Relations among Map Objects in Cartographic Generalization

Stefan Steiniger and Robert Weibel

ABSTRACT: Adequate representation of cartographic expert knowledge is essential if maps are to be created in an automated way. Part of this expert knowledge is made up by the structural knowledge embedded in the relations that exist among the objects depicted on a map, as these define the structures and patterns of the corresponding real-world objects that should be maintained and emphasized in the cartographic generalization process. With this article we aim to provide a foundation for the analysis and representation of such relations among objects in thematic and topographic maps, which we term horizontal relations. We start off by defining the terminology underlying map object relations and by discussing how these relations interact with map constraints and cartometric measures. We then present a typology of horizontal relations that may be found in a map with respect to map generalization. The typology is the result of a study of thematic and topographic maps, as well as an analysis of the literature on the use of map object relations. Five different types of horizontal relations are identified: geometric, topological, semantic, statistical and structural. Some of these can be based on standard operations available in commercial GIS or mapping systems, while others are less easily accessible. To demonstrate the use of our typology and show how complex horizontal relations can be formalized, we present an application of the typology to the grouping and generalization of islands. Subsequently, we discuss the various steps involved in the usage of horizontal relations in map generalization, as well as their associated roles.

KEYWORDS: Map generalization, map object relations, horizontal relations, structure recognition, data enrichment, cartometrics

Introduction

In the last decade, research in automated map generalization reached a point where automated methods were continuously introduced into map production lines. Reports on the successful and ongoing integration of automated map generalization procedures have been published, among others, for the production of topographic maps at the Institut Géographique National, France (Lecordix et al. 2005) and the Ordnance Survey of Great Britain (Revell et al. 2006). Most of the automated procedures used in operational production lines, however, are limited to rather isolated operations, or they are applied independently to individual map objects (e.g., shape simplification) or to objects of a single object class (e.g., typification of buildings).

While it is possible to achieve considerable productivity gains with such generalization operators (Lecordix et al. 2005), it is also clear that further progress can only be made if research delivers solid solutions for *contextual* generalization operators (i.e., operators taking into account their spatial context), as well as for the *concurrent* treatment of multiple object classes (i.e., operators considering the mutual relations among objects of more than one class). Although the development of contextual operators for individual object classes is on the way (e.g., Ware and Jones 1998; Bader et al. 2005), the development of methods that can deal with multiple object classes is still in its infancy. One of the rare examples is Gaffuri (2006) who reports on a first attempt to treat simultaneously different object classes. We argue that an agreement about the kinds of spatial and semantic relations that exist among objects in a map, as well as methods to formalize, detect, and represent such relations, will be essential prerequisites to the progress of research in this area.

A simple example of four lakes, shown in Figure 1, should help to illustrate the necessity of representing the structural knowledge embedded in contextual, inter-object relations. A legible map should meet several visual requirements, including that map objects should have a minimum size to be unambiguously perceived by

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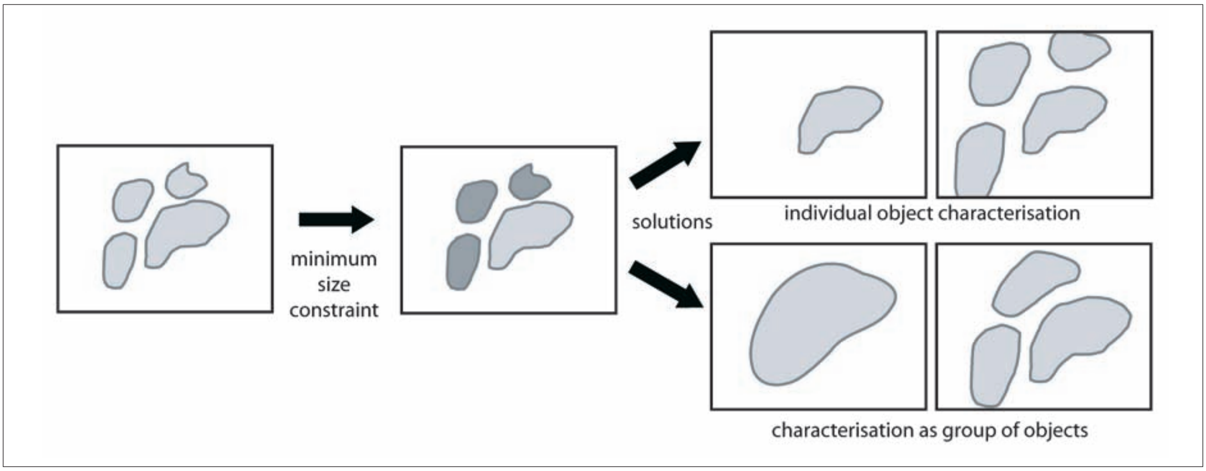


Figure 1. Different generalization solutions when contextual relations are ignored (top-right) and observed (lower-right).

the map reader. In our example we assume that three of the lakes would not meet this constraint for a particular target scale, and we have to decide how the problem can be solved. On the top right of Figure 1, two simple solutions are shown that ignore the contextual situation—deleting the three small lakes or enlarging them individually until they each reach the minimum size. These solutions both meet the basic perceptual requirement (of minimum size), but they do not necessarily represent a good cartographic solution from a structural point of view. A more adequate solution would be to maintain the typical structures or patterns that extend across map features and thus emphasize the specificities of the map. Such a solution can only be obtained by considering inter-object relations. Both solutions shown in the lower-right corner of Figure 1 better preserve the typical properties of the spatial arrangement, as well as the size and shape relations, among the objects involved.

In this article, we propose a typology of relations among map objects aimed to act as a foundation for future research on developing new methods for contextual generalization involving objects from multiple object classes. The typology should offer a basic set of elements to represent the structural knowledge necessary to characterize the types of relations occurring in both topographic and thematic maps, and inform the selection and parameterization of contextual generalization operators.

The idea outlined above, to characterize a map with relations and to store the characterization results to support subsequent decision processes, has been pursued by several other authors. In the map generalization community the idea is generally known today as “data enrichment” (Ruas and Plazanet 1996; Neun et al. 2004) and the sub-process of context analysis is known as “structure recognition” (Brassel and Weibel 1988) or “structure analysis” (Steiniger and Weibel 2005a). Even though data enrichment and associated processes have been around for a while, to our knowledge, no author has as yet attempted to establish an inventory of possible map object relations. Until recently, the discussion of (spatial) context relations in map generalization has either remained on the general level (Mustière and Moulin 2002) or it focused on the analysis of rather specific scenarios. Examples of the latter include the detection of groups of buildings and the modeling of relations between roads and buildings (Boffet 2001; Regnauld 2001; Duchêne 2004).

The remainder of the paper is organized as follows. The next section introduces the necessary definitions as a foundation of the subsequent sections. The third, central section introduces the proposed typology of horizontal relations. It starts off with a short review of existing, related typologies in order to derive the structure of the proposed typology. Following that, the set of relations is presented, and existing work is discussed. In order to demonstrate the utility of our typology and show how complex relations can be formalized, we then offer an example on the grouping and generalization of islands. This is followed by a section discussing the various steps of the utilization of map object relations, including directions for future research. Finally we summarize the main insights of the paper. Note also that an extended version of the proposed typology has been presented in Steiniger and Weibel (2005b).

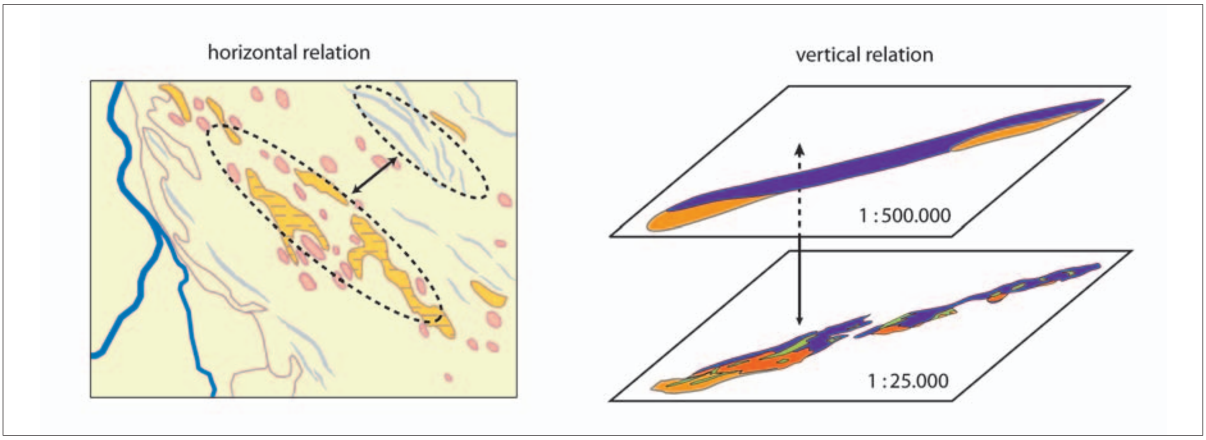


Figure 2. Horizontal relations (left) and vertical relations (right) in categorical maps. [Data: © FOWG (for an explanation of acronyms see Acknowledgments)].

Defining Object Relations in Maps

Before we present our typology, it is necessary to define the underlying terminology. We start with definitions of the different types of relations that are particularly relevant in the context of map generalization and multiple representations. Then, we discuss the interactions between relations, constraints, and measures.

Horizontal, Vertical, and Update Relations

In mathematics, "relations" denote arbitrary associations of the elements of one set with the elements of other sets. Depending on the number of sets involved, the relations are termed unary (involving only elements of one set), binary (involving associations of elements of two sets) or *n*-ary (involving elements of multiple sets). While we embrace the mathematical notion of the term "relation," we are only interested in those relations that are relevant for map generalization. In map generalization, the notion of scale, resolution or level of detail (LOD) plays a crucial role, leading to the definition of the first two classes of relations, termed *horizontal* and *vertical relations*, respectively. Because map generalization is a process leading to modifications of the content of a map or map database, we further define *update relations* as a third relation class.

Horizontal Relations

These relations of map objects exist within a single scale, resolution, or level of detail, and they represent common structural properties—e.g., neighborhood relations and spatial patterns (Neun et al. 2004). For instance in a geological map, polygons of a particular rock type that are close to each other form a group, while polygons of another rock type that are also close to each other form another group (see Figure 2). The rock polygons now have a relationship to the groups, being part of the group or not, and the two groups of rocks have a relationship to each other as well (e.g., an exclusion relation, and a distance relation).

