

# Relations and Structures in Categorical Maps

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**Abstract**— Focusing on thematic and more specifically, categorical maps, the paper emphasises the importance of horizontal relations in automated generalisation. A literature review shows that automated generalisation of thematic maps has not received ample attention. It shows further that the extraction and use of relations in topographic maps for generalisation purposes is making progress, but no systematic categorisation for relations has been made so far. Hence a typology of horizontal relations for categorical maps is proposed and links to existing generalisation constraints are identified. Besides this, methods for the storage of extracted relations are proposed. A selection of relations is discussed and illustrated in more detail. Subsequently a characterisation of alignments – here subsumed about the so-called meso structures – is presented using Wertheimer’s laws of organisation. Finally further research perspectives are proposed, with the aim of pattern detection and data enrichment in thematic maps.

**Index Terms**— horizontal relations, thematic maps, categorical maps, cartographic generalisation, structure recognition, structure metrics, data enrichment, Gestalt theory

## I. INTRODUCTION

IN the last decade research on the automation of cartographic generalisation mainly focused on problems of topographic maps. This is reasonable since topographic maps are in the main interest of national mapping agencies. Yet, taking a look at the maps produced in general it is obvious that most of the maps are special-purpose thematic maps. The spectrum of thematic maps used is very diverse and ranges from short-lived maps for the daily weather forecasts to maps that have a medium term life span such as hazard maps, and further to long term maps, represented for instance by maps of climate, geology or linguistic groups.

Common to thematic maps is the (geo)spatial reference of their content. But since the subjects covered are very diverse the main geometric data types are also variable. Thus, in maps of location based services, so called points of interest (POI) visualised by point symbols are dominant. For navigation charts showing routes and facility maps showing gas, power and water lines, the linear objects are emphasised. Finally, on climate or geologic maps polygonal subdivisions are used to present the continuous thematic variables.

The focus of this paper is on thematic maps with polygonal

subdivisions as the main data type and the relations that can be found between polygon objects. As mentioned above, only little work has been carried out on automated cartographic generalisation of this map type so far. Peter (2001) lists measures for the characterisation of polygonal subdivisions mainly derived from metrics of FRAGSTATS (McGarigal and Marks, 1995), a program widely used in landscape ecology for patch analysis. Galanda (2003) presents polygonal constraints for a constraint-based generalisation model. Further literature can be found on semi-automatic and interactive generalisation of soil maps (Fuchs, 2002), of geologic maps (Downs and Mackaness, 2002), and of different map types for the Atlas of Canada (Brooks, 2002).

Analysis of the cited literature reveals that the recognition of structures and patterns and the enrichment of data with the obtained information is essential as a basis for the automation of thematic map generalisation. This has, though in the context of topographic maps, also been argued by Ruas and Plazanet (1996) who state that ‘*data enrichment is necessary to perform generalization*’. However, they give only a vague description of data enrichment. Slightly more precisely Neun et al. (2004) describe data enrichment as ‘*necessary process to equip the raw spatial data with additional information about the objects and their relationships*’. Data enrichment itself should support different processes during the automated generalisation of maps. It concerns data characterisation, conflict detection, indirectly also the algorithm selection, and finally the evaluation of the map. Therefore, a need for extracting relations is given. Building a typology of relations for polygonal subdivisions is essential not only for thematic maps but also for topographic maps since certain relations are common to both map types. Mustière and Moulin (2002) discuss the meaning of spatial context in map generalisation and differentiate between three kinds of relations: *part of relation*, *being in area relation* and *being on ‘same-level’* of analysis with surrounding objects. Thus, they classify from a more spatial point of view. We will distinguish map object relations generally into horizontal and vertical relations. The typology proposed here will focus only on horizontal relations and divide them subsequently into spatial relations (geometry and topology), semantical, statistical and structural relations, whereas the latter relation type also includes groups of objects. Properties of vertical relations will be discussed in Neun and Steiniger (2005).

In Section II a description of horizontal and vertical relations is given followed by a review of the use of horizontal relations in map generalisation. The typology of horizontal relations is introduced in Section III and a selection of rela-

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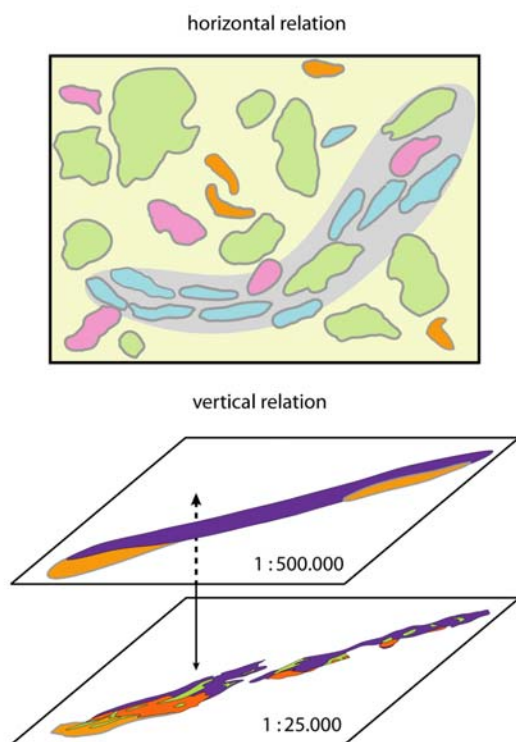


Fig. 1: Horizontal (top) and vertical (bottom) relations in categorical maps

tions described in more detail. The appendix of the paper provides the corresponding tables including a detailed characterisation of the relations. Section IV describes in detail the properties of meso structures as a special type of horizontal relation. Finally, further research objectives are presented.

## II. RELATIONS IN MAPS

### A. Horizontal and vertical relations

The differentiation of horizontal and vertical relations does not derive from a spatial property. Instead, it is implicitly given by the purpose of use and scale.

*Horizontal relations* of map objects exist within one specific scale or level of detail (LOD) and represent common structural properties – e.g. neighbourhood relations and patterns (Neun *et al.* 2004). In the top picture of Fig. 1 an example of a horizontal relation is presented by a curved polygon alignment. The extraction and use of horizontal relations is currently limited to the generalisation of topographic maps, which only requires a subset of the presented horizontal relations for the more general thematic maps.

*Vertical relations* are links between single objects or groups of objects between different map scales and LODs, respectively (see Fig. 1, bottom). They can be represented and stored in Multi-Resolution Databases (MRDBs) and can be exploited for paper map generalisation, incremental geo database updates (Harrie and Hellström, 1999) and for web and mobile mapping applications. An example of the use of previously

extracted vertical relations is the Adaptive Zooming algorithm by Cecconi and Galanda (2002), developed for web mapping applications.

### B. Why consider horizontal relations?

Data enrichment with horizontal relations fills the link between structural knowledge on the one hand and procedural knowledge on the other hand. Here, structural knowledge (Armstrong, 1991) denotes the information obtained by data characterisation and cartographic pattern and structure recognition. Hence, evaluation of horizontal relations supports answering the question *when* (and *where*) map generalisation is needed (McMaster and Shea, 1992). On the other side procedural knowledge is related to the question of *how* to solve a given generalisation problem. It holds information on suitable generalisation operators, available algorithms and their control possibilities. Here, identified horizontal relations constrain algorithm selection and processing order.

Ruas and Mackaness (1997) emphasised the need to consider spatial interdependencies (i.e. horizontal relations) for the generalisation of urban maps and presented working examples of building generalisation. With respect to categorical maps we give a simple example of four polygons (e.g. four lakes) presented in Fig. 2.

