ADVECTIVE HEAT TRANSPORT IN FROZEN ROCK CLEFTS

Advective Heat Transport in Frozen Rock Clefts – Conceptual Model, Laboratory Experiments and Numerical Simulation

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ABSTRACT

Advective heat transported by water percolating into discontinuities in frozen ground can rapidly increase temperatures at depth because it provides a thermal shortcut between the atmosphere and the subsurface. Here we develop a conceptual model that incorporates the main heat exchange processes in a rock cleft. Laboratory experiments and numerical simulations based on the model indicate that latent heat release can rapidly warm cold bedrock and can precondition it for later thermal erosion of the cleft ice by advected sensible heat. The timing and duration of water percolation both affect the ice level change because initial aggradation and subsequent erosion are of the same order of magnitude. The surplus of advected heat is absorbed by cleft ice loss and by runoff from the cleft so that this energy is not directly detectable in ground temperature records. Our findings suggest that thawing-related rockfall is possible even in cold permafrost if meltwater production and flow characteristics change significantly. Advective warming could rapidly affect failure planes beneath large rock masses and failure events could therefore differ greatly from common magnitude reaction-time relations.

KEY WORDS: bedrock; permafrost; advective heat transport; conductive heat transfer; laboratory experiment; numerical modeling; rockfall; climate change rock fall

INTRODUCTION

Mountain permafrost may be situated at high elevations above densely populated regions and in this steep topography it can condition debris flows or rockfalls which in turn may trigger greater hazards downvalley. Climate change is expected to modify this hazard potential (Haebeli et al., 1997) and to affect the links between permafrost-related mass movements and atmospheric conditions (Haebeli and Beniston, 1998; Geertsema et al., 2006; Harris et al., 2001; Harris et al., 2009). Key questions are whether an increasing number of extremely hot summers will lead to more rockfalls in steep bedrock terrain, as in the summer 2003 in the Alps (Gruber et al., 2004), and whether continued warming could cause large rock avalanches that differ markedly from historical events.

Both general thermal conditions and short-term thermal features must be comprehensively understood in order to investigate the temperature dependence of rock instability. In this context, it should be pointed out that: (1) the thermal conditions of recent rockfalls from permafrost areas are only known approximately and no correlation between these modelled rock temperatures and event frequency has been established (Noetzli et al., 2003; Fischer, 2010); and (2) much of the rockfall activity in the Alps in summer 2003 occurred earlier than the maximum active layer thickness predicted from heat conduction (Gruber et al., 2004). Given the hypothesis that these rockfalls relate to thermal conditions at the failure
planes, these two observations suggest that small-scale (cm–m) thermal anomalies caused by non-conductive effects may be significant.

Adective heat transport along clefts may be an important modifier of thermal and hydrological conditions and may influence the stability of ice-filled clefts in permafrost (Gruber and Haeberli, 2007). The quantification of this heat flux in steep, frozen bedrock is difficult because the physical processes are non-linear and natural conditions are heterogeneous and difficult to observe. For example, Hasler et al. (2011) measured significant thermal variability within the active layer of permafrost in steep bedrock, even on an annual basis, due to the cooling effects of thin snow cover and ventilation within clefts. Although short-term warming effects were measured in some clefts, they had minor impacts on average thermal conditions. Investigations of the local heterogeneity of rock temperatures using geophysical methods have also led to thawing corridors being detected and these were attributed to advective heat inputs from percolating water (Krautblatter and Hauck, 2007).

In this paper, laboratory and numerical experiments are presented in relation to a conceptual hydrothermal model of a rock mass containing a single cleft. These reveal the sensitivity of linear thaw (ice erosion) along clefts and corresponding local permafrost degradation, and form a basis for future investigations of cleft assemblies in fractured rock. The following research questions are addressed:

1) Which processes are important for advective heat transport in rock clefts at subzero temperatures?
2) What are the effects of advection on cleft-ice and on the temperatures around the cleft?
3) Which parameters govern the processes of advective heat transport?

The aim of the modelling is a semi-quantitative description of the thermal effects of advective heat transport in steep fractured bedrock with permafrost, rather than a direct prediction of the small-scale thermal field in nature. The latter cannot be undertaken at present because the necessary information (detailed surface and near-surface characteristics) is not available. However, the general sensitivity of clefts in bedrock permafrost to advective heat input can be assessed and this provides a means to understand the impact of extreme climatic events and mean annual temperature rise on the thermal, hydrological and mechanical conditions of steep bedrock with permafrost.

### BASIS OF THE CONCEPTUAL MODEL

Hydrothermal processes in permafrost and seasonally-frozen ground comprise the phase change of water and the transport of latent and sensible heat by the motion of water and water vapour. The importance of these processes varies with water content, hydraulic permeability and gradient, and depends on the scale considered. Hence, the near-surface characteristics and (micro-) topographic situation are significant factors.

#### Hydrological properties of steep bedrock permafrost

High-alpine mountain flanks consist to large parts of bedrock with average inclinations between 40° and 70°. These rock masses contain discontinuities formed by the orogenesis and weathering processes. Clefts, being macroscopic discontinuities, have typical apertures in the range of mm–dm, spacings of cm–m between each other in the shallow metres or decametres of steep bedrock (e.g. Hasler et al., 2011). These structural properties shape also the micro topography and lead to strong spatial heterogeneity of local aspect and slope angles. The hydrological properties of this steep bedrock differ from permafrost soils in gentle terrain, for which hydrothermal studies exist (cf. Kane et al., 2001; Boike et al., 2008), as the following: i) In unfrozen state, the water flow is dominated by the flow along open clefts and the water migration through the inter-cleft rock mass plays a minor role in low porosity rock (Dietrich, 2005). ii) Large hydraulic gradients (slope) and high permeability (macroscopic clefts) lead to mostly unsaturated conditions with preferential flow paths developed in the cleft system of the thawed near-surface layer. iii) In the permafrost body, the cleft-ice content is decisive for the hydraulic permeability. For point iii) it is often assumed that permafrost acts as an aquiclude in fractured bedrock or coarse-grained sediments as long as the pore space is saturated (Rist, 2007). Observations of ice filled clefts in the field (Gruber and Haeberli, 2007) indicate that this condition is often met in bedrock permafrost. Pumping experiments in fractured granite with and without permafrost support this hypothesis (Pogrebiskiy and Chernyshev, 1977). Nevertheless, thermally eroded and progressively deepening channels within the cleft ice cannot be excluded and deep-seated rock creep may render frozen fissures permeable by imposing geometry changes.
Our model assumes an initially ice-sealed permafrost body with percolation along preferential flow paths and lateral runoff on the cleft ice surface (Figure 1). Rapid percolation in relatively wide clefts results in minimal heat exchange for water within the unsaturated zone, causing an efficient heat transport to the cleft-ice level (cf. Rist and Phillips (2005) who observed similar effects in debris), which we can treat as a heat-transport short-cut between the surface (or atmosphere) and the ice level. The low depth of the water flow over the ice may, however, not be correct for very narrow clefts or clefts with sediment infill where heat exchange is expected to be less concentrated around the ice level. The temporal development of the ice level depends on available water and thermal conditions (see below). Water availability is limited during the cold season at high elevations, but in the spring, rock at several degrees below 0 °C may be subject to percolating meltwater (cf. Stähli et al., 1996; Boike et al., 1998; Scherler et al., 2010). For this reason, ice aggradation is expected during the early thawing season before ice erosion becomes dominant.