Vertical Relations

This class of relations links objects and groups among different map scales, resolutions, or levels of details. For instance, polygons of a particular soil type in a 1:25,000-scale geo-database are linked to the generalized soil polygons in a 1:500,000-scale database (see Figure 2, right). Note that the cardinality of such relations may vary between nullary, unary, and *n*-ary. Thus, a soil polygon at 1:25,000 may not have a homologous object at 1:500,000; it may have exactly one correspondent; or several polygons at 1:25,000 may be aggregated to one polygon at 1:500,000.

Update Relations

This relation class is used to describe changes of map objects over time. According to Bobzien et al. (2006), the update relation has three states: insert, remove, and change. As an example of the application of this relation, one might think of a building that has been newly constructed (action: insert), extended (action: change), or knocked down (action: remove), with the last revision of the corresponding map or spatial database having been published.

The concepts of horizontal, vertical, and update relations are not new. For instance, horizontal relations—though not termed that way—have been extracted and utilized by Gaffuri and Trévisan (2004) for the generalization of buildings and settlements in the form of towns, districts, urban blocks, building groups, and building alignments. Vertical and update relations are a well known concept used in Multiple Representation Databases (MRDBs). The use of vertical relations (commonly termed "links" in MRDB literature) has been demonstrated, for instance, by Hampe and Sester (2004) for the display of topographic data on mobile devices. Update relations that describe propagated updates of data within a MRDB were initially described by Kilpeläinen and Sarjakoski (1995).

A note should be made here on the naming of relation classes: We use the terms "horizontal relations" and "vertical relations" because we believe them to be intuitively (and linguistically) understood as terms that form a pair, yet are different. Obviously, these terms should not be understood in the geometrical sense; rather, as a stack of data layers (or maps) of different scales, where horizontal relations only affect a single layer (or resolution), while vertical relations extend across the entire stack of (resolution) layers. Other, equivalent terms have also been used, such as "intra-scale" and "intra-resolution" for "horizontal" and "inter-scale" and "inter-resolution" for "vertical" (Bobzien et al. 2006).

This paper intends to offer a more comprehensive and systematic discussion of horizontal relations in map generalization than available from previous research, which tended to focus on specific instances of horizontal relations, neglecting the more holistic view. Thus, the typology proposed below will focus exclusively on horizontal relations. As has been argued in the introductory section, we believe that a systematic analysis of the types of relations that exist among objects of a map (i.e., horizontal relations) will be instrumental to the further development of more complex, contextual generalization techniques. Vertical and update relations are not addressed further in this paper.

Relations, Constraints, and Measures

Together with the generalization algorithms, relations, constraints, and measures represent the fundamental parts of an automated generalization system. More specifically, the triplet relations-constraints-measures forms the basis for controlling the application of generalization algorithms, that is, the selection of appropriate generalization algorithms to remedy a given conflict situation, including suitable parameter settings. While it should be clear what (generalization) algorithms do, it seems to be useful to define measures and constraints and explain their interaction with relations.

Cartographic *constraints* are used to formalize spatial and human requirements that a map or a cartographic map feature needs to fulfill (Beard 1991; Weibel and Dutton 1998). Examples are the minimum size constraint of an object (e.g., a building) or part of an object (e.g., a building wall), or the maximum displacement constraint to preserve the positional accuracy of a map object. Certain constraints may be termed "hard constraints" (e.g., in generalization, a house must not change sides of the road along which it lies). Their evaluation will thus lead to a binary result (fulfilled / not fulfilled). Most constraints, however, will be "soft constraints," meaning that slight violations may be tolerated. A constraint can be described by a *measure* that appropriately captures the property expressed by the constraint (e.g., the area of a building as a measure of the size constraint). The degree of violation of a constraint can then be evaluated by calculating the value of the associated measure and comparing that value to a target value that should be met for an optimal map at the target scale. The deviation of the actual from the target value will then yield a normalized "severity" (or, conversely, satisfaction) score expressing the degree of constraint violation (Ruas 1999; Barrault et al. 2001).

While the interactions between constraints and measures have been studied by various authors (e.g., Ruas and Plazanet 1996; Ruas 1999; Harrie 1999; Bard 2004), we would like to extend this discussion by examining the roles and interactions in the triangle of constraints, measures, and *relations*, as shown schematically in Figure 3. We use the (simplified) example of a set of buildings that are aligned in a row, assuming that we would like to preserve this particular pattern in the generalization process.

The spatial arrangement of the buildings can be seen as a relation of the type "alignment," where every building is related to the group making up the alignment. Within the alignment, further relations can be found, such as distance relations (expressing the distance of the buildings from each other), angle relations (expressing the angular deviation from the alignment axis), size relations (expressing the area of the buildings compared to each other), shape relations (expressing the simi-

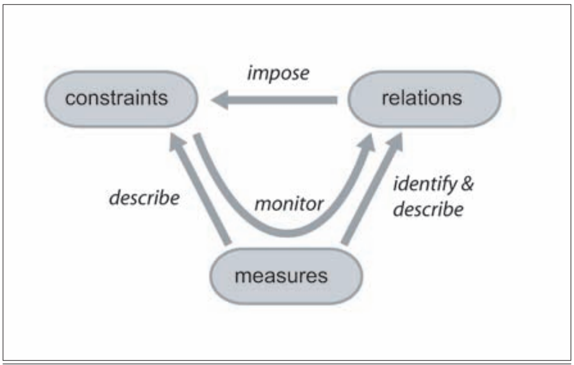


Figure 3. Interactions between constraints, measures and relations.

larity of building shapes), and semantic relations (expressing the similarity of the building types). To *describe and identify* these relations, appropriate measures are required.

Identifying the complex relation "alignment," for example, requires measuring whether the buildings are not located too far from each other (distance relation), whether they are sufficiently collinear (angle relation), whether they are similarly large or small, whether they are similarly shaped, and whether they belong to the same or similar building type. Once the relations have been established, they *impose* constraints on the generalization process, as one of the objectives of cartographic generalization is the preservation of structures and patterns represented in the relations. We have already mentioned that the role of measures with respect to constraints is to *describe* constraints. Hence, because relations are imposed on the generalization process as constraints, measures are used by the constraints to *monitor* the evolution of the relations and thus, constraint satisfaction, in the course of the generalization process. Classifications of measures and constraints affect the typology of relations presented in the next section.

A Typology of Horizontal Relations

General Structure Derived from Existing Classifications

A number of classifications of relations have been proposed in GIScience. Examples include the typology of topological relations by Egenhofer and Herring (1991) or the classification of spatial relations by Pullar and Egenhofer (1988), where the latter distinguish between direction relations (e.g., north, northeast), topological relations, comparative or ordinal relations (e.g., in, at), distance relations (e.g., far, near), and fuzzy relations (e.g., next, close). In the semantic domain, taxonomic (*is-a*) relations and partonomic (*part-of*) relations are commonly used in conceptual data modeling.

Although these classifications have proved to be very useful for GIScience applications in general, they are insufficient for cartographic purposes because they focus only on those relations that can be rigorously defined, leading to mutually exclusive and collectively exhaustive classifications. Maps, however, do more than simply portray an ideal world. Depending on their theme and purpose, they attempt to graphically represent a portion of the real world with its associated ambiguities. Also, maps are made by humans for humans who have to rely on their visual perceptions to "read" the messages conveyed by the graphics. Hence, it may be expected that a more comprehensive typology of relations among map objects has to go beyond rigorously definable types of relations, and include those relations that are associated with "human factors," including visual perception and, partially also, cognition. Note that even in some of the more rigorous typologies of spatial relations, such as the one by Pullar and Egenhofer (1988), there exist types whose instantiation will depend on the cognitive experience, such as in distance relations expressed as "far" or "near."

A typology of horizontal relations can be established from a functional perspective or from the scope of usage. Several authors have already proposed classifications of map *constraints* relevant for generalization from both perspectives. The first classification, proposed by Beard (1991) was a functional typology that distinguished between graphical, structural, application, and procedural constraints. This original classification has been revised later by other authors for specific applications (Ruas and Plazanet 1996; Weibel and Dutton 1998; Harrie 1999; Galanda 2003). For instance, the typologies of Ruas and Plazanet (1996) and Harrie (1999) focused on the graphical aspects of map generalization. A constraint typology with respect to the scope of usage has been presented by Ruas (1999), distinguishing between macro level (entire dataset or object class), meso level (group of objects) and micro level (associated with a single object) constraints.

In terms of existing typologies of *measures*, McGarigal (2002) has presented a typology in landscape ecology organized according to the scope of usage of measures. He distinguishes the scopes of patch, class, and landscape. Patch metrics are applied to a region of relatively homogenous environmental conditions, class metrics describe measures for all patches of one category, and landscape metrics are integrated over all patch categories of the entire dataset or a selected frame. In landscape ecology, the metrics are also classified into nonspatial and spatial categories, where the first group is called "composition metrics" and the second "spatial configuration metrics" (Gustafson 1998; McGarigal 2002). Finally, a functional classification for cartometrics has been presented by Peter (2001). He organizes the metrics into size, distance and proximity, shape, topology, density and distribution, pattern and alignment, and semantics.

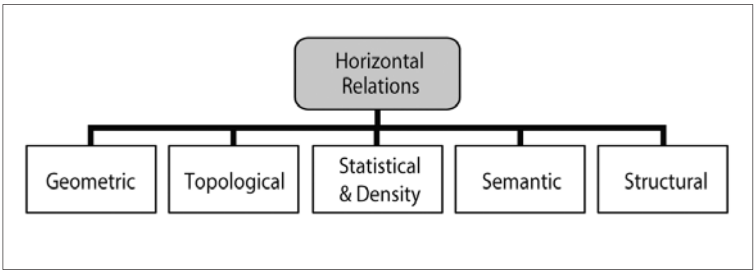


Figure 4. Typology of horizontal relations.

Figure 4 shows the organization of the top-level categories of our typology. It represents a fusion of the functional typologies discussed above, focusing on the commonly used categories. The "geometric" category can be linked to the "graphical" of Beard (1991) and Weibel and Dutton (1998); this category also represents an aggregation of Peter's (2001) categories of size, distance, and proximity. The "topological," "semantic," and "structural" categories are basic categories that have been used in all typologies, except by Beard (2001). The "statistics and density" category can be likened to the "density and distribution" category by Peter (2001). Beard's (2001) "application" and "procedural" categories only make sense when used with constraints, not relations, because relations describe states and not processes.