A well legible map meets several visual requirements, including that an object should have a minimum size on the map so that it can be perceived by the map reader. In our example three of the lakes would not meet the constraint. And we – or the computer – have to decide how we want to solve the problem. Fig. 2 shows two simple solutions on the left. Here, the link between structural and procedural knowledge is made by way of simple object characterisation without considering the relations to other, neighbouring objects. Thus, the possible solutions presented on the left of Figure 2 are either to delete the three small objects found to be too small or to enlarge them. These solutions do meet the basic requirement of visualisation (minimum size) but do not necessarily present a good cartographic solution from the structural point of view. A good cartographic solution of generalisation problems should also maintain the typical structures and emphasise the specificities of a map. Such a satisfactory solution can only be obtained by consideration of inter-object relations (see Fig. 2, right). Both solutions preserve the typical properties of the spatial arrangement.

### C. Research on map generalisation and horizontal relations

Today, detection and use of horizontal relations is in most cases restricted to the identification of proximity relations and proximity conflicts. Examples include the detection of overlaps and the consideration of minimum distances of buildings and road objects in topographic maps. Furthermore, proximity relations can be extracted to obtain free space for feature displacement and to aggregate area objects of the same class (e.g. buildings). The detection of more complex structures – so-called *higher order features* – in topographic maps is mainly focusing on the preservation of settlement character (Gaffuri

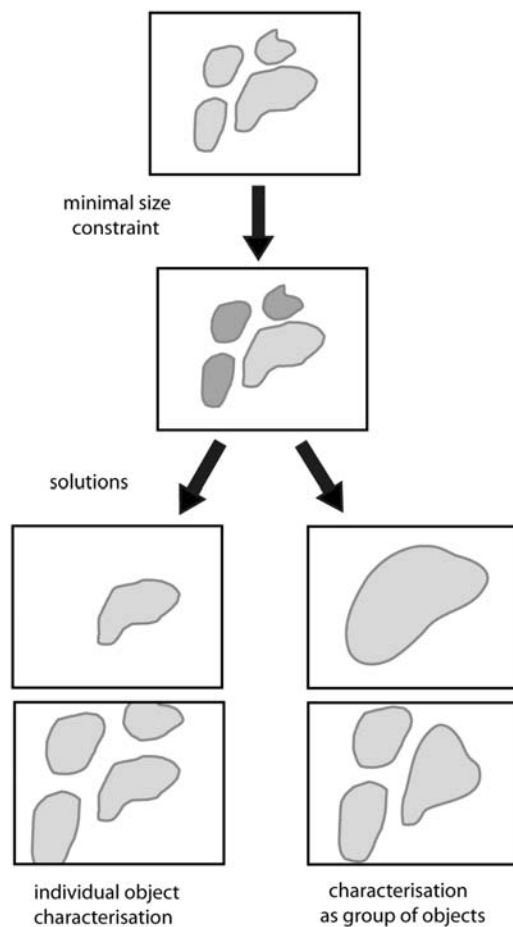


Fig. 2: Different generalisation solutions if horizontal relations are ignored (left) and if observed (right).

and Trevisan, 2004; Sester, 2005) or building groups and alignments (Regnauld, 1998; Christophe and Ruas, 2002).

Another view on the consideration of relations, especially the ones between objects of different categories, is offered by Fuchs (2002) in his derivation of a small scale soil map. Here, the focus is rather on semantic proximity of the different soil categories than on spatial proximity between the soil patches. These semantic relations have been extracted and used for the aggregation of polygons in a pre-generalisation phase of *structure analysis* (Steiniger and Weibel, 2005), which is somewhat similar to *model generalisation* (Weibel 1995) during the generalisation of topographic maps. Conversely, the other examples above are more or less part of *cartographic generalisation*.

The management of extracted horizontal relations during the generalisation process is carried out in different ways. Restricting the generalisation process modelling approaches to the constraint-based one, proposed by Beard (1991), we can distinguish between the agent-based approach, combinatorial optimisation, and continuous optimisation (Harrie and Weibel, 2005). For agent-based modelling two types of treatment of relations exists. The model by Ruas (1999) deals with micro,

meso and macro agents, whereby especially the meso agents – sometimes called group agents (Galanda 2003) – represent relations explicitly. Inter-object conflicts are resolved by orders which are passed by the responsible meso agent to the associated micro agents. In the agent model of Duchêne (2004) the relations are represented either by meso agents – as in the previous model – or by a relation object. Here, additionally to command hierarchies, solutions for inter-object conflicts may be obtained by negotiation between the micro agents. Some examples preserving higher order features in settlement structures by use of an agent system are given by Gaffuri and Trevisan (2004).

In the combinatorial and continuous optimisation models object relations have to be modelled explicitly but are not treated separately from single object conflicts during the solution process. With respect to inter-object relations the examples presented in the literature on application of optimisation methods are restricted to solutions of proximity conflicts or preservation of line crossings (Harrie and Sarjakoski, 2002). Apart from the article of Ware *et al.* (2003), modelling approaches for the preservation of higher order feature relations (e.g. building alignments) were not provided so far for the combinatorial and continuous optimisation methods.

### III. A TYPOLOGY OF HORIZONTAL RELATIONS IN CATEGORICAL MAPS

#### A. A typology derived from existing classifications

A number of classifications with respect to spatial relations have been proposed in the context of spatial reasoning (Egenhofer and Franzosa, 1991; Frank, 1996). An example includes the classification of Pullar and Egenhofer (1988) into: direction relations (e.g. *north*, *northeast*), topological relations, comparative or ordinal relations (e.g. *in*, *at*), distance relations (e.g. *far*, *near*) and fuzzy relations (e.g. *next*, *close*).

However, since the map relations should be used in automated generalisation and are not restricted to the spatial domain an obvious starting point is to organise them with respect to existing classifications of map constraints or measures used in generalisation. Starting with classifications of constraints two kinds of taxonomies can be distinguished. The first classification is organised by the function of the constraints whereas the second one orders the constraints by the scope of usage. An example for functionally oriented ordering is the typology into graphics, structural, application and procedural constraints originally suggested by Beard (1991). This grouping has been revised by different authors with respect to specific geometry types, functional purpose or generalisation process modelling (Weibel, 1996; Ruas and Plazanet, 1996; Weibel and Dutton, 1998; Harrie, 1999). A specific classification for constraints of polygon generalisation and categorical maps is proposed by Galanda (2003). He distinguishes four groups, namely: metric, topological, structural and procedural constraints. In contrast to this functional approach Ruas (1999) classifies constraints according to the scope (or levels) of usage. Here, the classification is made into macro level (entire

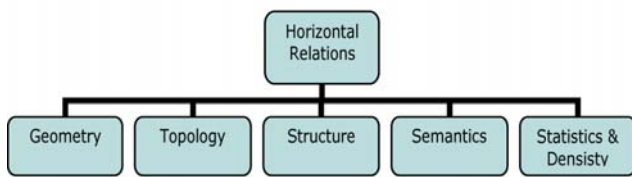


Fig. 3. Typology of horizontal relations.

dataset or type), meso level (group of objects) and micro level (constraint of a single object).

Another scope oriented typology, but applied to measures, is presented by McGarigal (2002) for landscape ecology. Here the levels are called patch, class and landscape. Patch metrics are applied to a discrete area of relatively homogenous environmental conditions. Class metrics describe measures for all patches of one category and landscape metrics are integrated over all patch categories of the entire dataset or selected frame. In landscape ecology the metrics are also classified into spatial and non-spatial categories, where the first group is called composition metrics and the second spatial configuration metrics (Gustafson, 1998; McGarigal, 2002). The classification of measures for automated map generalisation by Peter (2001) is closely related to the functional constraint typologies. He distinguishes between: (1) size, (2) distance and proximity, (3) shape, (4) topology, (5) density and distribution, (6) pattern and alignment, and (7) semantics.