Figure 1: Sketch of hypothetical flow paths in the active layer or a talik within of bedrock permafrost. Left: Cross section through a rock cleft with the 0 °C isotherm in summer (thawing front). Right: Profile along a cleft with preferential percolation paths and runoff on the surface of the cleft ice. Dashed boxes: Domain of physical and numerical modelling.

Heat flow and cleft ice evolution fractured rock
Advective heat transport to the subsurface comprises heat uptake in a source area at or near the surface, heat transport by water percolation, and subsurface heat release. Heat uptake at the surface depends on various factors, causing a wide temperature range for the percolating water. Manual water temperature measurements at the rock surface at different sites and with differing weather conditions showed a range of 0.2–20 °C for flow rates of 5 to several hundred L h⁻¹, which correspond to a sensible heat inputs of 20–10,000 W (Hasler, 2011).

Part of this heat is used to warm the surrounding rock while the water percolates to the ice level. The water that arrives at the ice level with temperatures >0 °C may advect sensible heat \( P_{\text{adv}} \) because the temperature at a macroscopic ice-water interface equilibrates at 0 °C. This results in (1) warming by \( \Delta T \) of the surrounding rock (and ice) mass with corresponding change in liquid water content of the porewater (\( \Delta LWC \)); and (2) geometry change (mass loss/gain) of cleft ice as a function of the average cleft ice level (\( \Delta z \)) if the cleft aperture (\( d_{\text{c}} \)) is constant (Figure 2). Accordingly, energy conservation in an advectively influenced cleft environment including conductive heat flux \( P_{\text{cond}} \) can be described as:

\[
(P_{\text{adv}} + P_{\text{cond}}) \cdot t = \Delta T \cdot C_1 + \Delta z \cdot C_2 + \Delta LWC \cdot C_3
\]

with the left terms being the heat input for a given duration \( t \) and the right terms being the internal energy of the rock mass and latent energy change of cleft ice change and pore-ice change. The constant \( C_1 \) is the heat capacity of the volume with changed temperature and \( C_2 \) and \( C_3 \) are the cleft geometry (width, length) and pore water content multiplied with the latent heat of fusion (the general heat transport equation as a physical formulation of these processes may be found in the supporting material). The advected sensible heat \( P_{\text{adv}} \) tends to zero when water that percolates into the cleft is at or close to 0 °C, but heat transport still takes place when latent heat of ice
aggradation is released ($\Delta z > 0$) and absorbed by cold rock ($\Delta T$) around the cleft. If ice is eroded by $P_{adv}$, latent heat is consumed and exported from the system as runoff ($\Delta z < 0$). Liquid pore water content ($\Delta LWC$) and cleft ice geometry ($\Delta z$) are both reservoirs of latent energy but differ in their reversibility and feedbacks: Liquid water that stays in place after melt will refreeze at the end of the thawing season and ground temperatures below the freezing transition indicate the enthalpy of the system and are inter-annually comparable. This is true for the pore water because we consider the average pore water content ($LWC$+pore ice) as constant due to the low hydraulic permeability of most rock types. Mass loss of cleft ice, which is much more mobile, reveals the enthalpy change and exports the advected energy with the eroded ice volume. Such geometry changes are essential because the melt-interface will be at a new depth for subsequent thawing events modifying hydraulic and thermal conditions and changing the sensitivity to future thawing events. Hence, permafrost degradation is not necessarily visible in ground temperature but changes in ground ice content need to be considered. Thereto a quantification of the heat release and its effect rock warming ($\Delta T$) and ice level ($\Delta z$) will be performed in the following.

**METHODS AND MODEL SETUP**

Our conceptual model of heat exchange was examined by focussing on a single cleft (Figures 1 and 2) using a combination of numerical analyses with a two-dimensional finite element model and physical experiments with an artificial cleft between two granite blocks in a cooling chamber.

The numerical model was formulated from the conceptual model with a heat conduction scheme and a moving ice-water interface. The fluid-dynamics of the flow on this interface and the corresponding heat exchange with the ice and surrounding rock were parameterized with heat transfer coefficients. The laboratory experiment dimensions were chosen based on typical cleft dimensions (Hasler et al., 2011) and initial numerical experiments to estimate the volume being thermally influenced by the cleft within the given time frame.

Experimental runs initially took place with constant heat inputs into the cleft (stationary experiments). These were used to calibrate the heat transfer coefficients to fit the observed ice level changes. The numerical model was then used to investigate ice erosion and rock warming by systematically varying advective heat input, rock temperature, and cleft size. In addition, a combined forcing by transient surface temperature and advection (cyclic experiments) was applied to the laboratory experiments to simulate thawing after a cold time period. Finally, scale effects and variable advective heat inputs were investigated numerically.

**Numerical model**

COMSOL Multiphysics software was used to solve the two-dimensional heat-conduction equation and the moving boundary of the ice-water interface (Stefan-Problem). The model geometry corresponded to a cross-section perpendicular to a cleft and consisted of the two rock sub-domains and simplified rectangular cross sections of the water flow-body and the ice within the cleft (Figure 3). The rock and ice sub-domains had an initialization temperature $T_{rock}$ (initial condition). The *driving variables* of the model were either the flow $Q$ and mean water temperature $T_w$ or the advective heat source $P_{adv}$, and the upper and lower boundary conditions ($T_{up}$, $T_{bot}$). Heat flux through the lateral boundaries was set to zero corresponding to the insulation in the laboratory experiments (Figure 3). To simplify the model, porewater was neglected ($\Delta LWC=0$) and the rock was considered homogeneous and isotropic, corresponding to low-porosity rock and the laboratory settings (see below). To achieve stable model operation and a correct solution of the heat exchange at the moving boundaries (ice-water interface), the mesh was refined toward the cleft and model time steps were limited to 10 or 20 seconds. Both stationary and cyclic experiments are transient model runs and the term *stationary* refers only to the conditions of the driving variables (no steady-state modelling).