Methodology

To populate the typology, we used a two-pronged approach. First, we studied the literature on a) existing guidelines on topographic and thematic mapping; b) sets of constraints proposed for topographic and thematic maps; and c) measures used for the evaluation of constraints. Then, we visually analyzed a number of topographical, geological and soil maps, as well as thematic atlas maps, so as to identify relations. If available, we used pairs of maps showing the same area at different scales to identify the steps cartographers had carried out in the map generalization process, and thus gain an understanding of the influence of horizontal relations on generalization decisions. The maps covered a wide range of scales between 1:10,000 and 1:25,000,000.

Before proceeding with the presentation of the typology, two comments seem warranted. First, while we seek to develop a typology of horizontal relations that is as comprehensive as possible, we do not claim it to be exhaustive, for the very same reasons outlined in the preceding subsections, most notably the difficulty of achieving rigor. Second, we assume that the horizontal relations present in topographic maps form a subset of those existing in thematic maps. This assumption is supported by the observation that thematic maps often make use of base maps that are indeed topographic maps, as is the case in geological maps and soil maps.

Horizontal Relations

In the remainder of this section we present a set of relations that should define a foundation for the characterization of geographic data for automated map generalization. Some of the relations and properties of objects are well known and, therefore, need not be explained in detail, while others are briefly discussed. If applications of the corresponding relations have been described in the generalization literature, we will give at least one reference. Because measures are used to describe relations, we also will give references to those, if available. We will make use of the classification of generalization operations proposed by McMaster and Shea (1992) whenever we describe what operations may be supported by a particular type of horizontal relation.

Geometric Relations

Geometric relations originate from the geometric properties or the position of a map object. As shown in Figure 5, within geometric relations one can distinguish between *comparative* and *direct relations*. Comparative relations are established by comparing the values of geometric properties (which themselves are unary relations) of real world objects or with idealized objects (thresholds)—e.g., the size of an area or the length of a line. In contrast, direct relations

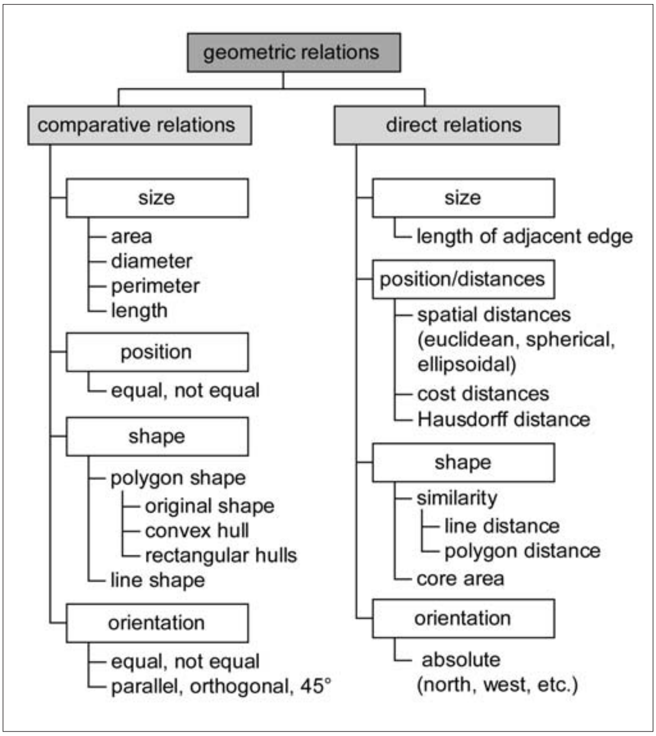


Figure 5. Geometric relations.

express binary relations between objects, such as spatial distances or shape difference measures.

In our analysis of comparative and direct geometric relations we identified four groups of geometric properties that describe a geographic object: *size*, *position*, *shape* and *orientation*. Most of these geometric properties and associated relations are well known in GIScience and in map generalization. Thus, we refrain from going into much detail and point to the literature instead.

Size properties and relations: Area, diameter, perimeter and length are basic properties that describe the size of geometries. They have been used in generalization to evaluate constraints that describe the minimum size of a geometry (or part of a geometry) to be visible on the map. An application of size relations (comparing a measured value to a threshold value) is given in Regnauld et al. (1999) who present generalization algorithms to ensure the legibility of buildings in topographic maps. *Length of adjacent edges* is a specific size relation which measures the length of the common border between two polygons and serves as a basis for the *border length* index. The border length index is a structural relation useful in the evaluation of the similarity among categories such as soils (see Figure 14).

Position relations / distances: Distance relations are used in generalization to evaluate the proximity of map objects. Usually, these relations are applied in map space to evaluate whether two objects can be visually separated, triggering generalization operations such as feature displacement. Alternatively, distances can be used in geographic space to form groups of objects (e.g., clusters of buildings that are close to each other). Distance relations can also be utilized in the so-called feature space, to identify objects with similar properties. Displacement algorithms for solving distance conflicts are described by Ruas (1999) and Bader et al. (2005). Approaches for the identification of building groups based on spatial proximity evaluation have been presented by Boffet (2001), Regnauld (2001), and Anders (2003). Note that most of these techniques use proximity-related, supporting data structures, such as the Delaunay triangulation or Voronoi diagram, to represent distance relations.

Shape relations: Comparative shape relations (e.g., comparing compactness and sinuosity values) and direct shape relations (e.g., angular distance) have diverse uses. They can be used to (a) describe visual similarity among objects or regions (e.g., for buildings, see Steiniger et al. 2008; Barr et al. 2004); (b) evaluate whether geometric transformations such as smoothing, simplification or typification are necessary (e.g., for roads, see Plazanet et al. 1998); (c) measure whether the shape deformation of a geometry is still acceptable when geometric transformations are applied (for buildings, see Bard 2004); and (d) guide the selection of appropriate generalization algorithms (for roads, see Mustière et al. 2000).

For polygons and lines, shape relations can be calculated for both the original and derived shapes, such as the convex and rectangular hulls (e.g., axes parallel envelope, minimum bounding rectangle). The reader is referred to the literature for more details on shape measures for polygonal and line objects. A comprehensive list of shape descriptors and other measures useful for generalization purposes is given in AGENT Consortium (1999). Further evaluation of polygonal shape indices has been presented by MacEachren (1985).

Core area (Gustafson 1998) is a specific shape relation (see Figure 5), which will be explained in more detail. The measure is calculated using a negative buffer operation, and it returns a geometry (Figure 6). Core area does not show a relationship to a specific map feature; instead, it embodies a relationship of a polygon to its environment. In landscape ecology the index is used to define a core zone, where a species is assumed to exist with 100 percent certainty. The area between core and polygon edge designates a transition zone between two species. Thus, the relation represents fuzziness, which is a common property for boundaries in a number of map types (e.g., in soil maps). Another application of core area is its use as an indicator of a necessary geometry type change, that is, to decide whether a river polygon should be collapsed to a line symbol. McGarigal (2002) advocates that core area integrates polygon size, shape, and edge effects into a single measure.

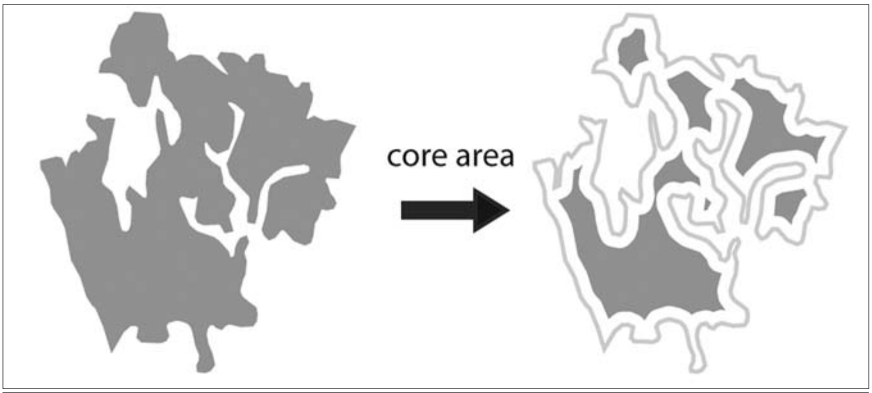


Figure 6. *Core Area* is calculated using an internal buffering operation. [Data: Digital Chart of the World (DCW)].

Orientation relations: Similar to shape relations, the relations among the orientations of diverse objects can be used to form groups of objects. An application has been presented by Burghardt and Steiniger (2005) for the grouping of buildings by comparing their orientation to the orientation of nearby roads, in order to form alignment patterns. Orientation relations, however, are not only used to group objects. Absolute orientations (north, east, etc.) and relative orientations among objects (parallel, orthogonal, etc.) are often emphasized to highlight the relations of objects to their neighbors or to facilitate map legibility. Examples are given in the generalization text by Swiss Society of Cartography (SSC 2005). Duchêne et al. (2003) present measures to calculate the orientation of buildings which may serve as a basis to derive the orientation of natural polygons.

To summarize, we showed that geometric relations are important in map generalization for four reasons. First, they are needed to evaluate whether geometric transformations of map objects are necessary to maintain the legibility of the map. Second, they help to calculate the degree of geometric transformation required to ensure map legibility. Third, they are used to evaluate whether a certain limit of deformation has been exceeded. Finally, they are used to identify perceptually similar and close objects that can be used to detect more complex structures such as alignments. Thus, geometric relations help to identify and manage generalization problems, while also serving as building blocks for the recognition of perceptual patterns. Both issues are treated in more detail in the application example of island grouping and generalization presented in the following section.

Topological Relations

In our analysis of the literature and maps we identified four types of topological relations: *intersection type*, *topological structure*, *neighborhood order,* and the so-called *ring configuration* relation (Figure 7). The essential purpose of these relations in map generalization is to prevent topological inconsistencies that are introduced in the generalization process and to preserve connectivity information. The four relation types are explained below in more detail.

