [2001]) or delete narrow points in an intermediate step. Unlike the TAFUS model the conventional snakes show no need for further restrictions and are stable for feature displacement and line smoothing on non-equidistant digitised data.

We tested the modified algorithm on 3 single lines with different granularity: a smooth river, a coast line (Fig. 3, top) and a river with strong sinuosity. Finally on 4 line datasets too: a small street network (Fig. 5, top) and political boundaries. All the results were satisfying.
Comparing the TAFUS model and conventional snakes the latter method shows some advantages in stability on one hand. Furthermore it represents a faster smoothing algorithm (using less iterations) since the parametrisation with high parameter values is nearly freely possible and we do not have to proceed an extra transformation to *arc length* representation. On the other hand the TAFUS algorithm can be a good choice if only a fine smoothing is recommended.

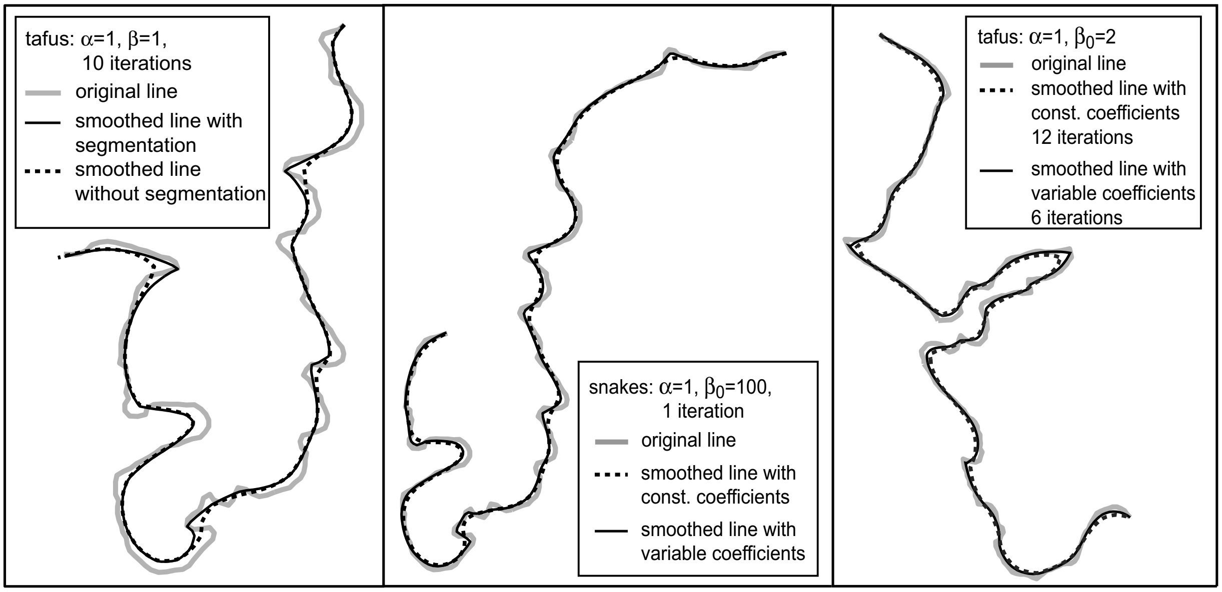

Figure 4: Smoothing of coast lines using TAFUS and the conventional snakes model. The effect of segmentation can be seen on the left picture. The results of the middle and right picture, respectively, show models with constant and locally driven parameter $\beta = \beta_0\,|\dot{\varphi}|^2$ using the curvature.

For both models a one step algorithm showed never a better graphical result than an iterative smoothing. It is further remarkable that line segmentation is not only preserving shape characteristics, it also highly reduces processing time on our test system because numeric operations are carried out using smaller filter matrices. In **conclusion** we prefer the iterative (conventional) snakes algorithm by Burghardt [2002] - partly modified - and using initial parameter values $\alpha = \beta = 1$, segmentation and a stopping criterion.

4 Combined algorithms

After detailed examination of the smoothing algorithm, as it has been done already with the snakes displacement models (Borkowski et al. [1999], Burghardt [2001] and Bader [2001]), both models can be combined.
To obtain a sophisticated map in cartographic sense the combination of different generalisation operators requires optimisation techniques. Therefore the AGENT system (Barrault et al. [2001]), an automated generalisation environment, uses a modified hill climbing algorithm and Harrie and Sarjakoski [2000] as well as Sester [2000] using Least Squares Adjustment.
Here, we only examined a simple combination of both algorithms for use in batch processing. The combined (optimisation) process has two aims. On the one hand we want to smooth the line object by minimising shape energy which is influenced by external forces. On the other hand we have line displacement, where a balance of shape preserving (internal) energy and shape deforming (external) energy is needed or, alternatively, the vanishing of displacement forces.
Since new displacement conflicts such as line intersection can be caused by line smoothing, side effects among both operations should be prevented. This is done by a series of small steps towards the desired line shape and place.