Flow characteristics (laminar or turbulent flow) along the ice surface in the cleft define the rate of heat exchange at the fluid-solid boundaries (water-ice and water-rock). Epstein and Cheung (1983) give a review of flow characteristics, heat exchange and interface instabilities on ice–water interfaces. Based on their model of simple phase change in an externally forced fluid flow, we used a heat transfer coefficient $h_{int}$ as input to the phase change boundary. For initial modelling, standard values for $h_{int}$ for a laminar flow of water were taken from literature (2000 W m$^{-2}$ K$^{-1}$). However, positive feedbacks between the interface geometry and the flow are known to lead to turbulence and higher coefficients (Gilpin et al., 1980). Because the geometry of the rectangular flow cross-section in the model does not accurately represent real flow heights and geometry,
the lateral heat transfer coefficient $h_{\text{side}}$ is different from $h_{\text{int}}$. This difference is considered by a scaling factor $sf$, which includes the effect of real flow height ($sf = h_{\text{side}} / h_{\text{int}}$).

The ice geometry change of the cleft ice is implemented via the moving mesh application mode of COMSOL. The melt-interface, that is the upper boundary of heat conduction in the ice sub-domain, is set constant at zero degrees Celsius and coupled to the water sub-domain via the heat transfer coefficient $h_{\text{int}}$. The resulting heat flux discontinuity ($\Delta q_i = q_{i,\text{up}} - q_{i,\text{down}}$), is divided by the volumetric latent heat of fusion ($L_f$) and defines the velocity of the vertical displacement of the melt-interface $dz/dt$. The displacement of the interface is averaged over the cleft width ($d_{cl}$) to avoid lateral mesh deformation:

$$
\frac{dz}{dt} = \frac{\Delta q_{i,\text{mean}}}{L_f} = \frac{dx}{2} \int (q_{i,\text{up}} - q_{i,\text{down}}) dx \left( d_{cl} \cdot T_f \right)
$$

(2)

Therefore the model reproduced the movement of the mean ice level but not the shape of the melt-interface. The water sub-domain was shifted according to the melt-interface velocity. The ice sub-domain was deformed by the interface movement at the top and kept stable at the lower boundary.

The output of the model consisted of the temperature field, heat flux density and ice geometry for each time step. Heat fluxes through the boundaries of the water sub-domain and the velocity of the melt interface ($dz/dt$) were extracted as single values once they were in equilibrium (constant) or as a time slice if they were changing significantly. Model operation was verified by a visual check of the distributed model outputs, by an evaluation of maxima and minima within the solutions and by cross-checking of the calculated heat fluxes and ice erosion rates.

### Laboratory experiments

The laboratory experiments were conducted in the climate chamber that can be controlled between outside air temperature and approximately $-20$ °C. It is subject to fluctuations due to regulation and defrosting of the refrigerator. Strong ventilation prevents air stratification inside the cooling chamber and increases heat exchange at exposed surfaces. Additionally, a basal cooling plate controls the bottom temperature. Harris et al. (2008) describe the cooling room with the basal cooling plate in detail.

The single-cleft model simulates the heat advection by water flowing along an ice surface within a frozen rock cleft and corresponds to the volume element shown in Figure 1. The vertically dipping cleft is formed by two inclined granite blocks with a spacing ($d_{cl}$) of approximately 4 and 8 mm, which is filled with ice (water frozen in situ) up to a defined initial ice level (Figure 4).
The ice level is kept parallel to the surface ($\alpha=12^\circ$) or at an inclination of $\alpha=3^\circ$ or $\alpha=30^\circ$ for the stationary experiments and at 12° as initial condition of the cyclic experiments. A controlled water flow $Q$ of temperature $T_w$ flows along the cleft. The granite blocks are insulated at their sides to minimize the lateral heat flux through these boundaries. The temperature at the surface ($T_{up}$) is controlled by the cooling room, and the bottom is kept at a constant temperature ($T_{bot}$) by the basal cooling plate. Granite with a low porosity (< 0.5 %) and homogenous characteristics was used to minimize anisotropy and latent heat effects of pore water ($\Delta LWC$ close to 0). Sawed cleft surfaces lead to a reduced roughness and a more constant cleft width compared to natural situations. This setting allows a quantification of ice volume changes by simple ice level measurements but may lead to more regular flow conditions than in natural clefts. Water was cooled to the input temperature $T_{win}$ by a looped copper tube in an ice-water bath and the flow rate was regulated by a mechanical dosing valve. The stability of $Q$ and $T_{win}$ was low due to feedbacks between flow and water temperature and could not be easily controlled to the desired input temperature.

Experiments are defined by input parameters specified for each run and result variables are recorded. Input parameters are the initial model temperature ($T_{rock}$) and driving variables $T_{win}$, $Q$, $T_{bot}$ and $T_{up}$. Result variables are the ice level changes $\Delta z$ and temperatures within the model. The vertical distance of the ice level to the block surface is manually measured at five positions (50, 150, 250, 400 and 450 mm from the inlet side) using a ruler with a precision of ±2 mm. These raw values are used to calculate the respective ice levels z1 to z5 and average ice level change $\Delta z$. The spatial distribution of rock temperatures is measured by a total of 50 thermistors placed in three transects (I=50 mm, II=250 mm and III=450 mm from inlet) perpendicular to the cleft, at 2, 40, 100 and 260 mm from the cleft.
(Figure 4). These temperatures are labelled according to the distance from the cleft and the height (z-axis) (Figure 4). Thermistors at the rock surface, the sides and bottom of the block provide boundary temperatures of block B (Figure 4; right block). Cleft aperture \(d_{cl}\) is surveilled with a crack meter at the top of the blocks and the output water temperature \(T_{wout}\) is measured in the funnel below the model (Figure 4).

A 60-channel Agilent 34970A data acquisition system was used together with negative temperature coefficient thermistors (YSI-44031) having an absolute accuracy of \(\pm 0.05 \, ^{\circ}C\) around the calibration point of 0 \(^{\circ}C\). Thermistors were calibrated before the instrumentation in a double coated ice-water bath. The measurement accuracy of the water temperatures \(T_{win}\) and \(T_{wout}\) is \(\pm 0.2 \, ^{\circ}C\) due to the influence of ambient temperature for the lowest flow rates and better if \(Q>10 \, L\, h^{-1}\). Water flow was measured using multi-purpose Digmesa FHKSC flowmeters. The sensors have an absolute accuracy of only \(\pm 20\%\) but better relative accuracy for temporal variations. Values are recorded every 10 seconds for stationary experiments and every minute for cyclic experiments.