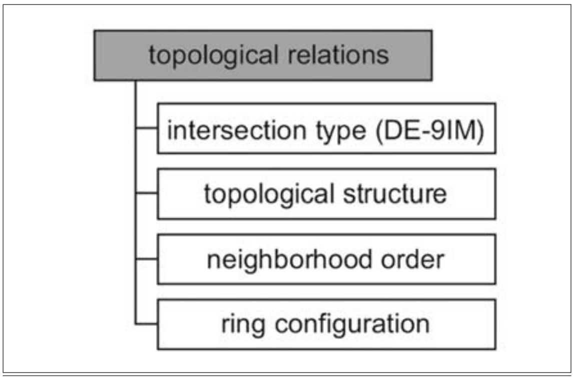


Figure 7. Topological relations.

Intersection type: To evaluate topological relations between two geometries, one needs to define a set of possible basic relations and describe how these can be determined. Such a set has been proposed by Egenhofer and Herring (1991), Clementini et al. (1993) and others for the 2-dimensional case, and its definition has evolved into a standard definition for GI systems in the OpenGeospatial Simple Features specification (OGC 1999). The basic set (DE-9IM) in the OGC specification describes the following topological relations between two geometries: disjoint, touch, cross, within, overlap, contain, intersect, and equal.

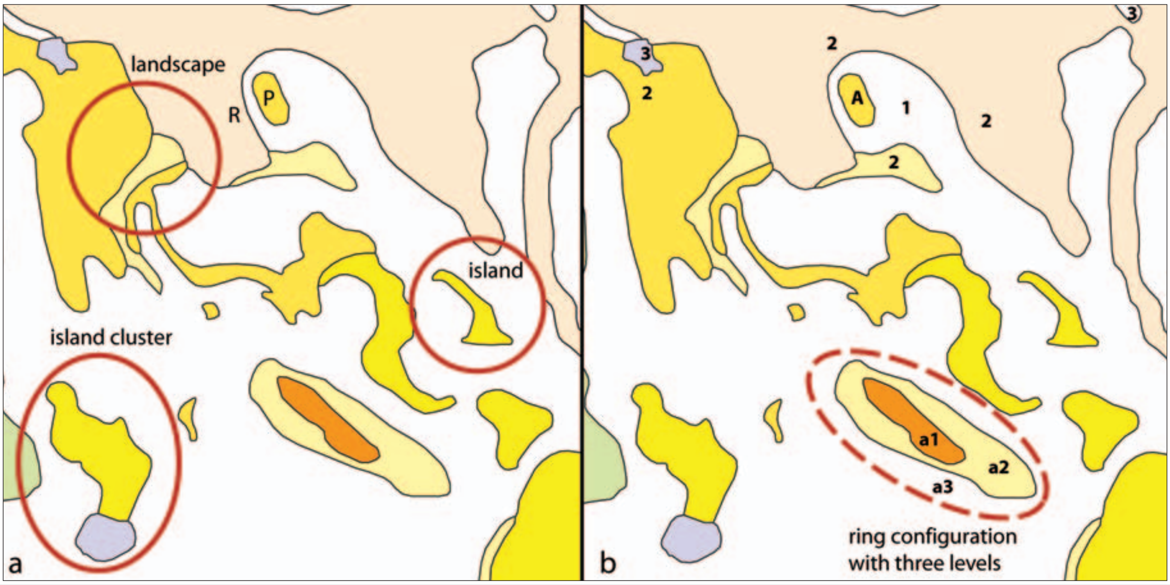


Figure 8. Topological relations. a) Circled in red are examples for the three *topological structure models*: island polygon, island cluster and landscape mosaic. b) Example of the *ring configuration*. Here, three ring levels a1, a2 and a3 (background polygon) exist. The *neighborhood order* is given for the island polygon denoted by A. The numbers 1, 2 and 3 refer to the order of topological neighborhood with respect to polygon A. [Data: © FOWG.]

This set of primitive topological object relations is a necessary condition to describe the other three topological models below. Additionally, the intersection type is directly utilized in generalization to check whether geometric generalization operations have introduced topological inconsistencies. For instance, following a displacement operation, a river and a road may cross each other where they did not before the operation.

Topological structure: This relation type distinguishes between three structure models: island polygon, island cluster, and landscape mosaic (Figure 8a). The naming of the structures *island polygon* and *landscape mosaic* is derived from the landscape ecology's perspective on patches (McGarigal 2002). The distinction of these three types is useful, on the one hand, to preserve the typical patch structure frequently found in polygonal maps (e.g., soil or geological maps), and on the other hand, to select and parameterize appropriate generalization algorithms. The latter purpose will be illustrated by an example.

The displacement model by Galanda and Weibel (2003) for the solution of proximity conflicts in polygonal maps requires the initialization of a deformation model. In this model, a polygon is either defined as rigid—and thus it will be displaced as a whole—or its outline is elastic and, hence, it can be deformed. After analyzing the topological structure of the map and the size relations, small islands (e.g., polygon *P* in Figure 8a) and small island clusters are typically assigned a rigid outline. Consequently, they will be displaced as a whole. In contrast, large polygons, polygons that are part of a landscape mosaic (e.g., polygon *R* in Figure 8a), or large island clusters are given an elastic outline to facilitate the resolution of proximity conflicts by partial deformation.

Neighborhood order: This topological index starts from a seed object (index = 0) and assigns every next neighbor visited an increasing order number (1, 2, …, *n*). An example is shown in Figure 8b where polygon A denotes the seed object. The order number is usually calculated by counting the minimum number of borders that have to be passed to move from the seed object to the current object. This index can be calculated for polygonal data as well as for points and lines. For points, the Voronoi regions (de Berg et al. 1997) are calculated first and then the number of Voronoi edges are counted which need to be traversed to move from one point to another. For lines in a line network, the neighborhood index is obtained by counting the number of nodes visited traversing the network. Topological ordering is well known in GIS analysis and elsewhere, and it has been applied in map generalization. In a displacement model for buildings, for instance, Ai and van Oosterom (2002) use the index to calculate the level of motion propagation for neighboring buildings.

Ring configuration: This particular configuration, where several polygons enclose each other like the peels of an onion (Figure 8b), is typical for maps of discretized continua, such as isarithm maps of temperature, heights fields, or snow depth. If only two polygons are involved, this relation is similar to the island structure mentioned above. As with all other topological relations, the usefulness of the ring configuration lies in being able to detect such ring-shaped patterns in order to preserve them in the generalization process.

Statistical and Density Relations

Although basic statistics and density relations are also used in topographic map generalization, the main source for the relations presented in this subsection has been literature on thematic mapping (particularly pattern analysis) in landscape ecology. In landscape ecology, the so-called landscape metrics have been developed to describe the heterogeneity and fragmentation of a landscape. They are usually grouped into two types of metrics, the non-spatial composition indices and the spatial configuration metrics (Gustafson 1998). The latter type of landscape metrics is discussed in the subsection on Structural Relations, because of its patch-, not category-based, computation. In our typology we will distinguish between four groups of indices: *statistical base indices*, *area relations*, *category relations,* and *diversity metrics* (Figure 9).

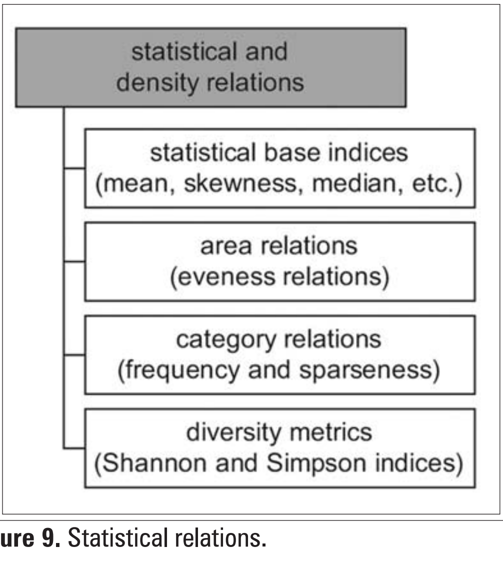


Figure 9. Statistical relations.

The use of these metrics has two main goals: 1) the preservation of overall map heterogeneity, which aims to maintain as much information as possible while ensuring a high level of map legibility; and 2) the detection of dominant or rare features. We refer to existing applications below. A comment regarding terminology: because most of the measures and theory discussed here have been developed in landscape ecology, we use the original terms “index” and “metrics.”

Statistical base indices: With these indices, we address statistical distribution parameters such as the n-th order moments (sum, mean, variance, skewness, etc.) and statistical indices (e.g., median, argmin, argmax etc.). These parameters have been used in topographic generalization to analyze, for instance, the homogeneity of city blocks or building groups (Boffet and Rocca Serra 2001). The analysis of the statistical distribution parameters is also used for the determination of classes for the display of a single phenomenon in simple thematic maps (e.g., population density maps). Such methods are described in Slocum (1999). Especially the analysis of attribute value distribution (variance) plays an important role in most clustering algorithms (Duda et al. 2000) developed for the classification of thematic datasets.

Area relations: The indices of this group (also called “evenness relations”) describe areal ratios. Example indices are the *item area probability*, which describes the area ratio between the current polygon and all polygons of the same category, or the *evenness index* (McGarigal and Marks 1995), which describes the area ratio between the polygons of one category to all polygons in the map or section. The area relations are useful for identifying rare categories in terms of occupied space and to measure the preservation of area ratios when geometric generalization operations are applied. A rather simple application for the latter case is the black-to-white ratio, which is used, e.g., in building generalization to determine the number of (enlarged) buildings to be retained in a building block (SSC 2005; Burghardt and Cecconi 2007). The ratio is based on the area that the buildings (black objects) will occupy on the target map, compared to the white space. This procedure should give the user a good impression of the settlement density, despite the condition that not all buildings can be displayed on the target map.

Category relations: Category-related indices measure the frequency of occurrence and, hence, level of sparseness. The *relative patch richness* measures the number of categories in a map section and relates it to all existing categories (McGarigal and Marks 1995). Thus, the index describes local homogeneity. The other index in this group is *category probability*, relating the number of items of one category to all items. As far as we know no use has been made of these indices in map generalization. However, we suggest that the latter index is useful for detecting rare categories, whereas the relative patch richness index can be used to evaluate whether the local heterogeneity has been preserved after applying a reclassification operator.