4.1 Methods of combination

Two simple methods of simultaneous line smoothing and displacement were examined which we will call *integrative* and *alternating method* respectively. The calculation specifications can be found in Table 2.

Remarks on displacement The variation of external energy $\frac{\partial E_{ext}}{\partial x}$ and $\frac{\partial E_{ext}}{\partial \varphi}$ of the basic snakes and TAFUS models are now specified as the variation of *displacement energy* E_x and E_φ in the table. For a vertex of a line the energy holds a non zero value if at least one other point is within a circle distance of radius h_{min}. The radius will be defined by human perceptual limits (i.e. the minimum distance) of two different map objects and varies for paper maps, screen maps and other output media. Positive and negative values for the variation of energy are possible since it represents the vectored translation which has to be done for the point to solve the conflict. The variables $\delta x, \delta\varphi$ are equivalent to changes of internal energy, which is used in the displacement model as *restoring energy* to prevent excessive point translations and hence distortions of the line shape.

Table 2: Combined Algorithms

	SNAKES	TAFUS
	alternating method	
smoothing	$\underline{x}^t = \underline{A}_P^{-1}\underline{x}^{t-1}$	$\underline{\varphi}^t = \underline{\varphi}^{t-1} + A_T^{-1}\Delta\underline{\varphi}^{t-1}$
displacement	$\underline{x}^t = \underline{x}^0 + \underline{A}_P^{-1}(\delta\underline{x}^{t-1} - \underline{E}_x)$	$\underline{\varphi}^t = \underline{\varphi}^{t-1} + \underline{A}_T^{-1}(\underline{E}_\varphi^{t-1} - \delta\underline{\varphi}^{t-1})$
	(analogous for y)	$\delta\underline{\varphi}^{t-1} := \underline{\varphi}^{t-1} - \underline{\varphi}^0$
	integrative method	
	$\underline{x}^t = \gamma\underline{x}^0 + \underline{A}_P^{-1}[\underline{x}^{t-1} - \gamma(\underline{x}^0 + \underline{E}_x)]$	$\underline{\varphi}^t = \underline{\varphi}^{t-1} + \underline{A}_T^{-1}\underline{c}$
	(analogous for y)	$\underline{c} := \gamma(\underline{E}_\varphi - \delta\underline{\varphi}^{t-1}) + (1-\gamma)\Delta\underline{\varphi}^{t-1}$

A_T and A_P are the filter matrices for TAFUS and the conventional snakes model. They are obtained by the discretization of *Eulerian Equations* using *Finite Differences*.

(1) Alternating Method The method changes between the smoothing and displacement algorithm either after one iteration or after multiple iteration steps. Experiments have shown that it is recommendable to take one step of displacement on one step of smoothing to (partly) remove conflicts caused directly by line smoothing or prevent them in advance. Both processes do not have to use the same parameter values. For the smoothing operation the parameter β can be driven locally to anticipate the knock-on conflicts mentioned.

(2) Integrative Method This method combines both systems in linear fashion to one system of equations. A new control parameter γ with $\gamma \in (0,1)$ is introduced. For $\gamma = 0.5$ smoothing and displacement are weighted equally. Using the boundary value $\gamma = 0\,(\gamma = 1)$ the equation system for smoothing (displacement) is obtained. The value of γ can be changed from step to step. However, a local control of the parameters α, β is not possible, since only one iteration matrix exists. Hence the alternating model is more flexible.

4.2 Preliminary results for combined methods

In our evaluated examples displacement conflicts among line objects were often solved by the smoothing process. However, for the selected examples of Figure 5 this is not the case, since the conflicts are located on smooth parts of the curves. The displacement conflicts can be solved by exclusive use of the displacement operation assumed that space between map objects is available. In patterns of dense object allocation, however, the displacement problems can be not resolvable and other kinds of generalisation operations (e.g. object elimination or aggregation) have to be chosen.