**Calculation of heat input \(P_{adv}\) and latent heat flux \(P_{lat}\)**

To link laboratory experiments and numerical modelling, the two main heat fluxes need to be defined considering the geometrical setting and the measured parameters. The sensible advected heat flux \(P_{adv}\) is calculated by the cooling of the water within the cleft \((T_{win} - T_{wout})\) multiplied with \(Q\), the heat capacity of water \(C_{w}\) in J \(m^{3}\)\(K^{-1}\).

\[
P_{adv} = Q \cdot (T_{win} - T_{wout}) \cdot C_{w} \tag{3}
\]

If \(Q\) is in L \(h^{-1}\) (as here) a divider of 3.6*10\(^6\) should be added to (3) to obtain Watt. For not constant fluxes and temperature differences such as the case in some laboratory experiments, \(P_{adv}\) needs to be calculated for each point in time and averaged subsequently.

The latent heat flux \(P_{lat}\) is deduced from the erosion rate \(dz/dt\) multiplied by the cleft width \(d_{cl}\), the length of the cleft \(l\) and the latent heat of fusion \(L_{f}\):

\[
P_{lat} = L_{f} \cdot 0.917 \cdot d_{cl} \cdot l \cdot \frac{dz}{dt} \tag{4}
\]

For the non-standart units used here, (4) needs to be multiplied with 6*10\(^{-5}\) to obtain Watt. The factor 0.917 applied on \(L_{f}\) arises from the lower density of ice. For the 2-dimensional modelling these values of the heat fluxes are divided by the length \(l\) of the experiment.

**Model runs and calibration**

The advection experiment consists of two main phases (stationary and cyclic) with several runs each. Detailed information in the settings of the laboratory experiments is provided in the supporting material (Table 1). For the stationary laboratory experiments this variation is not strictly systematic due to limitations in the control of \(Q\) and \(T_{win}\) however rock and water temperatures and heat input cover the following range: \(T_{win}\) varies between 0 and 6 \(^{\circ}C\), \(Q\) is in the order of 4 to 60 L \(h^{-1}\) and \(T_{rock}\) is in the range of -1 to -6 \(^{\circ}C\). This does not span all possible values in nature at the near-surface but is realistic for the conditions at some meter depth in alpine permafrost. The numerical simulations of the stationary experiments reproduce these settings for model calibration before systematic variation was applied to analyze the sensitivity of the ice erosion and rock warming on these settings. A detailed list and graph of the cyclic experiment settings is also included in the supporting material. For these experiments the surface temperatures fluctuated for all cycles between -4 and +4 \(^{\circ}C\) and water flow \((Q)\) of a total volume \(V_{w}\) was applied after a time delay \(\Delta t\). \(Q\) is limited to approximately 5 L \(h^{-1}\), hence the duration of the advection event depends mainly on the applied water volume \(V_{w}\). The water temperature was kept as constant as possible around 2 \(^{\circ}C\) for all experiments and only \(V_{w}\) and \(\Delta t\) are variable for the cyclic experiments (supporting material).

The heat transfer coefficient \(h_{int}\) and the scaling factor \(sf\) of the numerical model depend on the flow conditions, which result from the flow rate, the cleft width and the inclination of the ice surface. Assuming Reynolds Numbers in the order of 10\(^2\) to 10\(^3\) derived from average flow velocity observations in the laboratory, we expect laminar flow for this open channel situation. However, the observed step formation of the melt-interface and the retrograde erosion at the model outlet indicate the presence of local turbulence. To estimate the scaling factor we neglect these features and used a simple approximation of the flow height based on the flow velocity measurements. A linear function is used to parameterize this scaling factor dependent on the flow \(Q\) and the cleft width \(d_{cl}\) (supporting material). The value of the scaling factor has minor influence on the model results. The heat transfer coefficient \(h_{int}\) was determined by two different calibration procedures (supporting material). Both methods provide parameters in the same order of magnitude. A linear function for the flow dependent heat transfer coefficient is used for each cleft size
(see supporting material). If $T_w$ is used as driving variable, errors of $h_{int}$ propagate proportionally to the resulting erosion rate and warming of ice. For model runs driven by $P_{adv}$ the influence of $h_{int}$ is only significant when erosion rates are close to zero. $P_{adv}$ is used as driving variable for all experiments, except for the analysis of the scale effects.

**RESULTS**

**Laboratory experiments**

Several qualitative and semi-quantitative observations were made during the experiments that are important for interpretation of the quantitative results. One to four steps, 2-5 cm high, formed in the cleft ice surface with the exception of when the ice was inclined at only 3°. These irregularities affected those stationary experiments with strong erosion as well as cyclic experiments. Frontal erosion of the ice occurred at the lower side of the cleft (water outlet; close to $z_5$) and an over-deepening of the ice-water interface developed at the water inlet (at $z_I$). This required sealing the upper side of the cleft to avoid water loss. Some ice remained on the lateral cleft surfaces at low rock temperatures so the entire cleft ice volume within $dz$ was not subject to melt. These observations indicate the importance of flow conditions for local ice erosion rates and the potential for warm water to cause rapid ice-erosion. For each stationary experiment, water temperatures within the cleft were measured intermittently, indicating a linear temperature decrease along the cleft. The cleft width ($d_{cleft}$) did not change significantly as long as the model remained frozen.

The results of one stationary experiment are shown in Figure 5a. The advective heat flux according to equation (3) and the erosion rate in the middle ($z_3$) of the experiment were constant (black line). The higher erosion rate at $z_5$ resulted from frontal erosion at the experiment outlet. The near-cleft temperatures (at $z = 300$ and 280 mm) increased until the ice level and corresponding water flow reached the level of the thermistor. Subsequently, the temperature dropped again with a slightly lower gradient. When two thermistors were passed by the erosion surface, the time between two temperature peaks indicates the erosion rate at this transect (Figure 5a). For the experiment shown, this value corresponds well with the manual measurements of ice erosion.

The average ice erosion rate based on all ice level measurements was calculated for stationary experiments. These rates depend on cleft width, advective heat input and initial rock temperature (Figure 5b). The number of experiments is not sufficient to quantify these relations empirically, but Figure 5b indicates that the erosion rate correlates with the applied advective heat for similar cleft widths and initial rock temperature ($T_{rock}$). The lines in Figure 5b indicate the maximum erosion calculated by equation (4) assuming that all heat is transformed into ice melt ($P_{int}/P_{adv} = 1$). Erosion rates exceeded this theoretical limit in one experiment (Figure 5b; *stat_13*: red square at 47 W). The water temperature had a further influence on the erosion rate because higher water temperatures at the same $P_{adv}$ (larger $T_{win}$ and smaller flow Q) resulted in more ice erosion ($P_{int}/P_{adv}$ is larger). However, this effect was minor compared to the influence of $P_{adv}$ and $T_{rock}$. The inclination $\alpha$ also had an influence on flow conditions ($h_{int}$ in the model) and modified the heat input $P_{adv}$ (supporting material). This explains the observed frontal erosion and may control the along-cleft form of the ice level.