The typology for horizontal relations proposed in this paper is based on Peter's classification of measures but merges the first three groups into one called geometric relations. The other four classes remain as separate groups. The resulting typology is organised by functional purpose and shown in Fig. 3.

### B. Horizontal relations and their characterisation

Relations between map objects can be established by simple and complex methods. An example for a simple extraction method would be the comparison of attribute values of two objects. This can simply be done during the generalisation process as it does not require major computation power. Conversely, relations may require more complex methods, such as the use of auxiliary data structures – e.g. a minimum spanning tree (Regnauld, 1998). Usually such data structures need more computing resources and therefore are established in a pre-processing phase. Since object relations can change during the generalisation process, however, dynamic data structures.

In the Appendix of this paper 5 tables can be found which contain an inventory of relations organised by the typology of Figure 3. The listing does not claim to be complete since not every type of categorical map has been studied. Here, starting point for the definition of horizontal relations was a number of geological, soil and thematic atlas maps covering a range of scales between 1:10'000 and 1:25'000'000. Most of the relations listed in the Appendix can be found in geological maps and soil maps. Such maps typically contain topographic information in addition to thematic information.

TABLE I  
USED DATA TYPES AND CHARACTERISATION

data type	origin	differences*	ranking	example
Boolean	No	No	No	yes / no (exists / not exists)
Nominal	No	No	No	shape: star, circle, rectangle, cross,
Ordinal	No	No	Yes	satisfaction: bad, ok., good
Interval (Integer)	Yes / No (artificial)	Yes	Yes	number of objects, temperature
Continuous	Yes (natural)	Yes	Yes	size of area
Geometry	---	---	---	point, line, poly- gon,

\*) Does the calculation of differences give a useful result?

Every relation in the tables is characterised by a possible kind of measurement as well as the data type resulting from the measure or definition. Data types used are: *boolean*, *nominal*, *ordinal*, *interval (=integer)* and *continuous* values (Table I). These are supplemented by the *geometry* data type, since the core-area relation (see below) can not be described by the other data types in an appropriate way. Furthermore, the relations are specified by a proposed storage type for later exploitation during the generalisation process. Three different storage types are possible:

- It is recommendable to store measurement results as **attribute values** if the data type offers the possibility and if computation of relations can be deferred until run-time.
- Furthermore, storage of extracted or pre-defined relations can be implemented by **relation matrices (RM)**. Here, a distinction is made between symmetric (RMs) and asymmetric (RMa) matrices. The matrix storage type provides fast access to bidirectional relations of one map object or category to others and is particularly useful if all map objects or classes have more or less a relation to each other (i.e. in the case of densely populated matrices).
- The third storage type, the **relation object (RO)**, is used if only few map objects have something in common. This type has already been used as a relational object in the agent model of Duchêne (2003) and as so-called meso structure in the agent model of Ruas (1999). Besides this kind of modelling as a “real” object, a relation object can also be implemented by a graph structure (e.g. a neighbourhood tree).

Some storage details in the Appendix are followed by a specification in parentheses, explaining to which map entity – *map*, *map section*, *polygon*, *class (equal terms: category, legend unit)* – the storage type is assigned.

Finally, a fourth characterisation of relations is made by linking them to the constraints of polygon generalisation, specified by Galanda (2003). He proposed altogether 17 constraints organised in four groups (see previous section). Twelve of them could be linked to the relations proposed in this paper, whereas *linking* means that either the attribute val-

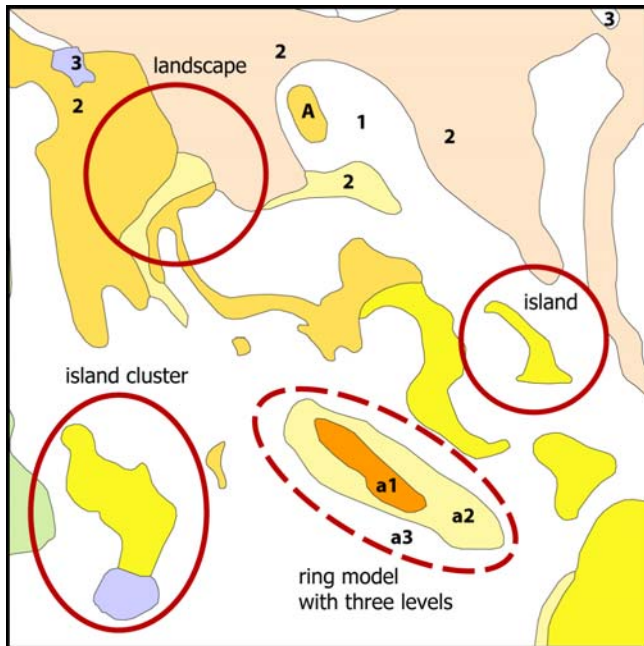


Fig. 4. Topological relations. The red rings enclose examples for the three *topological structure models*: island polygon, island cluster and landscape mosaic. Close to the centre of the picture is an example for the *ring model relation*. Here, three ring levels a1, a2 and a3 (background polygon) exists. The *topological neighbourhood* is given for the island polygon denoted by A. The numbers 1, 2 and 3 refer to the order of neighbourhood with respect to polygon A.

ues (especially the metric ones) or the relations are involved in the evaluation of constraints.

In the following section a selection of relations from Tables 1-5 is explained in more detail.

### C. A selection of relations

#### 1) Geometric relations

Most of the metric variables in Table 1 are well known in GIS and generalisation since they describe object-specific properties such as area, perimeter, etc. Four groups of variables have been identified which are **size variables**, **position variables**, **shape and orientation variables**. Here, the term *variable* is used since most of the geometrical relations are inferred only at runtime by simple comparison of the values of such variables to each other or against a threshold. Subsequently a further distinction is made into relative and absolute variables. Using *absolute variables*, the metric relations are obtained by comparison of property values. In contrast, *relative variables* such as distance measures already represent relations between two objects. Their property values can be compared as well. Measures exist for all variables. For storage either the use of attributes, attached to the polygon or relation object, or the use of matrices is proposed.

Table 1 reveals that a lot of measures exist for the same metric relation group. However, while it may be easy to describe a relation by a measure nothing is said about how representative a specific measure is. Hence, there is a need to compare the existing measures if they are an appropriate representation of an object property and if they give an appropri-

ate representation of the generalisation constraint objective. A comparison with respect to object property representation has been done by MacEachren (1985) for **shape measures** which describe the complexity of the polygon shape. An example for a shape measure is the *compactness index*, which calculates a value describing the shape difference to a circle. Shape indices might be important since a certain number of objects with similar shape could be perceived as a pattern by the map reader (Wertheimer, 1923; see section 3). As indicated in Table 1 a shape index might be useful as well for a constraint which observes and prevents excessive shape distortion during the generalisation process. Here, the relation is built between the object shape before and after generalisation; a relation with respect to temporal changes.

**Core area** (Gustafson, 1998) is the second index highlighted here. The measure is calculated by use of a negative buffer operation and returns a geometry (Figure 5). This index seems not to fit so well in the typology of horizontal relations since it does not reflect a relation to a specific map object. Instead, core area presents a relation of a polygon to its environment. For generalisation the relation can be used to indicate a necessary geometry type change (e.g. feature collapse). This especially holds for use on line elements (e.g. rivers) and their symbolisation. In landscape ecology the index is used to define a core zone, where for instance a species or soil class exists to 100 percent. In contrast to this, the area between core and polygon edge designates a transition zone between two species or soil classes. Thus it represents fuzziness, which is a common property for a number of categorical map types. The core area integrates polygon size, shape and edge effects into a single measure (McGarigal, 2002).