Non-spatial diversity metrics: This group of metrics encompasses composite measures of evenness and richness (McGarigal 2002), which have been described in the two previous categories. The landscape metrics of this group are, for instance, the *Shannon diversity index*, the *Shannon evenness index*, the modified *Simpson diversity index* and the modified *Simpson evenness index* (McGarigal and Marks 1995). These indices can be applied to the whole map or to a map section. Both Shannon indices characterize the amount of information, the so-called *entropy*, as a concept that originated in communication theory (Shannon and Weaver 1949). The original Simpson indices are not entropy measures; they are probability measures. According to McGarigal and Marks (1995), the *modified* Simpson and Shannon diversity indices are similar in many respects and have the same applicability for the characterization of landscapes.

A possible application of the Shannon diversity index in map generalization is in measuring the loss of information resulting from the generalization process. The Shannon evenness index, on the other hand, can be useful in identifying dominant categories, since evenness is the complement to dominance (*evenness* = 1 - *dominance*; Gustafson 1998). A practical application of entropy measures to soil maps has been reported by Ibáñez et al. (1995) to assess pedodiversity, i.e., the variation of soil characteristics. According to Fuchs (personal communication, 2004), entropy measures are used by the German State Office for Geosciences and Resources Brandenburg to evaluate the quality of their soil maps, which have been derived through generalization processes. Bjørke (1996) proposed two applications of entropy measures, one for evaluating automated map design and another for eliminating point symbols while preserving point cluster structures (Bjørke and Myklebust 2001).

As a final comment in this subsection we have to admit that while we did advocate the use of metrics developed in landscape ecology for generalization purposes, no practical applications to generalization exist so far, to our knowledge, except for the non-spatial entropy-based measures. We clearly see a need for generalization research to evaluate the potential and expressiveness of such metrics.

Semantic Relations

The structural analysis, and with it the study of semantic relations, represents the first stage of map compilation. Especially if categorical maps are directly derived from GIS data or if, for instance, a small-scale soil map should be derived from a medium-scale soil map, then the number and structure of the categories needs to be defined. This differs from topographic map generalization, where the map content and classification schema are often clearly defined by the mapping authorities. In topographic maps, the classification usually differs only from country to country; for soil maps, the map legend units may differ from map sheet to map sheet. Therefore, the semantic analysis needs to address *priority relations* of categories and object groups, *resistance and attraction relations* between individual polygons, and *causal and logical relations* among classes (all of which can be found in topographic maps), as well as *similarity relations* to define the legend units of thematic maps (Figure 10).

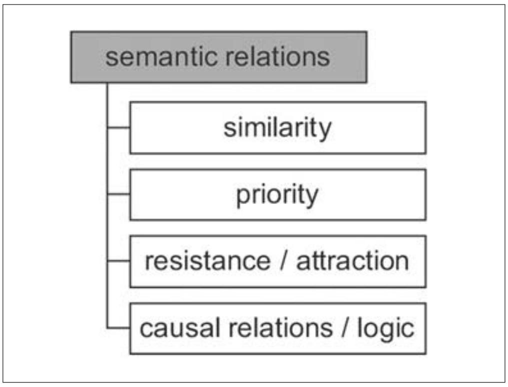


Figure 10. Semantic relations.

Semantic similarity relations: As noted above, similarity relations are needed to assign map objects to the categories of the new map. If the classes are not known beforehand, they have to be inferred from the data. Every object is first described by several properties that characterize it and may help to distinguish it from other objects. For instance, a building can be described by its area, and its squareness and perimeter, that is, the geometric properties of the building. These properties span an *n*-dimensional *feature space* (n denotes the number of properties).

Figure 11 shows such a feature space spanned by 10 properties of buildings, but it is transformed to a 2-D space for visualization purposes. Every dot in this image represents a building in the feature space, the position of which is defined by the values of its geometric properties. The similarity between two buildings can now be obtained by measuring the distance that separates them in the feature space. Buildings with similar properties will be located close together.

If we decided to classify the buildings into categories that represent similar building types (and thus also similar urban settlement structures), we could apply unsupervised pattern classification methods such as clustering (Duda et al. 2000). Such methods would probably identify three building structure categories in our example, corresponding to the three point clouds (clusters) shown in Figure 11. In contrast to similarity between two individual buildings, the similarity between two categories is expressed by a probability model that accounts for the shape of the point cloud making up the categories, as well as distance in the feature space. Were we to use the pure distances only, as the semicircle around the center of category 1 indicates, then the objects of category 2 would belong to category 1.

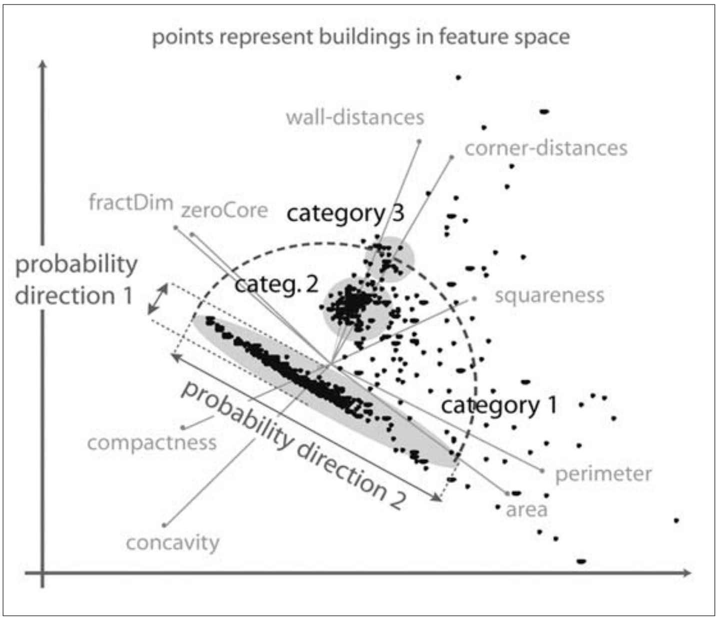


Figure 11. Buildings described by geometric properties depicted in feature space. The buildings form 3 natural categories (clusters). The definition of *similarity* in feature space encompasses distance *and* probability. Otherwise the objects of category 2 would belong to category 1.

Sometimes, categories are known in advance, and the task will then consist of assigning new observations to these categories. Let us assume that for the purposes of a planning map, the categories “inner city,” “urban,” “suburban,” “industrial,” and “rural” have been defined to classify a study area into zones of different structure types based on the characteristics of the buildings they contain. We start again by characterizing the buildings by means of geometric properties such as area, squareness, perimeter, etc. Every category is then defined by selecting a set of representative buildings for each category (i.e., a training sample). These representative buildings can then be used in classification methods, such as discriminant analysis, to assign the remaining buildings to the prototype classes (Duda et al. 2000). Because we use prior knowledge (i.e., the training samples), the classification is called “supervised;” similarity in this case is defined by distances in the features space.

A second approach to defining semantic similarity is to establish classification rules which assign objects to categories based on their properties. In the above example of settlement classification, for instance, all very large buildings may be defined as industrial and separated from the rest. The remaining buildings are further analyzed to identify buildings that are alone within a 100-m buffer. These single buildings are then separated and defined as rural buildings, while the remaining buildings are again analyzed further. This approach results in what is usually called a “decision tree” (Duda et al. 2000). The similarity in this case is expressed by the rules.

A third approach to expressing similarity can be applied if the data are already organized in a set of categories, and this set needs to be reduced. For instance, if the five categories of our planning map are to be reduced to the two categories “rural” and “urban,” then the similarity can be defined by the user. This is preferably done by assignment rules that relate each category to its super-category.

Applications of similarity analysis in map generalization have been presented by several authors. Bregt and Bulens (1996), for instance, discuss three approaches to aggregating soil areas using a classification hierarchy defined by an expert, the border–length index (see the description of configuration metrics in the following sub-section), and a self-developed similarity index. Based on this work, van Smaalen (2003) later developed an approach to derive an aggregation schema for the land-use layer of topographic maps. Fuchs (2002) used properties of soil patches as input values for a cluster analysis to generate a new set of soil categories. Steiniger et al. (2008) present a discriminant analysis approach for the classification of urban blocks into predefined urban structure classes, based on representative buildings. Approaches to derive rules for a decision

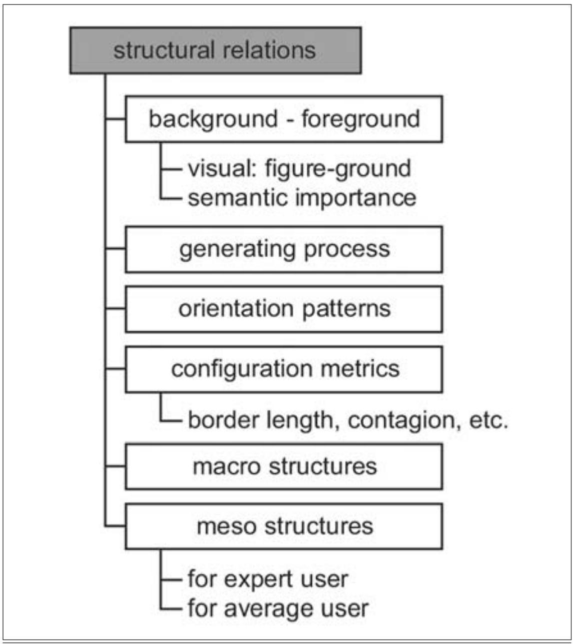


Figure 12. Structural relations.

tree to classify roads for generalization purposes are reported by Mustière et al. (2000).

Priority relations: Like similarity relations, priority relations focus on the category level, but additionally also on the object group level. Priority is used in the generalization process to give more importance to a special object class or category than to others. For instance in topographic maps, roads have a higher importance than buildings. Thus, roads push buildings away if they are widened for visualization purposes. The priority of roads over buildings also implies that roads are generalized first. In thematic maps, the theme or purpose of the map basically decides the priorities of object classes or categories, respectively. For instance in a vegetation map, rare plant societies are emphasized over other categories, even if the corresponding polygons are too small to be displayable. Explicit modeling of priority for object groups over non-grouped objects has already been realized. This was shown by Gaffuri and Trévisan (2004) for the preservation of building alignments.

Resistance and attraction: The resistance and attraction relations focus on the individual object level. They define whether neighboring polygons are aggregation candidates or not. The resistance relations can be defined by the user, or they can be calculated as a compound index based on semantic similarity, class priority, and/or statistical relations. The relations are, for instance, evaluated when the generalization system needs information about whether it may aggregate two forest polygons across a small area of another land-use type. Here the resistance relation will probably return a positive value (attraction) if the small area is grassland. But the aggregation would be rejected if the area between the forest polygons is a river (resistance).