The input parameters and other variables of interest in one cyclic experiment are shown in Figure 6a. The near-cleft temperatures at the ice level showed an abrupt increase when water was applied. A significant modification of the conductive warming was also observed at the thermistors at 40 mm distance to the cleft. The applied advective heat per cycle is proportional to the water volume $V_w$ (light blue area) if the water temperature remains constant as intended for all experiments. This was not the case and $T_{win}$ was significantly different from 2 °C for some cycles (Figure 6a). However, $V_w$ and $\Delta t$ were used as common settings that define cycle classes and the variability in $T_{win}$ affected the spreads (boxplot) in ice erosion per cycle within these classes (Figure 6b; supporting material). Step formation in the ice surface accumulating over several cycles may have been a further reason for the large spreads. Nevertheless, a qualitative dependency of the ice erosion on $V_w$ and $\Delta t$ is apparent, with the timing of the applied advective heat and the amount of heat of comparable importance (Figure 6b).
Figure 5: Results from the stationary laboratory experiments: a) measured driving variables ($T_{up}$, $T_{bot}$, $T_{win}$, $Q$), rock temperatures of transect II near the cleft (2_300, 2_280, 2_260), and ice levels ($z_1$-$z_5$) at the example of experiment stat_5 (4 mm cleft, $Q$=12 L h$^{-1}$, $T_{win}$=4.7°C). The line for $z_3$ indicates an erosion rate of 1.6 mm/min. The time difference between the two peaks indicates 20 mm erosion in 11 minutes (1.8 mm/min); b) ice erosion rates for different cleft sizes (point shape) as a function of applied advective heat flux $P_{adv}$; point colours indicate rock temperature at experiment start; the lines correspond to the maximal erosion rates when all applied energy is used for ice melt only: $P_{adv}$ of one experiment is proportional to the distance from 0 mm/min erosion and the distance to the corresponding line is proportional to the advective warming of the rock (example by arrows).

Figure 6: Results from the cyclic laboratory experiments: a) boundary conditions, flow ($Q$), water temperatures ($T_{win}$, $T_{bot}$), and near-cleft rock temperatures for warm period of cycle 30; b) ice erosion per cycle, ordered by applied water volume $V_w$ and time delay $\Delta t$. The box plot indicates the spread (variance (box) and extreme values (whiskers)) of all the ice level changes ($z_1$ to $z_5$) of the different cycles with similar settings.
Numerical model results
Figure 7 shows the near-cleft rock temperatures for a model run with $P_{adv}$ and $T_{rock}$ settings similar to those of the laboratory experiment stat_5. The amplitudes and time lags for two modelled temperatures (m_300 and m_280) correspond well with those for the thermistors 2_300 and 2_280 in the experiment. It should be noted that these temperatures were not used for the calibration of $h_{int}$ (supporting material) so this accord indicates that the model is operating correctly. In experiment stat_5, the ratio $P_{lat}/P_{adv}$ was 20 W/33 W, while the modelled ratio for the corresponding settings is 19.2 W/33.3 W.

The evolution of the modelled ice level ($\Delta z$) for different $P_{adv}$ and $T_{rock} = -3 \, ^\circ C$ is shown in Figure 8. For large heat inputs ($P_{adv} > 20$ W) the ice erosion starts within seconds after model initialization. If $P_{adv}$ is low, however, significant ice aggradation occurs. Similar aggradation was observed in the laboratory experiments (Figure 5b; negative erosion rate). The aggradation is greatest at the start of the advective heat flux due to the large sensible heat flux toward the sub-zero cleft environment. For simulations with moderate heat input (10 W > $P_{adv} > 5$ W; for 8mm cleft) ice aggrades during the first seconds or minutes, and is subsequently eroded or remains at a constant level if the heat flow for rock warming ($\Delta T$) balances $P_{adv}$ (Figure 8). The time until the heat fluxes are in equilibrium and $dz/dt$ becomes constant is referred to as equilibration time ($\Delta t_{eq}$).

SYNOPSIS AND DISCUSSION

Ice erosion, aggradation and stable conditions for constant heat input
A first step towards quantitative understanding of heat advection in frozen rock clefts is to quantify $\Delta z$ and $\Delta T$ for a constant $P_{adv}$. Figure 9 shows the modelled equilibrium ice erosion rates $dz/dt$ that result from a heat input $P_{adv}$ to a rock cleft at a given bulk temperature $T_{rock}$. Equilibrium is reached after $\Delta t_{eq} = 200-4000$ seconds. Ice erosion increases nearly linearly with the heat applied for rock temperatures ranging from 0 to -5 °C. The relative difference of $dz/dt$ between different rock temperatures decreases with $P_{adv}$ (Figure 9). This suggests that cleft ice is eroded independently of the temperature of the surroundings for large $P_{adv}$ once the erosion equilibrium is reached (after $\Delta t_{eq}$). The decreasing influence of $T_{rock}$ is caused by the larger ratio $P_{lat}/P_{adv}$, which results from smaller thermal gradients between the water and the warmed cleft surface at the water-rock interface while the ice surface remains at 0 °C. The erosion rates are of the same order of magnitude as those for the stationary laboratory experiments (Figure 5). In cases where the cleft orientation deviates strongly from the modelled vertical case, less heat is consumed by ice melt (smaller $P_{lat}/P_{adv}$) due to a larger contact area of the flow cross-section and the cleft surface, and possibly because of ice remaining at the upper cleft surface.

A multiple linear regression analysis was used to describe the ice erosion rate $dz/dt(P_{adv}, T_{rock})$ after $\Delta t_{eq}$ for an 8 mm near-vertical ice-filled cleft of 0.5 m length:

$$\frac{dz}{dt}[\text{mm/min}] = (0.05 + 0.04 \cdot P_{adv} + 0.1 \cdot T_{rock})$$ (5)

with the numerical values of $P_{adv}$ in W and $T_{rock}$ in °C. This linear approximation is represented by the lines in Figure 9 for the results from the numerical model. The quality of both the approximation and the numerical model results is low for small erosion rates and heat inputs due to larger relative errors. Even though an extrapolation with equation (5) to stable conditions of the ice level
and 13 of net erosion takes place the applied fluxed time until the aggraded ice erosion (ture of analyzed of these experiments is limited, these results were of heat advection (tween the start of surface warmi depends on the sum of applied advective heat but depends on the length of the considered cleft, the heat input per length unit (W m⁻¹) can be calculated by replacing the coefficient of 0.04 by 0.02 (multiplying by 0.5 m). In the supporting material the effect of variable heat input is discussed and can be summarized that a variable heat input causes slightly more ice erosion than a constant one and that this variability is more important if the heat input is small (close to stability conditions).