#### 2) Topological relations

The extraction of these relations (Table 2) requires an implementation of a topological data structure in the generalisation system. This can be done by use of an intersection model (see Egenhofer and Herring, 1993; Clementini *et al.*, 1993) and additionally either by the use of a planar graph (Bobzien and Morgenstern, 2002) or of auxiliary data structures such as the Delaunay triangulation (de Berg *et al.*, 1997). After establishing the topological model four types of topological relations should be distinguished: **topological structure**, the order of **topological neighbourhood** (see Fig. 4), the **intersec-**

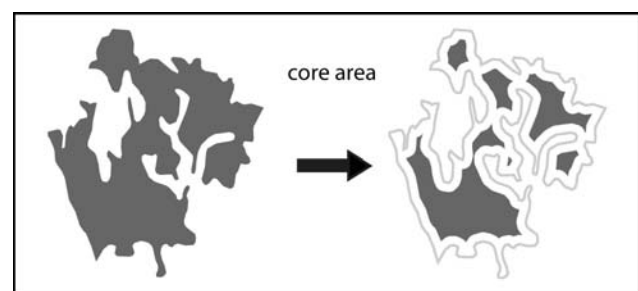


Fig. 5. *Core Area* is calculated using an internal buffering operation.

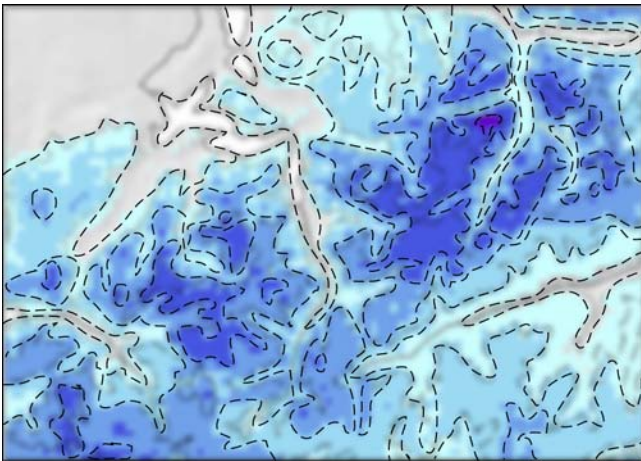


Fig. 6. Portion of a map with snow depths. It shows the topological *ring-model* relation and the *inter-thematic* relation between snow depth and mountain topography. (Base data from SLF, Davos)

**tion type** (e.g. overlap, touch, cross, etc.) and the **ring model**. Although all four types represent explicit relations they require different data types (nominal, interval, boolean) and storage methods. For topologic relations all three proposed storage types are used: attribute of map or polygon, matrix and relation object.

The **topological structure** relation separates three types of structure models. These are: island polygon, island cluster and landscape mosaic (see Fig. 4). The naming of structures *island polygon* and *landscape mosaic* is derived from the landscape ecology perspective on patches (McGarigal, 2002). The distinction into three types has a two-fold meaning for generalisation. First, it is necessary to preserve the typical patch structures in the map and second it can be used for displacement models like the one described by Galanda and Weibel (2003). This polygon displacement and deformation model needs information whether a polygon is rigid – will be displaced as whole object – or if the polygon outline is elastic and hence can be deformed. This kind of information could be derived from the different structure models. Such that islands or island clusters would be assigned a rigid outline and consequently be displaced as a whole.

The **ring model** relation applied on two polygons is similar to the island model mentioned above. But considering the case where more polygons are involved which enclose each other like the peels of an onion, then we obtain a pattern which is typical for maps of discretised continua. Examples are isarithm maps of temperature, heights fields, or snow depth (Fig. 4 and 6). The key use of this type of relation is the maintenance of structures and the prevention of illogical outputs of generalisation processes – e.g. an island being turned into a peninsula.

### 3) Statistical and density relations

Most of the relation measures in Table 3 are adopted from landscape ecology. Originally they were developed to describe the heterogeneity of a landscape. While in landscape ecology

a general grouping into spatial configurations and non-spatial composition indices is made (Gustafson, 1998) we distinguish between five groups of indices. These are: **statistical base parameters** (mean, variance, etc.), **area relations**, **class relations**, **diversity metrics** and **configuration metrics**. Apart from one index (*richness*) all measures yield a continuous result type and storage of these values is proposed as attribute of the corresponding map entity (map, map section, category or polygon). Only the *border length index* (Fuchs, 2002) which describes the length of the common edge between the polygons belonging to two classes, should be stored in a symmetric relation matrix.

The use of these relations has two main goals. First, the preservation of map heterogeneity and second the detection of dominant or rare objects. To achieve the first goal the group of **diversity indices** can be employed. Both Shannon indices characterise the amount of information, the so-called *entropy*, either of the whole map or of a map section. The Simpson indices are not entropy measures; instead they can be seen as probability measures.

Entropy as a measure of information has its origin in communication theory (Shannon and Weaver, 1948). The *Shannon diversity index*, given in Table 3, is the entropy measure applied in landscape ecology (McGarigal and Marks, 1995). Since the absolute magnitude of the index is not very meaningful it should rather be used as relative measure to describe the loss of information as a consequence of the generalisation process. The *Shannon evenness index* is based on the previous index but is normalised by the maximum diversity. Thus, the evenness index is the complement of dominance of a class. An application of entropy measures to polygon maps has been reported by Ibáñez *et al.* (1995) for the characterisation of pedodiversity in soil maps. Bjørke (1996) proposed the index as well for evaluating automated map design and eliminating point symbols by preserving point clusters (Bjørke and Myklebust, 2001). Linking the diversity metrics (i.e. the Shannon and Simpson indices) to constraints (see Table 3), they can be used in polygon generalisation to preserve the relative configuration by detecting dominant classes. Further they are useful to ensure an optimal balance of map information content and legibility by helping to determine the categories of the map legend.

The group of **configuration metrics** consists of three different measures which quantify the configuration and fragmentation of a landscape. Typically these metrics are based on a matrix of pairwise adjacencies between all patch types. The elements of such a matrix are the proportions of edges in each pairwise type (McGarigal, 2002; see also Fig. 7). Configuration metrics are useful in map generalisation on one hand to preserve the spatial fragmentation, that is, to preserve the 'picture' of patch structure. On the other hand they are useful to determine the number and composition of (new) legend categories. If old categories should be combined to new categories the border length index (Fuchs, 2002) can be used. The index gives a probability for the common appearance of two classes/categories and consequently a type of similarity meas-

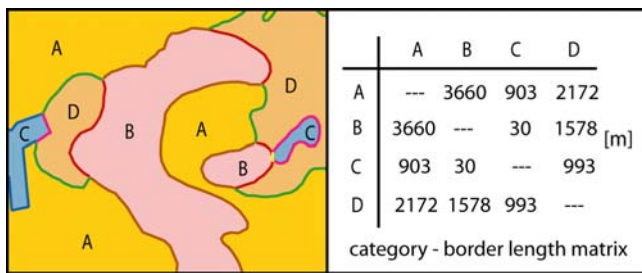


Fig. 7. A section of a landscape and its configuration described by the category-border length matrix. The matrix is used for different *spatial configuration* indices.

ure. The *interspersion and juxtaposition* index (IJI) by McGarigal and Marks (1995) measures the interspersion of a landscape and is not necessarily affected by patch size, contiguity or dispersion. While the IJI can be used for raster and vector analysis, the *contagion* index (Li and Reynolds, 1993) can only be applied to raster analysis. IJI and the contagion index could be used for maintaining the typical patch structure (though the latter only in the raster case).