Causal relations: Causal relations describe dependencies among categories. These relations are used when there is a need to eliminate map features or aggregate classes during the generalization process. An example for the use of causal relations has been reported by Duchêne (2004) for topographic maps. In her generalization system, the categories of "road" and "building" have been linked with a causal relation. If a road is deleted, the system searches for nearby buildings that would lose their connection to the road network. If such buildings are found, then the system has two choices. Either it deletes the buildings as well, or it restores the road if one building is marked as important (e.g., a hotel).

Structural Relations

As the word "structure" suggests, the relations of this group should denote types of structural patterns that are perceived in maps. In this sense, most of the relations discussed in this sub-section are linked to human perception and cognition. We have identified six relation types as being part of this group: the background–foreground relation, generating process, orientation patterns, spatial configuration metrics, macro structures, and meso structures (Figure 12). Apart from the background–foreground relation, the other relations of this group should be identified in maps before the generalization starts so as to preserve important patterns during map generalization.

Background–foreground relation: With this type of relation, we want to ensure that problems can be addressed that concern the definition of a visual order in maps. Therefore, two issues must be considered: 1) the elimination of figure–ground effects (Dent 1999), which can be provoked by an unskillful choice of colors and can lead to a wrong user perception of the map content; and 2) the agreement between the semantic importance of an object class (given by the map purpose) and its visual weight. For instance, a disagreement exists if roads in a topographic map are overlapped by forest polygons. Research in automated map generalization has paid only scant attention to these figure–ground problems, which we think is due to two reasons. The assignment of visual weight is not a problem in topographic map generalization because the symbols, colors, and the order of the thematic layers of topographic maps are usually fixed and appropriately defined. However, despite some advances, such as the work by Chesneau et al. (2005) on automated color contrast enhancement, research in thematic map generalization is still far from being able to establish a ready-to-go map production system. Thus, there is always a manual post-processing stage during which a designer or cartographer can revise figure–ground problems and assign the correct visual weight.

Generating process: This relation should describe whether a map reader may gain the visual impression of the underlying process that generated the displayed real-world objects. We propose to distinguish three types: *without structure*, *artificial structure*, and *natural structure*. The characterization should be applied to: a) the complete map or a map section, b) groups of map objects, and c) the object and its parts. The upper image of Figure 13 shows examples for an artificial and natural structure of soil site borders (type c). On the object level, shape measures such as sinuosity and squareness may be helpful to identify the type of the generating process relation. However, apart from early work (Buttenfield 1985), measures have not yet been developed sufficiently to reliably detect such particular structural relations. The use of configuration metrics from landscape ecology (see following section) should be evaluated for use on entire maps, map sections, or at group level. For point distributions, the well known nearest neighbor index (Haggett 2001) can be applied.

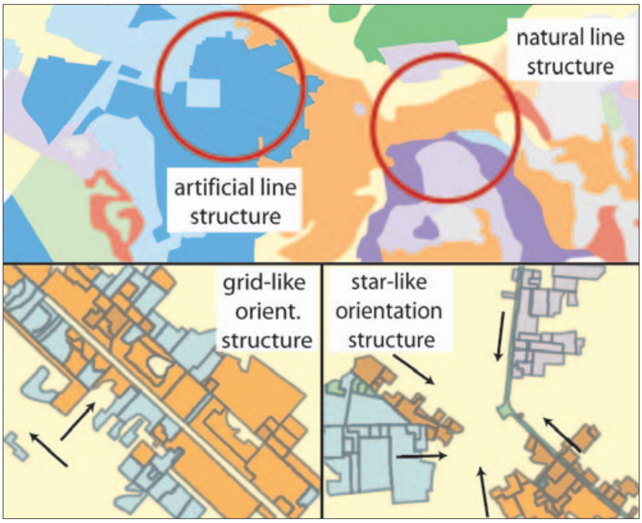


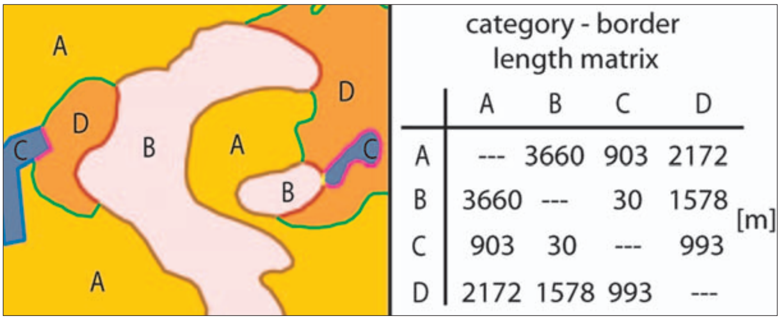
Figure 13. *Structural relations* and properties. The upper picture shows artificial and natural polygon structures from a German soil map. The lower pictures present two examples of orientation patterns in a land-use dataset from New Jersey. Here, the orientation patterns are induced by the road network. [Data: © LGRB, NJDEP.]

Orientation patterns: This relation type corresponds to the extension of the simple orientation relation of two objects (cf. geometric relations) to more complex patterns. Examples of such complex configurations include star-like patterns and grid structures (see Figure 13), as well as circular arrangements. Orientation patterns of road networks have been described by Zhang (2004) and Heinzle et al. (2006) for generalization purposes and by Marshall (2005) for transportation analysis. Heinzle et al. (2006) also describes a method to detect circular road patterns.

Spatial configuration metrics: Four different measures belong to this group: the *border length index*, *contagion*, *interspersion and juxtaposition index* (IJI), and *lacunarity analysis*. Typically, configuration metrics are based on a matrix of pairwise adjacencies between all patch types. The elements of such a matrix hold the proportions of the edges in each pairwise combination (Figure 14; McGarigal 2002). While the border length index and IJI can be applied to vector data, the contagion index (Li and Reynolds 1993) and the lacunarity analysis (Plotnick et al. 1996) can be applied to raster data only. Note that lacunarity analysis differs from the other indices in that it is a multi-scale method with a binary response.

We suggest using these metrics to measure the change of fragmentation before and after generalization to quantify the changes. However, experience with these measures in map generalization is limited, and research is required to evaluate their explanatory power. An exception is the application of the border length index reported by Fuchs (2002). The index gives a probability value for the common appearance of two categories and, consequently, it provides a kind of similarity measure. Fuchs (2002) used this measure of similarity to obtain a reduced set of legend units for the generalization of a soil map. Apart from the work by Fuchs (2002), it is also worth mentioning that the border length index is implemented in sliver removal procedures available in commercial GIS software (e.g., ESRI ArcGIS, “Eliminate” tool). The removal is achieved by merging sliver polygons with that particular neighboring polygon with which they share the longest common edge.

Macro structures: Macro structures are not directly manifested and visible on a map of a given scale because they relate to a different (macro scale) level and resolution. As a result, they can only be recognized if the map reader has particular



	A	B	C	D
A	---	3660	903	2172
B	3660	---	30	1578
C	903	30	---	993
D	2172	1578	993	---

Figure 14. A section of a landscape and its configuration described by the category-border length matrix. The matrix is used for different *spatial configuration* indices. [Data: © LGRB.]

information about them. An example is given in Figure 15, which shows geological patterns of the Black Forest north of the Swiss–German border. The patterns can hardly be perceived on a map at the scale of 1:100,000 (left), but they become obvious on a map at the scale of 1:500,000 (right). A detection of such structures in high-resolution map data by pattern recognition methods is difficult to accomplish, because the granularity is too high ("one cannot see the forest for the trees"). Nevertheless, the influence of such large structures on map design is considerable since cartographers use them as structuring components. A person who knows about macro structures will tend to abstract them, even on large-scale maps. Research on the treatment of macro structures in automated generalization has not yet been reported.

Meso structures: Unlike macro structures, meso structures cover visible and detectable patterns. Examples of meso structures are given above in Figure 2 (left), showing alignments of soil patches of the same category. Meso structures can be grouped into those where visual patterns are obvious to every map-reader (e.g., four aligned lakes) and those with thematic patterns, which are only obvious to the experts familiar with the particular topic. The structures visible to every map reader are perceptual patterns—which have been described by Wertheimer (1923) in his *Laws of organization in perceptual form*—and correspond only to a lesser degree to patterns formed by the reader's background knowledge. How perceptual patterns are formed is briefly discussed in the next section (cf. Figure 18). Besides being distinguished on the basis of expert and non-expert patterns, meso structures can also be grouped into structures composed of entities of a single or multiple object classes. A sub-classification is possible (if the shape of the pattern is considered) into *parallel* or *curved* alignments, and *clusters* or *layers*.

Approaches reported for the recognition and preservation of meso structures focus on the analysis of building structures in topographic maps. Several researchers presented methods to detect either building alignments (Christophe and Ruas 2002) or other building groups perceived as "intuitive" (Regnauld 2001; Boffet 2001; Anders 2003). Another typical example for the consideration of meso structures in topographic maps is the recognition of major roads or water network structures. The detection methods are often based on the perceptual principle of good continuity (Wertheimer 1923; Thomson and Richardson 1999), but other methods, such as traffic simulation analysis in the case of road networks, have also been used (Ruas and Morisset 1997). As the final example of meso structure recognition, we like to refer to Downs and Mackaness (2002). They identify fault line structures in geologic maps to preserve them during the generalization process. Further meso structures are discussed in the generalization book of the Swiss Society of Cartography (SSC 2005) for the case of topographic maps.

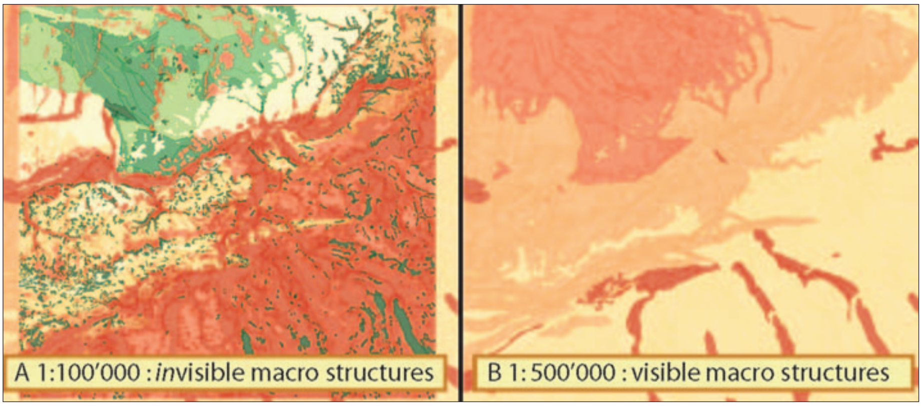


Figure 15. *Macro Structures*. Macro structures are concealed in the original map scale if the reader has no information about them, but they are clearly visible at smaller scales. In picture B a geological macro structure extends from SW to NE. Maps not shown to scale. [Data: © FOWG.]