Figure 5 shows results of the alternating (1) and the iterative (2) model. In process (1) one displacement step is done on two smoothing steps and in process (2) the operations were set equivalent ($\gamma = 0.5$). The number of iterations was selected as stopping criterion. This results in a different degree of smoothing for both methods. A larger number of steps does not show big differences between the results of processing with fixed and locally driven parameters. Indeed a variable parameter β is to prefer since it better prevents follow-up conflicts during the first iterations.

The processing time of displacement is higher than of line smoothing (ratio 1:4 to 1:10). This is caused by the complex evaluation of conflict energy for all the vertices of a line. For use of snakes displacement in real time generalisation the improvement of the algorithms should focus first on this aspect. Comparing the computation time needed by TAFUS model and the conventional snakes only negligible differences were detected. The combined snakes algorithm is robust without any further requirements and therefore preferable.

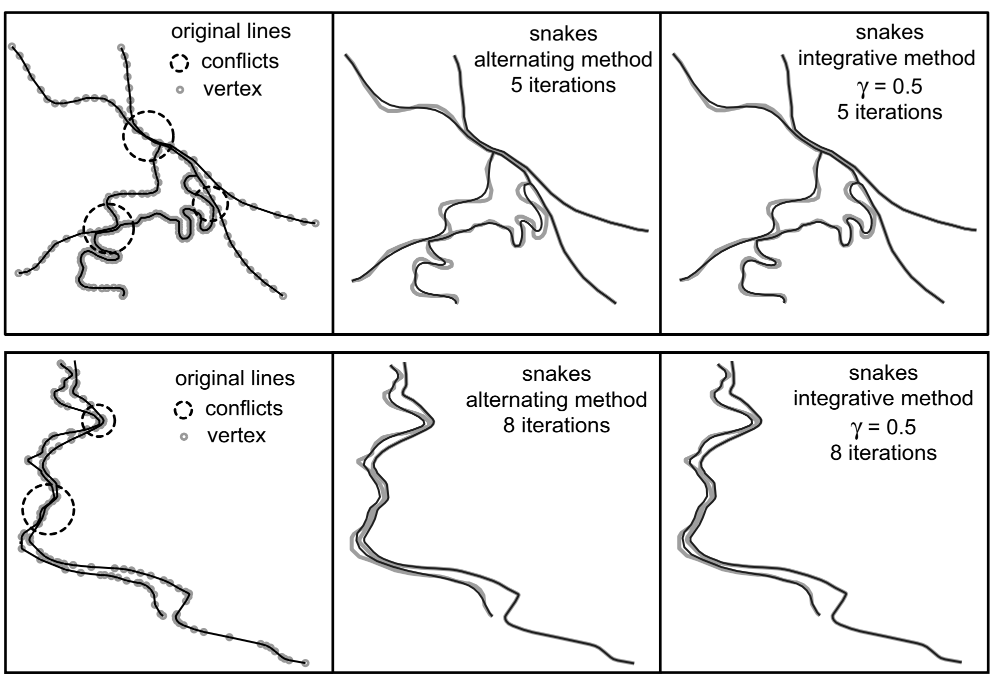

Figure 5: Line patterns with local conflicts. Smoothing and displacement with snakes ($\alpha = 1, \beta$ variable with $\beta_0 = 1$) using two different combination methods. See section 4.2 for discussion.

5 Conclusions

Smoothing of line objects is possible using different methods like Gaussian filtering (Badaud et al. [1986], Plazanet et al. [1998]) or splines. A good cartographic quality can be obtained using shape consistent filters. With respect to combined generalisation operations in a uniform concept and cartographic software with a clearly structure we prefer a solution based on snakes. The snakes algorithm using constant parameters ($\alpha = \beta = 1$) is a simple smoothing filter whereas locally driven parameters (e.g. β depends on curvature) represents a shape consistent filter algorithm. The use of the latter requires continuously differentiable curves and is suggested for the alternating method of combined algorithms. The segmentation of lines on breakpoints (constraint points) using curvature as criterion shows benefits in terms of cartographic quality and processing time.
The solutions based on the TAFUS model are equivalent to the conventional snakes only if stabilising arrangements will be performed. Since TAFUS requires a data pre- and postprocessing they do not show the expected advantages for numeric calculations.

Regarding the development of combined algorithms for smoothing and displacement we prefer the so called alternating method using conventional snakes. This method is more flexible and shows better results for complex conflict situations than the integrative method. But here further investigation has to be done in terms of evaluation for typical and also extreme conflict situations in topographic maps. Furthermore tests in automated generalisation environments versus interactive control are needed. The smoothing algorithms have been tested exhaustively and can be recommended for use in commercial software. We are optimistic as well to do so for the combined methods since our tests show promising results.

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