#### 4) Semantic relations

Consideration of semantic relations (Table 4) is important for categorical map types. If a choropleth map is directly derived from GIS data or if a small scale map should be derived from a medium scale map, then the first stage of generalisation is *structure analysis* (Steiniger and Weibel, 2005b). In contrast to the generalisation of topographic maps no fixed classification scheme is given which defines the number of categories in the final map. Instead, a number of categories have to be defined during the stage of structure analysis. Therefore it is necessary to define or to derive – by use of data analysis methods – semantic relations between classes. Four types of these relations could be elaborated: **similarity** – for aggregation of categories, **priority** – for generalisation process modelling, **resistance / attraction** – for aggregation of polygons and **causal and logical relations**. Saving defined values or measurement values for the mentioned relations can be done by use of attributes for each category or polygon, respectively. **Similarity** between categories can be represented in three ways. One way is to decide if a class is equal or not equal to another class. Here, the similarity relation of two categories/classes expresses whether both can be aggregated (or not) in the case of excessive information density (see previous section). Another way of similarity representation is to define an aggregation-tree structure. An example for use of such a tree structure is the sorting of fruits by colour, size, shape and taste as given in Duda *et al.* (2000, p. 395). The third possibility to define similarity is by use of pattern classification methods (e.g. clustering techniques). A condition, therefore, is that every legend unit can be described by a number of different properties (shape, colour, etc.). These properties span a *n*-dimensional *feature space* where the distance between two categories can be seen as measure of semantic similarity (*n* denotes the number of properties). Usually the

feature space is linked to a probability model, since the property values of the objects of one map unit have a different variability in each property direction (Duda *et al.*, 2000, p.538). Fig. 8 shows why a probability model should be applied. A viewer of this figure would probably percept 3 clusters in the feature space, which can be seen as 3 categories (grey round areas in Fig. 8). Had we used the pure distances only, as the semi-circle around the centre of cluster (3) indicates, however, then at least the cluster (2) would belong to the class of the lower cluster (1).

**Priority** relations are used in the generalisation process to give some special object class more importance than others, e.g. a small polygon of a rare soil category should be exaggerated instead of being eliminated. Thus, the neighbouring soil polygons with lower priority are down-scaled. The priority value of an object class does not act like the importance or priority values of constraints in the agent modelling approach (Galanda, 2003), rather the value initialises constraints and affects their importance and priority values.

The **resistance and attraction relation** is necessary if two map objects should be aggregated. It defines whether neighbouring polygons are aggregation candidates or not. The resistance relations can be either defined by the user (stored as object class matrix) or obtained from semantic similarity, class priority and statistic relations.

**Causal and logical relations** describe category dependencies of object classes within a single theme. Causal relations are used if map objects should be eliminated or classes aggregated during the generalisation process. An example for causal relation is that the definition of an island is bound to a surrounding water area. Another one would be that an area denoted as beach needs as well a water area next to it. The storage of such relations can be accomplished in a class relation matrix which is usually not symmetric, since for example a water area does not necessarily end at a beach. This relation

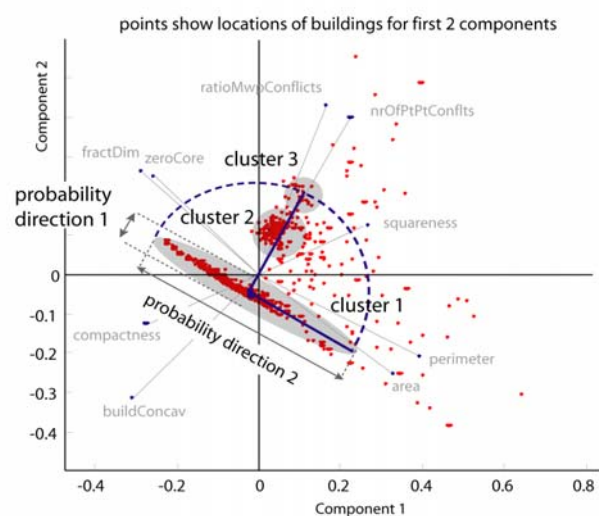


Fig. 8. The definition of *similarity* in feature space should contain distance and probability. Otherwise the objects of cluster 2 would belong cluster 1.

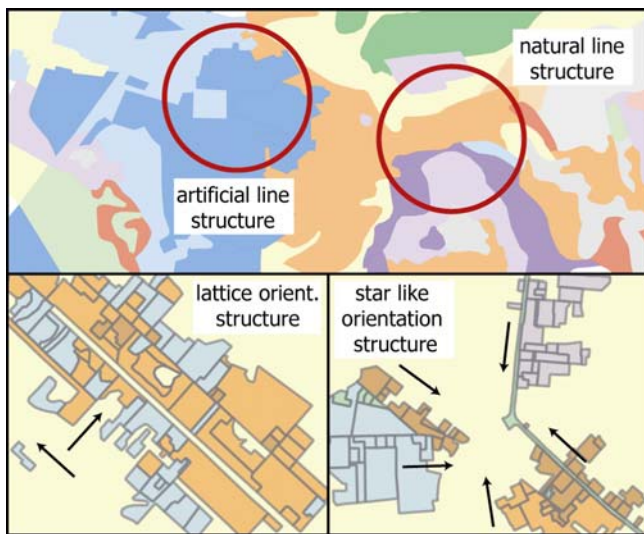


Fig. 9. *Structural relations* and properties. The upper picture shows artificial and natural polygon structures from a German soil map. The lower pictures present two examples for orientation patterns in a land-use dataset from Delaware. Here, the orientation patterns are induced by the road network.

meets two processing constraints. First, it prevents illogical results and second it can avoid further generalisation conflicts.

##### 5) *Structural relations*

These relations listed in Table 5 are intimately connected to Gestalt theory (Wertheimer, 1924) and visual perception. As the word “structure” suggests the relations of this group should classify types of patterns which are perceived in maps. A distinction is made into six relations. One is the **background-foreground** relation, describing the visual order of adjacent objects (e.g. visual weight by colour; cf. Dent, 1990). Furthermore we distinguish the structural relations: **nature of origin**, **orientation patterns** (see Fig. 9), **inter-thematic relations**, **macro structures** and **meso structures**. The use of relational objects for modelling and storage is proposed in Table 5, but not for nature of origin. This relation is expressed by three nominal values: *without structure*, *artificial structure* and *natural structure* (see Fig. 9), which should be saved as attribute value of the specific map entity that the nature of origin relation is associated with. Linked to a specific constraint of Galanda’s (2003) list for polygon generalisation are only orientation patterns and meso structures. This particular constraint should preserve the relative configuration but is not sufficiently formalised by Galanda (2003) in checking the number of polygonal objects of a group agent. Thus, a refinement of the polygonal constraints for consideration of higher order features is necessary (cf. Section IV).

To define a constraint for **inter-thematic relations** might be difficult since the possible relations between two themes can be very diverse. Examples for inter-thematic links can be found in soil maps – e.g. the dependence of soil class on the underlying rock category or the alignment of gravel soils to rivers. Thus, for the latter situation, movements of the river during generalisation induce also changes on the soil entity.

However, inter-thematic relations also appear in weather or snow depth maps (cf. Fig. 6), where a strong connection to topography exists.