**Dependency of erosion rate on timing of advection and conduction**

Advective heat inputs into a cleft are usually accompanied by conductive heat from the surface. Depending on the geometric situation and the surface condition (bare rock vs. snow cover) this conductive warming has differing influences on the rock temperature around the ice level. The cyclic laboratory experiments indicate that the erosion of the cleft ice during a thawing event depends on the sum of applied advective heat but is also sensitive to changes in the time lag Δt between the start of surface warming and the start of heat advection (Figure 6b). As the data quality of these experiments is limited, these results were analyzed using equation (5). The small applied heat flux (P_{adv} = 14 W) results in a rock temperature of -5.7 °C for stability. The rock temperatures around the cleft ice level before percolation (Figure 6a) were in the range of -2 to -5 °C and erosion occurred after initial aggradation. The time until the aggraded ice was completely eroded is in the order of 3 to 30 min and is very sensitive to changes in P_{adv} and T_{rock}. If the duration of the applied flux is short, a net aggradation instead of net erosion takes place (see cycles 33–38, 8–12 and 13–20 in Figure 6b).

The amount of heat conducted through the rock that reaches the cleft ice level after a given time depends on the latter’s depth. If the ice were to aggrade over several cycles it would reach a level where heat conduction warms the rock more rapidly and aggradation would decrease or cease. Erosion does the opposite, moving the thawing front in the cleft to colder levels and resulting in a reduced rate of erosion. That leads to stabilization of the ice level after repeated similar cycles as was observed in the laboratory for cycles 8–12 (while a decrease in erosion per cycle occurred for cycles 39–42, as shown in the supporting material). In the field, this means that the ice level represents a hydrothermal equilibrium if conditions are similar on an inter-annual basis and if the ice level is at a depth of a few meters where annual cycles are present and the temperatures decrease with depth when sensible heat advection occurs. If the cleft ice erodes to a level where thermal gradients become reversed, similar hydrothermal conditions would lead to progressive thawing over the years until the lateral heat loss of the percolating water (Figure 2) is large enough to reduce the heat input at the cleft ice level. This indicates that relatively small climatic variations resulting in changed inputs of water could significantly alter cleft ice conditions.

Figure 9 Modeled ice erosion rates dz/dt in dependence of the advective heat input (P_{adv}) and for different rock temperatures (legend) in an 8 mm wide cleft. The points are the results from the numerical model, the lines indicate the approximation by the linear function (5). The relative difference of the erosion rate decreases with larger advective heat input. This corresponds with more heat used for ice melt regarding the applied heat (P_{adv}/P_{adv}). The residuals between linear approximation and model results appear larger for small erosion rates due to the logarithmic representation.

(6.25 W) coincides with the erosion rates of small P_{adv} values in Figure 8, much time is needed for these rates to reach equilibrium. The laboratory experiment with small advective heat inputs and cold rock temperatures indicates a similar range (equation (5)) with -5 °C: 11.3 W; Figure 5b: interpolation from squares ≈ 10 W). Even though equation (5) is based on only a few results from the numerical model it describes the dependency of the two main parameters P_{adv} and T_{rock}. A Pearson’s correlation coefficient of 0.94 was obtained when it was applied to all laboratory experiments (all cleft sizes). Hence, equation (5) can be used to provide a rough estimate of ice erosion for clefts approximately 1 cm wide, and to eliminate the length of the considered cleft, the heat input per length unit (W m⁻¹) can be calculated by replacing the coefficient of 0.04 by 0.02 (multiplying by 0.5 m). In the supporting material the effect of variable heat input is discussed and can be summarized that a variable heat input causes slightly more ice erosion than a constant one and that this variability is more important if the heat input is small (close to stability conditions).
Scaling of conductive and advective heat transport

The cleft size \(d_c\) influences advective heat exchange by modifying the flow cross-section for a given discharge (changed \(sf\) in the model) and the ice erosion rate changes because the latent heat used for erosion depends on \(d_c\) (ice volume). In the laboratory, a constant \(P_{adv}\) causes less ice erosion and less sensible warming if \(d_c\) is increased because more ice volume needs to be eroded for a given \(\Delta z\) (see Figure 5b).

If the whole experiment is scaled by a factor \(f\) and \(Q\) is multiplied by \(f^2\) (proportional to the surface supplying the cleft with water of temperature \(T_{win}\)), the ice erosion rate remains in the same order of magnitude. For a modelled cleft with \(d_c = 80\) mm, for example, the erosion rate \(dz/dt\) increased by only \(50\%\) compared to one with \(d_c = 8\) mm. This change is caused by an increase of \(P_{bud}/P_{adv}\) at larger dimensions. If the heat input in the up-scaled experiment lasts for factor \(f\) longer (time scaling of \(f\)), the ice level change relative to the experiment scale remains in the same order of magnitude (e.g. increased by the \(50\%\) from the above example). For the situation including conductive heat flux (cyclic experiments) the time scaling of the two processes is not similar: in an up-scaled situation, the conductive heat wave needs \(f^2\) more time to propagate from the surface to a depth increased by the factor \(f\) (cf. model result in Figure 3c). This different time scaling of the advective and conductive regime (\(f^2\) vs. \(f\) respectively) does not allow a direct scaling of the cyclic experiments to annual cycles of a larger volume. The cyclic laboratory experiments, therefore, illustrate the processes but do not provide a quantitative basis for up-scaled conditions. In contrast, the stationary experiments can be transferred to other scales and allow estimates of ice erosion and rock warming in larger clefts.

CONCLUSIONS

Several specific conclusions can be reached from this study:

1) Advective heat transport by percolating water is a highly efficient process to transmit heat from the surface to the level where clefts become impermeable. The conceptual model suggests that heat exchange with the surrounding rock takes place mainly in the area of the cleft ice level as the water accumulates there and runs off laterally. Progressive thaw of cleft ice conceals the effect of warming events by export of latent heat making them not directly detectable in ground temperature records. As a consequence the local degradation of permafrost along rock clefts may be at least partially hidden from thermal monitoring.

2) More than half of the advected sensible heat is consumed by phase change of the cleft ice erosion while the minor part results in rock warming. The findings from the laboratory experiments and the numerical models indicate that ice erosion occurs even in cold rock if the applied heat along a cleft is large enough (>28 W m\(^{-2}\) for -6 °C and >12 W m\(^{-1}\) for -3 °C in a 8 mm cleft). This ice erosion occurs only after the cold rock has locally warmed due to the latent heat released by ice aggradation during initial percolation. For large advective heat inputs the rock temperature has only a minor effect on the erosion rate. Conversely, percolation with only a small heat input into a cold rock cleft results in ice aggradation.

3) The main parameters that govern ice erosion and rock warming in a frozen cleft are the advective sensible heat input (resulting from water flow rate and water temperature) and rock temperature. Other factors, such as scale effects, water temperature alone (for the same heat input) and the variability of the flow, slightly alter the ratio between energy used for ice erosion and rock warming but do not strongly modify the absolute values. Laboratory experiments simulating melt water input during a thawing event indicate that the timing of the initiation of the sensible heat advection relative to conductive warming from the surface is important. Given the small heat fluxes applied during these experiments, this effect is attributed to ice aggradation during initial water percolation.