Utilizing Relations to Characterize a Group of Islands

At the beginning of our research and of this paper we set out as our overall goal that the proposed typology should help to identify relations facilitating the generalization of topographic and thematic maps. We will demonstrate this on a concrete example. The case we have chosen deals with the generalization of a group of islands. It was selected for several reasons. First, islands need to be generalized for thematic and topographic maps, although the particular goals and constraints might be different. Second, it is a simple example in that we need to consider only one object class and only one geometry type (polygons). This has the effect that not all generalization operations are applicable, and relations among object classes do not have to be considered. However third, and perhaps most importantly, generalization of a group of islands highlights the necessity of preserving perceptual patterns (i.e., the meso structures).

The island data that we use in the example were extracted from the ESRI Data & Maps media kit. The islands are part of an archipelago in the Baltic Sea, located between the Åland Islands and the Finnish southwest coast (Figure 16). Formed during the ice age, the archipelago consists of "skerries" (small rocky islands too small to be populated) and larger islands with diameters up to a few kilometers. The resolution of the map data corresponds to a nominal scale of roughly 1:350,000.

To our knowledge, only two previous studies can be said to have reported on the generalization of islands. In fact, both studies used lakes rather than islands, but islands and lakes are often considered to be structurally similar for the purposes of generalization. The first study is by Bertin (1983), who describes a manual and stepwise approach for generalizing clusters of small lakes while preserving the spatial and structural configuration of the lakes. In the second study, Müller and Wang (1992) present an algorithm for the generalization of area patches exemplified on lakes (Bertin's lakes, as a matter of fact). They note, however, that their implementation was not able to preserve archipelago structures.

Cartographic Constraints for Island Generalization

Before we start to work through the list of relations relevant for island generalization, it is worth discussing which constraints necessitate changes to the island data. There are, in general, two reasons why data are generalized. We wish to obtain a legible map when the map scale is reduced, and, we may wish to reduce the amount of data for storage or data transfer reasons (e.g., in web mapping). Galanda (2003) presented a list of cartographic constraints for the generalization of polygons. An analysis of Galanda's list delivered a set of five *active* constraints applicable to our island data. They are termed "active" because they aim to fulfill the requirements of map legibility and low data volume thus:

- C1: An appropriate distance between consecutive vertices on the polygon outlines to reduce data volume;
- C2: A minimum width of an island (or parts of it, e.g., bays or headlands) to be visible on the map;
- C3: An appropriate outline granularity, e.g., delete imperceptible bays or headlands;
- C4: The minimum size of an island to be perceptible in terms of the area; and
- C5: Good visual separation of nearby islands.

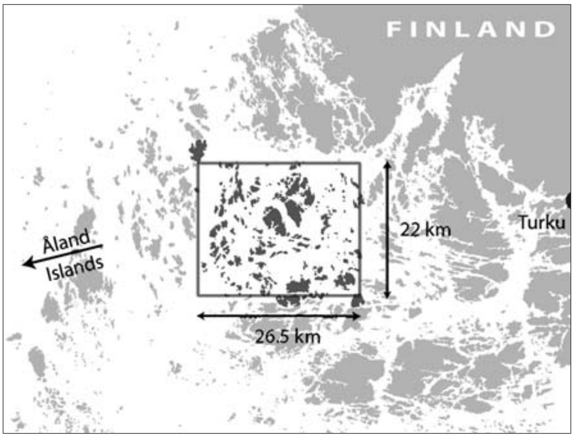


Figure 16. The box covers the islands data set used for the example of this Section. The islands are part of an large archipelago south west of Finland. [Data ® by ESRI.]

All other constraints reported by Galanda (2003) as being applicable to islands are *defensive* constraints, i.e., they are used to prevent strong changes of an island's position or the distortion of an island's shape and to preserve the spatial configuration.

Evaluating the Relations for Island Characterization

We will now organize the relations of our typology into four groups. The first group includes the non-applicable relations. The remaining

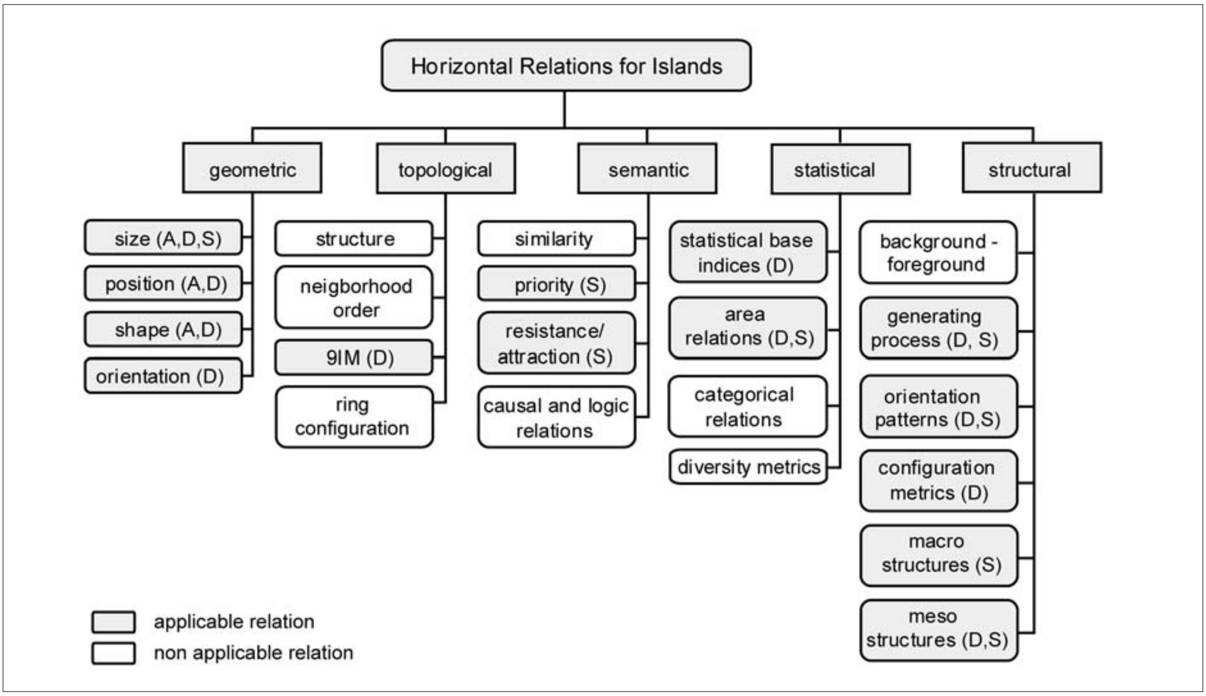


Figure 17. Applicability of horizontal relations to island generalization. A, D and S denote whether a relation is useful for Active constraints, **D**efensive constraints or operator and algorithm **S**election.

three groups comprise relations relevant to our problem, i.e., relations that help to evaluate active constraints; relations that can be assigned to defensive constraints; and relations that support the selection of operators and algorithms. The resulting classification is summarized in Figure 17, with the relevant relation types highlighted in gray.

Non applicable relations: Eight relation types have been identified as not applicable to islands. Among the topological relations that are not applicable are ring configuration (no concentric polygons can be found), neighborhood order (because islands are disjoint), and topological structures (again, because islands are disjoint). Similarity, causal and categorical relations and diversity metrics are not applicable because we have only one object class. Background–foreground relations do not play a role if the islands and the sea are assigned colors depending on the purpose of the map and cartographic tradition.

Relations supporting active constraints: Only three types of relations induce the generalization of a map; they all belong to the group of geometric relations. The size relations are used to evaluate the constraints C1 (vertex distance), C2 (minimum width), and C4 (minimum size). The position relations are used to evaluate whether two islands can be visually separated (constraint C5). The shape relations, e.g., in the form of a bend analysis (Plazanet et al. 1998), can help to evaluate the granularity constraint (C3).

Relations supporting defensive constraints: Defensive constraints are supported by most of the relations described in this paper. We begin our explanations with the geometric and topological relations and then move on to the structural relations, as knowing the latter is important for identifying other relations.

- **Geometric and topological relations**: The size, position, and shape relations have previously been identified as relations that support the evaluation of active constraints. In addition they can be used to evaluate the effect of geometric transformations, e.g., displacement, enlargement, or smoothing, and can thus subsequently help to identify excessive deformations. This also holds for orientation and intersection type (DE-9IM) relations. The orientation relation is necessary to evaluate whether absolute and relative orientations have been changed in an unacceptable manner during generalization. The intersection type relation specifically serves the purpose of detecting cases where operations involving displacement lead to an overlap of island, or a merger of two island groups that were previously considered as perceptually distinct.
- **Structural relations**: Four structural relations are applicable in terms of defensive constraints

and island generalization. Generalization operations may destroy patterns on two levels. At the global level, the distribution of islands may change from a natural structure to a more undesirable, ordered structure. At the object level, generalization operations may change the outline of islands from a natural smoothness to an artificial straightness or *vice versa* for port areas. Meso structures are useful as well, in that they describe natural, perceptual groupings of islands, which have to be identified to either preserve them during generalization, or to emphasize them. Figure 18 shows "perceptual groups" within island groups identified in a pencil-and-paper experiment described in Steiniger et al. (2006). Based on their experimental results, Steiniger et al. (2006) showed that Wertheimer's (1923) "laws of organization in perceptual forms" (i.e., the principles of Gestalt theory) can be used to describe perceptual groupings of islands. In Figure 18, the large groups of islands marked by people are based on Wertheimer's Gestalt principle of spatial proximity. In contrast, the smaller groups are described by the spatial proximity principle, as well as the principle of similarity of island shape, orientation, and size, and the principle of dominance of a large island in a smaller group.