**Macro structure** relations are visible patterns if the map reader has information about them. They can not be compared to the macro objects for agent based map generalisation (Ruas, 1999). Macro structures are often not directly manifested and visible on a map of a given scale as they relate to a different (macro scale) level and resolution than the given map. Examples are topographic landscape structures formed during ice age. They can hardly be perceived in soil maps of scale 1:25’000 but are obvious in soil landscape maps of scale 1:500’000. A detection of such structures in high-resolution map data with data analysis methods is difficult to accomplish, since the object granularity is too high (‘one cannot see the forest for the trees’). Despite this, the influence of such large structures on map design issues is high since the cartographer will try to use them as structuring component. Consequently, a person who knows about such macro structures will try to abstract them, even on large scale maps. Information on macro structures is especially needed for small and medium scale map derivation from large scale maps. The integration into the generalisation process could be achieved by defining special constraints and by keeping the model generalisation process interactive (e.g. using workflow models instead of agent based modelling; cf. Steiniger and Weibel, 2005).

In contrast to macro structures, **meso structures** are visible and detectable by the use of pattern analysis methods applied to structural characteristics. Examples of meso structures are shown in Fig. 1 (top) and Fig. 10. For meso structures, a differentiation can be made into patterns that are obvious to every map reader (e.g. four aligned lakes) and visual thematic patterns, only obvious to the experts, familiar with the particular theme. Besides this grouping a classification can be done into meso structures composed of entities of one class or of multiple classes, respectively. Furthermore, a sub classification is possible by consideration of the meso level shape of a pattern, whereby *parallel* or *curved alignments*, *clusters* or *layers* can be distinguished. Since the identification and preservation of typical structures and special feature alignments is very essential to obtain good cartographic generalisation results, a more detailed characterisation of meso structures follows in Section IV.

A remark on differentiation between the relation types *causal and logical relations*, *inter-thematic relation* and *meso structures* might be necessary. Whereas causal and logical relations describe dependencies among map categories within one theme meso structures are defined as relations on the map object level. Inter-thematic relations can be defined on both, the category level and on the map object level as well.

#### IV. A CHARACTERISATION OF MESO STRUCTURES USING GESTALT THEORY

Meso structures represent a special property within the map image. They represent a visual weight (Arnheim, 1954) and with it an attraction on the viewer. This leads to the question:

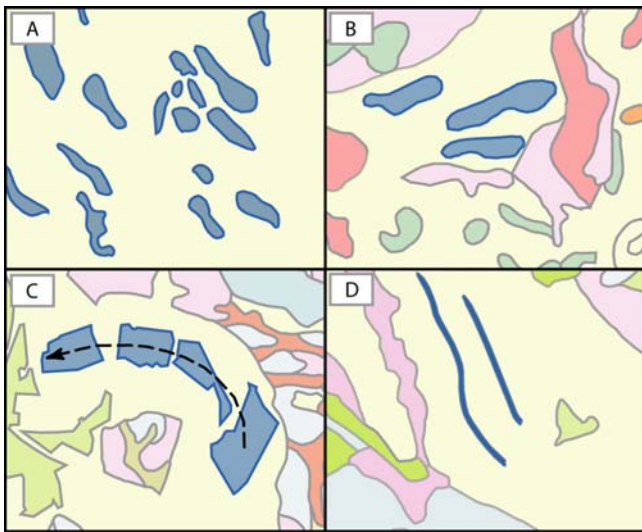


Fig. 10: Examples of laws of organisations and consequently the principles forming meso structures. A) A cluster of polygons in the top right formed by the *law of proximity*. B) Three polygons identified as a meso structure using the *principle of similarity*. The polygons are of same category and also nearly similar in shape, orientation and size. C) The *direction of an alignment* has not necessarily to be straight. It also might be circular. D) The effect of the *law of Prägnanz* makes the two long and thin polygons appear as parallel objects. Of course they are only nearly parallel if orientations are calculated.

#### What forms a visual weight?

We know from theoretic cartography that visual weight can be generated by visual variables. The variables listed by Bertin (1983) are (1) shape, (2) orientation, (3) colour, (4) pattern, (5) size and (6) brightness – and extensively described in his book. For thematic map making the visual variables play a major role, since the user's attention has to be guided to the important information. Dent (1990) gives rules for thematic maps on how to use the visual variables to form a visual weight. For the detection of structures from GIS data these rules are not so relevant since only some of the visual variables can be associated to the data. **Shape, orientation and size** are visual variables that can be connected to geographic data. Replacing the visual variable colour by class, or category respectively, results in four characteristic properties, the TOSS (type/category, orientation, size, shape) structure (Atwood, 2004). These four variables describe possible ways of how a *single* map object can attract the attention of the map reader. However, the TOSS structure does not describe how cognitive structures ('mental maps') are built during the process of perception and how we can use the four TOSS variables for the detection of groups. Thus we have to ask: *What laws form a group (meso structure) and which properties does such a group have?*

Gestalt theory has extensively dealt with this question, beyond the perception of visual groups. Especially Wertheimer (1923) developed a list of *laws of organisation in perceptual form*. Some of these will be discussed below and linked to the horizontal relations. Hereby, linking means to put the law in relation to the characterisation of map structures and map generalisation. As we will see, these laws or properties, respec-

tively, of groups will make use of the four visual variables mentioned above.

1) The law of **proximity** expresses that objects being part of one structure have usually short (spatial) distances to each other with respect to other objects (c.f. Fig. 10, picture A). This law can be linked to two sets of relations, including the *position variables* of the metric relations and the topological relations *order of neighbourhood* and *ring model*. Hence, proximity among members of a group can be characterised by these relations.

2) The second law identifies **similarity** among the individuals of a group. Similarity can be defined with respect to the four visual variables: type (category), size, shape and orientation (c.f. Fig. 10, picture B). Thus, using the similarity principle for structure analysis the geometric relations including *size, shape and orientation variables* could be used. Further the *similarity* of categories may be given by the semantic relation with the same name.

3) The **common destiny** specifies the effect of an action on groups. The possible effects are the distortion of a structure or the preservation of a structure. Linking this law to horizontal relations is hard to achieve since it rather represents the constraint for preservation of relative configuration. Therefore it expresses a goal (or constraint) of generalisation process.

4) The law of '**Prägnanz**' describes the classification of recognised structures into natural configurations. An example is the consideration of an alignment as a vertical one which has an orientated structure with an average near-vertical direction of 95°. Another example of two lines which are considered as parallel is shown in Fig. 10, picture D. This law connects structure analysis on one hand to template matching of predefined *orientation patterns* and on the other hand to *nature of origin*. Besides this, *Prägnanz* can be seen as a certain degree of flexibility in map object modification during generalisation. It provides an indication of the acceptability of small changes. Since *statistic and density relations* can be used either to describe pattern homogeneity and vice versa heterogeneity, or to describe changes in patterns, *Prägnanz* links structure analysis as well to this kind of relations.

5) The structure property **objective set** refers to the issue that structures might be seen differently from a different perspective. Most importantly it provides a hint for structure detection to consider that specific map objects can be part of different meso structures.

6) **Direction of alignment** can be regarded as a property of the whole meso structure. The overall orientation does not necessarily have to be straight since curved structures can appear as well (c.f. Fig. 10, picture C). For structure analysis different kinds of orientation and alignment patterns have been identified (see *meso structure relation* in Table 5).