A general conclusion is that rockfall, if related to the warming and ice erosion in clefts, is not necessarily limited to areas with relatively warm permafrost. This may account for the low correlation observed between modelled permafrost temperature and rockfall occurrence. Further the study shows that heat applied from the surface by advection may reach failure planes at depth more rapidly and cause events of a greater magnitude than would be expected based on the assumption of conductive heat transfer.

Studies with hydro-thermal models, as well as geophysics in steep bedrock may contribute to a better understanding of permafrost degradation relating to ground ice loss. Of particular interest would be the coupling of such models with meteorological data so that the sensitivity of clefts to different meteorological situations could be assessed and compared to observed rockfall activity.
LIST OF TERMS

\( \alpha \) inclination of the ice surface along the cleft \([\degree]\)
\( \Delta LWC \) change in liquid pore water content [%]
\( \Delta t \) time delay between start of warming and applied advection in cyclic experiments [h]
\( \Delta t_{eq} \) equilibration time needed until the ice erosion rate remains constant [s]
\( \Delta T \) temperature change of rock (and ice) in the considered volume \( \left[{^\circ}C\right] \)
\( \Delta z \) cleft ice level change [mm]
\( C_v \) heat capacity of water (volumetric; liquid water): 4.18 MJ/m\(^3\) K
\( d_i \) cleft width (aperture) [mm]
\( \frac{dz}{dt} \) ice erosion rate (positive values for \( \Delta z < 0 \)) [mm/min]
\( h_{lat} \) heat transfer coefficient for water–ice interface \( [W/m^2\cdot K] \)
\( I_f \) latent heat of fusion (volumetric; liquid water): 334 MJ/m\(^3\)
\( l \) length of cleft in experiment; constant: 0.5 m
\( P_{cond} \) conductive heat flux [W]
\( P_{adv} \) advective heat flux [W]
\( P_{lat} \) latent heat flux due to cleft ice change (\( \Delta z \)) [W]
\( Q \) water flow (flow rate) [L/h]
\( q \) heat flux \([W/m^2]\)
\( s_f \) scaling factor for the sideward heat transfer coefficient (water–rock) \( (s_f = h_{side} / h_{lat}) \)
\( T_m \) mean water temperature in the cleft (empirically: \((T_{win} + T_{wout})/2\)) \( [^\circ C] \)
\( T_{win} \) water temperature at the inlet \( [^\circ C] \)
\( T_{wout} \) water temperature at the outlet \( [^\circ C] \)
\( T_{rock} \) initial or undisturbed rock (and ice) temperature \( [^\circ C] \)
\( T_{rot}, T_{rot} \) measured rock temperature within the physical model (laboratory) \( [^\circ C] \)
\( T_{up} \) temperature of upper boundary (surface in laboratory) \( [^\circ C] \)
\( T_{bot} \) temperature of lower boundary (bottom) \( [^\circ C] \)
\( V_o \) applied water volume in the cyclic experiments (L)
\( z_{1–5} \) cleft ice level along the cleft at 50–450 mm from the inlet [mm]
\( z \) ice level (in general, modelled, averaged)

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Supporting Material

General heat transport equation

Equation 1 derived by the time can be written as the general heat transport equation for a porous rock mass (neglecting pore water migration) containing partly permeable clefts with $Q$ being the water flow through the cleft, $T_w$ the water temperature, $C_v$ the volumetric heat capacity of water, $K_r$ the thermal conductivity of the rock, $L_f$ the latent heat of fusion of water, $V_{por}$ the pore volume, $sat$ the water saturation of the pores and $C$ and $T$ the heat capacity and temperature of the considered volume element:

$$C_v \cdot Q \cdot V_T + V \cdot (K \cdot \nabla T) = \frac{\partial T}{\partial t} + L_f \cdot V \cdot Q + L_f \cdot V_{por} \cdot sat \frac{\partial LWC}{\partial t}$$  \hspace{1cm} (6)

In (6) the cleft ice mass change is expressed by mass gain or loss in the cleft water flow assuming that the liquid cleft water is not retained. This formulation does not address the Stefan Problem because the heat exchange between the water and the ice is not expressed. Thereto we need a formulation, how efficiently sensible heat is transformed into latent heat. This is expressed by (2) in the formulation of the numerical model.

Sensitivity analysis, parameterization and calibration of the numerical model

The model parameters, influencing the model output to different extents, are grouped in four categories: The internal model parameters and the material parameters, are set to constant values for all experiments. The heat exchange parameters being the heat transfer coefficient $h_{int}$ and the scaling factor $sf$. Finally there are the input parameters (initial condition ($T_{rock}$, $\alpha$, $d_{cl}$) and initial ice level) and the time dependent driving variables being the heat input ($T_w$ and $Q$ or $P_{adv}$) and conductive boundary conditions ($T_{w, n}$ and $T_{def}$)). Their effect on the model output is presented in the results section and they are not subject to this technical sensitivity analysis.

Internal model parameters have no physical meaning and are prescribed with high/low values ($K_w$=100 W m$^{-1}$ K$^{-1}$, $C_w$ = 1000 J kg$^{-1}$ K$^{-1}$) to set the whole water sub-domain to uniform temperature. The parameter $h_{con}$ couples the input variable $T_{w, mass}$ to the sub-domain and is set to 10$^8$ W m$^{-3}$ K$^{-1}$. The model results are not sensitive these values. The material parameters, the heat capacity and the thermal conductivity were set to $C_i = 1.93$ MJ m$^{-3}$ K$^{-1}$ and $K_i = 2.2$ W m$^{-1}$ K$^{-1}$, for ice and to $C_r = 2.4$ MJ m$^{-3}$ K$^{-1}$ and $K_r = 3.2$ W m$^{-1}$ K$^{-1}$ for rock (Cermak and Rybach, 1982). Rock parameters varying $\pm$30% cause no significant change of ice erosion rates but the sensible heat flux from water to the rock $P_{slikr}$ is slightly modified. This may influence the results in situations where the melt-interface is nearly stable and when the model is driven by $P_{adv}$. Ice parameters are regarded as invariable for the temperature range considered.

The scaling factor was derived from the estimated flow heights of the laboratory experiments. Travel-time measurements from inlet to outlet are converted into flow velocities ($v_f$). This values in the order of 0.1–0.5 m/s is used to estimate the flow height and scaling factor correspondingly:

$$sf = \frac{Q}{(v_f \cdot d_{cl}^2)}$$  \hspace{1cm} (7)

A linear parameterisation function for $sf$ is fitted through estimated ratios of the flow height over cleft aperture (Figure S1a).