For the automated recognition of the large island groups identified visually by humans, Steiniger et al. (2006) have presented algorithms that formalize Wertheimer's principles by means of a set of horizontal relations, more specifically, using the geometric relations of distance, shape, and orientation. The third applicable relation is the orientation pattern, which can be used to evaluate the defensive constraints. In Figure 18, the meso structure G1 in the lower left corner is an orientation pattern shaped as a banana. Other meso structures, such as the group G2, exhibit a straight orientation to the North. The spatial configuration relation is the fourth relation type supporting the evaluation of defensive constraints. With the configuration metrics, excessive changes in the land–sea configuration could be detected.

- **Statistical relations**: If meso structures are found, then two statistical relations become relevant to describe them and subsequently support the defensive constraints. Basic statistical parameters can be used to describe a group of islands in terms of their area, distribution, extent, and position properties. This can be done before and after generalization. Area relations, on the other hand, should be used to evaluate whether the black-to-white ratio (i.e., the ratio of the area covered by the islands to the area of the background) has changed for the map partition occupied by the particular island group. Both statistical relations and the spatial configuration relation require that limits be specified for changes that are still considered acceptable. If these thresholds are exceeded, then the generalization actions should be rolled back and adjusted.

Relations supporting algorithm selection: An important application of the relations is that their identification and characterization can inform the selection of generalization algorithms. As an example, one will usually not apply a smoothing operator to the part of an island outline that represents the docks of a port. For macro structures, the case is very similar. An example of a macro structure is illustrated in Figure 18, on a curved arrangement of islands leading from North to East. The structure can be recognized more easily when the view is extended to a larger area of the archipelago (see Figure 16). A macro structure can form a constraining generalization element that will force the algorithms working on a more detailed scale to emphasize this pattern.

If a separation constraint C5 is violated, then the area relation (statistical) can be used to support algorithm selection. For instance, if the island density is very high, as is the case in the middle of

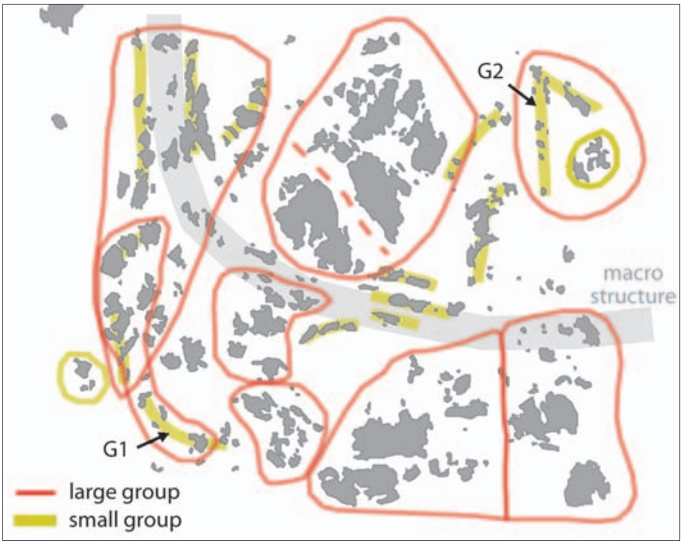


Figure 18. Meso structures in the archipelago identified by participants in a pencil-and-paper experiment.

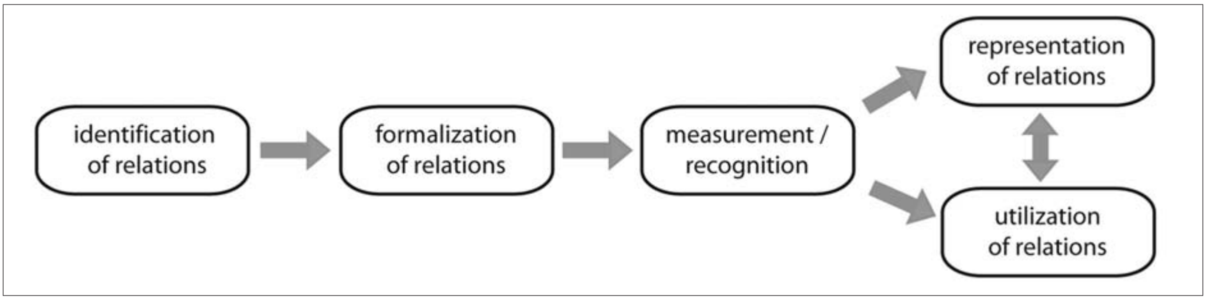


Figure 19. Utilization of relations.

the large cluster of islands in Figure 16, then we have to use typification instead of displacement operations, because there is no space to displace all islands without removing some. Another relation that may be used for the selection of an appropriate displacement algorithm is the size (geometric, comparative) relation. For example, if a small island is located too close to a large neighboring island, then we need to find a solution by using displacement operations. If the large island is treated as a mainland object and the small island as an island object, then we will only displace the small island while fixing the large island's position and making the boundaries of both islands rigid.

Two relations supporting algorithm selection are left to discuss. These are the priority relation and the resistance/attraction relation. Priority is used to enforce that island groups that have been detected are preferred over other islands that are not part of any structure, and that they are preserved in displacement, amalgamation, or elimination operations. The use of the resistance relation may be explained if we assume that additional road data are available. The resistance relation may allow a merging operation if two islands are connected by a bridge, but it will reject merging proposals if the islands are not connected by transportation lines.

Discussion

The discussion of the previous section has shown how horizontal relations can be used to formalize and evaluate constraints and to support algorithm selection for a specific example. We hope to have thus clarified the utility of the proposed typology. However, it is still largely an open issue how we can "quantify" the relations themselves. This problem and related other issues that should be addressed in future research are discussed in this section.

The utilization of relations in cartographic generalization can be represented in the form of a schema consisting of five stages (Figure 19). In the first stage, relations are identified that may exist in a data set, with a focus on those relations that need to be preserved and should be emphasized. The typology presented in this paper can serve as an initial check list on the kind of relations that may exist.

The second step aims to formalize the relations, that is, describe their elements in a sufficiently formal way so that rules or algorithms can be developed in the subsequent step for the detection of relations. For many of the relation types, the formalization can build on the literature cited in this article. This holds particularly for the relation types that are of a more generic nature, including the geometric, topological, and statistical relations. Semantic and structural relations, on the other hand, are often more specifically linked to the characteristics of the given object classes and map themes.

Thus, while it is perhaps possible to benefit from experiences reported in the literature, the formalization has to be specifically adapted to each particular case. For instance, let us assume that visual exploration of a series of soil maps has established meso structures that relate gravel soils to river beds. We can then try to describe that type of meso structure by means of geometric relations (e.g., both objects seem to always have a similar orientation of the polygon segments involved) and topological relations, e.g., the gravel soil is adjacent to or overlapping the river bed. This formalization will help us to later develop, in the third stage, measures and pattern recognition algorithms for the more complex relations. Note also that the formalization step can be assisted by a variety of knowledge acquisition techniques, such as interviews with experts and observations of experts, as well as the pencil-and-paper exercises that were used in the island grouping example discussed in the previous section (Steiniger et al. 2006).

The third step consists of transforming the formalization of relations into actual rules and/or algorithms for the measurement and recognition of the corresponding relations. The review of the typology of horizontal relations provides useful links to the pertinent literature. Indeed, a plenitude of measures and algorithms exists that might be used to implement the recognition of certain relations. Thus, as it has been pointed out in the discussion of landscape metrics, often the real problem will not be to find indices in the literature that can potentially describe a particular relation or measure a particular property of an object. Rather, the difficulty will be to identify whether the measure does exactly describe what we want it to describe. Linked to that is the problem of interpreting the values that are delivered by the measures, in order to make qualitative inferences from quantitative values.

Apart from these two issues, the measurement/recognition stage should also address a further problem that arises if several measures are required in association to describe complex relations, such as perceptual meso structures. In this case we need to ensure that the various metrics involved do indeed measure different object properties. As an example for the necessity of an evaluation of measures that can be found in the literature, we refer to the study by Riitters et al. (1995). They evaluated the (dis)similarity of 55 measures commonly used in landscape ecology using correlation analysis and factor analysis. Twenty-nine measures (i.e., more than half the measures) could be discarded preceding the factor analysis after a simple correlation analysis had established very high correlation coefficients ($r > 0.9$).

Once the measures and structure recognition methods have been developed and applied, the representation and storage of the relations found is addressed in the fourth step. Possible representations to store horizontal relations have been presented in Neun and Steiniger (2005) and Neun et al. (2006), including saving values as simple attributes in tables, or over relation matrices for class dependencies, and such complex data structures as triangulations and other graph data structures. Data structures suitable for representing horizontal relations are well known in the computing literature; in fact, they do not go beyond graphs. The precise method of implementation, however, may depend on the specific case at hand, including algorithmic requirements such as space efficiency and computational efficiency.

The final step in the chain focuses on the utilization of horizontal relations. Application scenarios need to be developed for the horizontal relations, with a focus on the interaction between relations and constraints, as shown for the island example in this paper. These scenarios should cover three aspects of the utilization of horizontal relations. First, constraints should be defined from the identified relations, such as the specific gravel soil-river relation. Second, the relations should be linked to established generic constraints to support the constraint evaluation for specific object classes in the generalization process. Finally, the third usage is to develop rules for the selection of generalization operations and algorithms based on the information provided by the relations.

Conclusion

In proposing our typology of horizontal relations for thematic and topographic maps, we hope to strengthen research on an important part of the cartographic research agenda, automated generalization. We have shown in our example of island generalization how horizontal relations can be used to characterize map data, support the detection of conflicts, and assist in the choice of generalization operations appropriate for the resolution of these conflicts. Furthermore, we deem the typology crucial for the development of more and better generalization algorithms that take into consideration the context of map objects and that are able to act over multiple object classes, rather than being restricted to a single object class, as is still frequently the case for existing generalization algorithms. However, the island generalization example was a conceptual one and has only partially been implemented. In order to accomplish the full task, many further problems will need to be solved. We have addressed some of these open issues in the Discussion section. A full-scale solution is needed to link the various elements—constraints, measures, relations, and algorithms—together in a comprehensive system that is capable of controlling the interaction of these elements in the generalization process. Such systems have been reported in the literature, albeit so far only for specific generalization problems, as exemplified by the AGENT system for the generalization of urban zones in topographic maps (Barrault et al. 2001, http://agent.ign.fr).

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