7) The law of **good Gestalt** (good continuation) describes different properties which support the visual recognition of a meso structure. These are simplicity, closure, equilibrium and symmetry. It is important to note that these four properties are features of the structure as a whole and not properties of group individual. Furthermore should be mentioned that the circle is

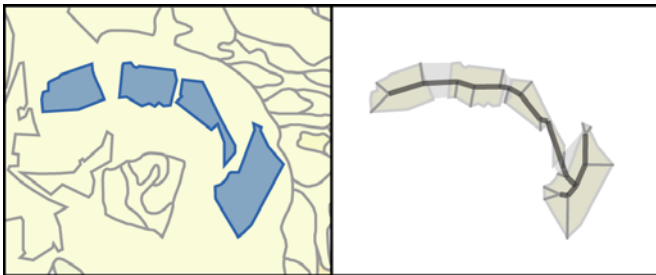


Fig. 11. A polygon alignment (left) and its *structural skeleton* (right). The skeleton algorithm is described in Petzold *et al.* (2005) and is accessible as web generalisation service (Neun and Burghardt, 2005).

the simplest shape (see Arnheim, 1954) and fulfils all four requirements of good gestalt. Hence, circular meso structures tend to be discovered first by the map reader.

*Good Gestalt* and *direction of alignment* can obviously both be linked to metric relations, particularly to the groups of *position* and *orientation*. Now, one might ask: Why not the group of shape measures? An answer is given by Arnheim (1954): ‘Although visual shape of an object is largely determined by its outline, the boundaries cannot be said to be the shape.’ The intended *real* shape is ‘the *structural skeleton* created in perception by the material shapes’ (Arnheim, 1954). Thus, the shape measures of the metric relations describe a different kind of shape which characterises single objects and not structures. For organisations an extraction of the structural skeleton is necessary to evaluate the criteria of good Gestalt and to compute the direction(s) of an alignment. Fig. 11 shows an example of a structural skeleton for an alignment. An approach for extraction of the structural skeleton is, for instance, given by Gold (1992) in conjunction with the use of Voronoi regions and pattern recognition methods.

8) A further law specifies a possible grouping of objects based on **past experiences** of the map reader. The detection of structures using pattern templates (see Table 5: *orientation patterns*) is one link to horizontal relations. Further, the relations *subject specific meso structures* and *macro structures* do likewise reflect the past experiences of map users. The law of past experiences emphasises the need of these three relation types for map generalisation if the information content should be preserved.

9) Finally, the **figure and ground differentiation** describes the issue of visual weight. According to Wertheimer the background is an object which continues under the figure. Whereas the meso structure, the organisation, represents the figure (see also Dent, 1990). The corresponding horizontal relation used for evaluation of the principle during structure analysis is the *background-foreground relation* (Table 5).

In conclusion one can say that the laws of similarity and proximity and their corresponding relations are the **building blocks** of organisations and of meso structures, respectively.

Thus, we **define meso structures** as a group of at least two single map objects (plus a background object) which have at least two similar visual properties (category and/or geometry)

and an orientation relation which fulfils the principle of *Prägnanz* or past experience. An example is given in Figure 10 (picture D), where two thin polygons are of the same category, have similar shape characteristics and have a quasi parallel orientation. Does a structure consist of three or more single map objects then the requirement of a prominent orientation relation is replaced by the law of proximity.

The laws of proximity and similarity have already been used by several authors for topographic map generalisation. Especially Regnaud (1998), Boffet (2001), Christophe and Ruas (2002) and Li *et al.* (2004) used these for the detection of building alignments. The laws of *Prägnanz*, direction of alignment and good continuity, as well as their corresponding relations describe the **quality** of a meso structure (Thompson and Richardson, 1999). Further, the basic statistic relations can be used for the evaluation of candidate structures to assess their homogeneity. The remaining laws either pay attention to specific issues of processing (objective set, past experience, figure to ground) or describe the objective of pattern detection for generalisation (common destiny).

## V. CONCLUSION

In presenting a comprehensive typology of horizontal relations for categorical maps, a first step to automated generalisation of such map types has been made. It has been shown that horizontal relations can be used to characterise map data and that they describe structural information. Additionally, the possible use of horizontal relations to detect meso structures has been proposed by linking Wertheimer’s laws on organisations to horizontal relations.

The immediate objectives for future work are the definition and the identification of further group structures and the assessment of measures for the relations defined in this paper. As mentioned in Section (IV.C.1) on metric relations this may include a comparative evaluation of different measures to find the most appropriate one(s). Establishing a set of basic meso structures may be useful to study possible methods of pattern recognition either by bottom-up approaches (e.g. clustering or buffering in feature space) or by top-down approaches (e.g. template matching by use of search trees).

Finally, generalisation constraints on pattern preservation should be developed, the correlations between these constraints studied, and corresponding measures analysed. Initial investigations on the latter tasks with respect to characterisation of settlement structures have already been accomplished (cf. Burghardt and Steiniger, 2005).

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## APPENDIX – RELATIONS IN CATEGORICAL MAPS

## A. Possible storage types for relations

- attribute value, where (..) describes the object to which the attribute should be attached to
- RM = Relation Matrix: RMs = symmetric matrix, RMa: asymmetric matrix
- RO = Relation Object

## B. Relation data types:

(1) geometry, (2) boolean, (3) nominal, (4) ordinal, (5) interval = integer, (6) continuous

## C. Constraints for Polygon Generalisation (Galanda, 2003)

- **Metric constraints:** **M1** - Consecutive vertex distance, **M2** - Outline granularity, **M3** - Distance between boundary points, **M4** - Minimal area, **M5** - Respect spatial context – prevent follow up conflicts, **M6** - Object separation, **M7** - Number of categories
- **Topological constraints:** **T1** - Self-intersection, **T2** - Intersection of different polygons
- **Structural constraints:** **S1** - Shape distortion, **S2** - Absolute position, **S3** - Relative configuration, **S4** - Size ratios
- **Procedural constraints:** **P1** - Illogical results, **P2** - Child entity's constraints, **P3** - Aggregation similarity, **P4** - Equal treatment

**bold:** constraints connected to relations

GALANDA, M.; 2003: Modelling Constraints for Polygon Generalization. *Fifth Workshop on Progress in Automated Map Generalization*, Paris (France).

type of metric variable	metric variable	Measurement		possible storage of measurements	related constraints (Galanda, 2003)
		possible measures	data type		
Size variables	comparison of absolute variables*				
	Area	algorithms of computational geometry	continuous	attribute (polygon)	M4: ensure minimum area, S4: preserve size ratios
	Diameter		continuous	attribute (polygon)	X
	Perimeter	summed length of edges	continuous	attribute (polygon)	X
	Length	Minimum Bounding Rectangle (MBR), longest edge	continuous	attribute (polygon)	X
	comparison of relative variables**				
	length of adjacent edge among 2 polygons	topologic query and measure of edge length	continuous	RMs (polygon)	X
position variables	comparison of absolute variables*				
	absolute position (Cartesian, polar)	basic	continuous	attribute (polygon)	S2: preserve absolute position

	comparison of relative variables**					
	distance in 2d and 3d space (euclidean, spherical, ellipsoidal)	basic geometry algorithms (centroid or Face to Face)	continuous	RMs (Map)	M6: ensure object separation, S3: preserve relative configuration	
	Hausdorff distance	see Hangouet (1995)	continuous	RMa (Map)	X	
	cost distance (not necessary metrically)	case dependent	continuous	RMa (Map)	X	
shape variables	comparison of absolute variables*					
	shape of polygon area	original shape	compactness, fractal dimension, squareness (see Agent DC1, 1999)	continuous	attribute (polygon)	S1: prevent shape distortion
		Convex hull	area of convex hull, squareness	continuous	geometry and attribute (polygon, RO)	S4: preserve size ratios (for group of objects)
		squared hull (envelope, MBR)	width, length, area	continuous	attribute (polygon)	S1: prevent shape distortion
	shape of polygon boundary	sinuosity, variance of curvature, no. of inflexion points (see Agent DC1, 1999)	continuous	attribute (polygon)	M2: ensure appropriate outline granularity	
	comparison of relative variables**					
	polygon area distances	Turning distance, radial distance	continuous	RMs (map)	S1: prevent shape distortion, S3: preserve relative configuration, M6: ensure object separation	
	polygon boundary distances	angular distance, Hausdorff distance, Fréchet distance (see Agent DC1, 1999)	continuous	RMs, RMa (Hausdorff) (map)		
	special relation to environment					
	core area	negative buffer	nominal (geometry)	geometry	X	
orientation variables	absolute orientation	MBR orientation, wall statistical weight, longest edge (see Duchêne <i>et al.</i> 2003)	continuous	attribute (polygon)	S3: preserve relative configuration	
	special relations between two objects					
	equal, not equal, parallel, orthogonal, 45°	basic	nominal	RMs	S3: preserve relative configuration	

\*) *Absolute variables* are properties of a single object. A relation is obtained by either comparing the variable values of two map objects or by comparing the property value against a threshold value. Thereby a classification of the comparison result into 'equal', 'smaller' and 'larger' could be done (see also the *similarity relation* of Table 4).

\*\*\*) *Relative variables* describe properties among two objects, thus they already express relations. They can be compared to each other or against thresholds like absolute variables. A classification of the comparison results may be useful as well.

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type of topological relation	sub-class or description	Measurement		possible storage	related constraints (Galanda, 2003)
		possible measure	data type		
topological structure	island polygon, island cluster, landscape mosaic	identification via neighbourhood graph	nominal	attribute (map, polygon)	S3: preserve relative configuration
order of neighbourhood	n'th-neighbour	Topological adjacency query	interval	RMa	X
intersection type	9 Intersection Model or DE-9IM: (1) disjoint, (2) touch, (3) cross, (4) within, (5) overlap, (6) contain, (7) intersect, (8) equal	(see Clementini <i>et al.</i> 1993; Egenhofer and Herring 1991, OGC Simple Features Specification 1999)	nominal	RMs (matrix of intersection matrices)	T2: prevent polygon intersection
ring model	for 2 map objects (background and foreground object): Island	number of nodes	boolean	attribute (polygon)	M6: ensure object separation
	for no. map objects $\geq$ 3: Ring model of continua	identification via neighbourhood graph	boolean	RO	S3: preserve relative configuration

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type of statistical relation	index	measurement		possible storage	related constraints (Galanda, 2003)
		calculation & references	data type		
statistical base parameters	count (number of polygons, <i>richness</i> = number of categories)	landscape ecology: McGarigal and Marks (1995); soil sciences: Finke <i>et al.</i> (2001)	interval	attribute (map, map section)	X
	mean		continuous		
	variance				
	skewness				
	median				
area relations (evenness relations)	item area probability	area of polygon to all polygons of all categories	continuous	attribute (polygon)	S4: preserve size ratios
	item area-category probability	area of polygon to all polygons of own category		attribute (polygon)	
	evenness	area of own category to all categories (McGarigal and Marks, 1995)		attribute (category)	
	patch richness density	number of categories to overall-area, (McGarigal and Marks, 1995)		attribute (map, map section)	

category relations (frequency and sparseness)	relative patch richness	categories of map section to all categories (McGarigal and Marks, 1995)	continuous	attribute (map section)	X
	item-category probability	1 / number of items of own category		attribute (polygon)	
	category probability	number of items of own category to all items		attribute (category)	
diversity metrics	Shannon diversity index	entropy in general: Shannon and Weaver (1948); entropy in data mining: Duda <i>et al.</i> (2000) entropy in landscape ecology: McGarigal and Marks (1995); entropy in soil science: Ibáñez <i>et al.</i> (1995); entropy in cartography: Bjørke (1996)	continuous	attribute (map, map section)	M7: ensure appropriate number of categories, S3: preserve relative configuration
	Shannon evenness index				
	modified Simpson diversity index				
	modified Simpson evenness index				
configuration metrics	border length index	length of perimeter between two categories (Fuchs <i>et al.</i> 2002)	continuous	RMs (category)	supports P3: minimum aggregation similarity
	contagion index	raster based, Li and Reynolds (1993)		attribute (map, map section)	S3: preserve relative configuration
	interspersion and juxtaposition index	vector based: McGarigal and Marks (1995);			

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TABLE 4 - SEMANTIC RELATIONS

type of semantic relation	sub class or description	Determination		possible storage	related constraints (Galanda, 2003)
		method	data type		
similarity (for aggregation of legend units)	equal / not equal	per definition	boolean	attribute (category)	P3: minimum aggregation similarity
	in intervals from: weak to strong	per definition, for nominal data as tree structure (see Duda <i>et al.</i> , 2000, p. 395)	interval		
	weak to strong, possible transfer probability	probability model and/or metric dependent distance of n-dimensional feature space (see Duda <i>et al.</i> , 2000, p. 538)	continuous		
Priority (on category and meso level; for process modelling)	not important to very important	per definition for category, thematic layer or structure (see Table 5)	ordinal or interval	attribute (category, RO) or RMa (category)	process constraint

resistance / attraction (for aggregation of polygons)	from high resistance over neutral to attraction	per definition	ordinal or interval	attribute (category, polygon)	P3: minimum aggregation similarity
		calculated from other relations (especially statistics)	continuous		
causal and logical relations	examples: island and water; beach and sea	per definition	boolean	RMa (category), attribute (category)	P1: prevent illogical result M5: respect spatial context

DUDA, R. O., P. E. HART and D. G. STORK; 2000: *Pattern Classification*. 2nd edition, John Wiley, New York.

type of structural relation	sub class or description	Determination		possible storage	related constraints (Galanda, 2003)
		possible definition / measuring	data type		
background-foreground relation	visual appearance, Figure to Ground segregation (see Dent, 1990)	Possible measure: area relation within a moving window	ordinal	RO (polygon level)	X
	semantic importance	defined by priority or by order of data layer	ordinal	RO (class level)	
nature of origin	map level				
	macro level (map, map section)	per definition or use of homogeneity (statistic) measures	nominal: (1) without structure, (2) artificial structure, (3) natural structure	attribute (map, map section)	X
	meso level (meso structure, pattern, alignment)			attribute (RO)	
	micro level (polygon, polygon outline segment)	per definition or use of sinuosity and squareness measures		attribute (polygon, outline segment)	
orientation patterns	possible patterns: star like, circular shape, fan, grid, etc.	to be defined	nominal	RO	S3: preserve relative configuration
inter-thematic relation	a) on category level b) on micro level examples: gravel soils next to river, snow level with topography and elevation	per definition	boolean	RO (for map objects), RMa possible for categories	X
macro structures	Visible with background knowledge, but not detectable since data resolution is too high. Important for map design. example: ice age influenced terrain structures on medium scale maps	per definition, (needs to be introduced for follow up generalisation)	X	RO	X
meso structures	Obvious and detectable structures for everyone. map objects of c) same category (alignment, cluster) d) different category (layers, parallel alignments, curved alignments)	detection by rules of Gestalt theory (Wertheimer 1923, Regnauld 1998) and use of pattern detection techniques	boolean or nominal (nominal, needs identification and definition of primary structure types)	RO	S3: preserve relative configuration
	not obvious structures for non specialists	specification by specialist	X		

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