The heat transfer coefficient $h_{int}$ is deduced in two ways: 1) For a given $T_w$, $h_{int}$ is adjusted until the modelled interface velocity in the model corresponds to the ice erosion rate $dz/dt$ (averaged over the middle three measurements $z_2$, $z_i$ and $z_3$); and 2) $P_{adv}$ is used as driving parameter and $h_{int}$ is adjusted to reach the measured values for $T_w$. The first procedure is sensitive to errors in the ice level measurements, whereas the second depends on $sf$ and provides inaccurate values in case of strong frontal erosion (erosion of the cleft ice at outlet). For the experiments with 3° inclination, we used only the first procedure because of significant frontal erosion. For all other experiments, method two provided more consistent results, which were used to calculate linear regressions (Figure S1,b). The heat transfer coefficient increases with inclination and $Q$ except for the 3° inclination and are approximated by the linear function: $h_{int} = 2300 + Q\times90$ (Figure S1,b). The accuracy of the absolute values of $h_{int}$ is limited due to the data quality of the lab measurements but this is relevant only if $T_w$ is used as driving variable.

Overview of experimental runs

The laboratory experiment consists of two phases with several runs each (Table 1). Prior to the actual advection experiment, test and calibration runs where conducted (Table 1) to check sensor calibration in situ and to evaluate rock thermal parameters for use in the numerical model. In the first stationary phase, ice erosion is observed and the warming of the rock is recorded every 10 seconds. The cyclic experiments of the second phase simulate the combination of a conductive heat wave from the rock surface with advective heat input at the melt interface. Values are recorded
every minute. Aggradation and erosion of the cleft ice during these thawing cycles are observed only by their integral effect on the ice level as it is evaluated once per cycle. In Figure S2 all the ice level measurements at the end of each thawing phase. The ice level change for cycles with same settings (Vw and Δt) vary strongly because the water temperature was different between cycles (1.5–3.5 °C). Further the ice level change depends on the depth of the ice level within the experiment because of the damping and latency of the conductive heat wave with depth. this may lead to a stabilization of the ice level after many cycles with similar parameters. This effect is adumbrated for the cycles 9–13 and 29–33 (Figure S2).

**Variable versus constant heat input – numerical sensitivity study**

The applied water flow and temperature does not remain constant over time in nature and the question, if variable \( P_{adv}(t) \) has similar effects as its temporal mean (\( P_{adv\_mean} \)), is essential for a transfer of results to natural situations. As an extreme case of variable advection we discuss the effect of a pulsed heat input in comparison to a constant \( P_{adv} \): \( \Delta t_{adv} \) is the duration of water being applied to the cleft and \( \Delta t_{dry} \) is the time period with no water in the cleft and with possible negative temperatures at the melt interface. Comparisons of the modelled pulsed heat input with \( P_{adv\_mean} \) (averaged advective heat input) show that \( P_{lat} \) and the corresponding ice erosion are larger in the pulsed case (Figure S3). This is explained by the fact that the ratio \( P_{lat}/P_{adv} \) does not remain constant: With \( P_{adv} \) being larger than \( P_{adv\_mean} \) less of the heat applied is used to warm the rock and, consequently, more heat is available for ice melt. This causes a large relative difference of \( dz/dt \) if the ice level is nearly stable (Figure S3; const5W vs. puls10W). With cyclic heat input where \( P_{adv} \) remains positive (\( cycl = 10 \pm 8 \) W) and water is always available for refreezing, there is no significant increase of \( dz/dt \) compared to constant

![Figure S1](image1.png)

**Figure S1**  Numerical model parametrization: a) estimated scaling factor between downward and sideward heat transfer coefficient (\( sf = h_{int}/h_{side} \)) based on flow height estimates; b) heat transfer coefficient \( h_{int} \) is fitted to the observed with values deduced from the lab experiments.

![Figure S2](image2.png)

**Figure S2**  Results of the cyclic experiments: Ice levels of all cyclic experiments with corresponding applied water volume \( V_w \) and time delay from start of warming to start of water discharge \( \Delta t \).
heat input (Figure S3) The faster start of ice erosion of the simulations with cycles is due to the higher initial values of $P_{adv}$ during the first half period. As water percolation and sensible heat supply under natural conditions will show clear diurnal and annual cycles, these thermal effects of variable advection have to be considered. If water percolation stops completely for some time, larger erosion rates compared to the ones of $P_{adv\,mean}$ have to be expected. To the current state of knowledge, we also have to expect this effect for longer time scales than modelled. This may significantly modify the ice erosion in cases where $P_{adv\,mean}$ is in the order of stable conditions. In these cases, the interruption of the advection has to be represented in a model of heat advection. For simulations with significant average erosion or variations without ebbing of the flow, the use of an averaged heat input is appropriate.

| Table 1. Experimental runs of the laboratory experiments |
|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| phase           | run            | $d_{cal}$   | $\alpha$ | $Q_{a}$ ($V_{a}$) | $T_{win}$   | $T_{bot}$   | duration, (time delay) | $dz/dt$ |
| therm. test     | 3              | 12          | 0        | -                | 0           | -           | 0               | 1 d   |
| temp. step      | 3              | 12          | 0        | -                | -10         | -10         | 1 d             | 1 d   |
| initial freeze  | 3              | 12          | 0        | -                | -10         | -10         | 1 d             | 1 d   |
| stat3_1         | 3.6            | 12          | 4.5      | 5.92            | -1          | -1          | 21 min          | 1.33  |
| stat3_2         | 3.6            | 12          | 4.4      | 0.97            | -1          | -1          | 73 min          | 0.11  |
| stat3_3         | 3.6            | 12          | 3.9      | 1.7             | -1          | -1          | 38 min          | 0.25  |
| stat3_4         | 3.6            | 12          | 9.8      | 2.2             | -1          | -1          | 24 min          | 0.54  |
| stat3_5         | 3.6            | 12          | 11.7     | 4.69            | -1          | -1          | 20 min          | 1.78  |
| stat3_6         | 3.6            | 12          | 15.8     | 3.4             | -1          | -1          | 11 min          | 1.59  |
| stat3_7         | 8.7            | 12          | 8.3      | 0.79            | -5          | -6          | 32 min          | -0.23 |
| stat9_2*        | 8.7            | 12          | 11.5     | 1.71            | -5          | -6          | 42 min          | 0.22  |
| stat9_3         | 8.7            | 12          | 11.6     | 1.61            | -5          | -6          | 30 min          | 0.02  |
| stat9_4         | 8.7            | 12          | 19.7     | 5.29            | -5          | -6          | 25 min          | 1.47  |
| stat9_5*        | 8.7            | 12          | 21.9     | 5.07            | -1.5        | -1.5        | 8 min           | 2.83  |
| stat9_6         | 8.7            | 12          | 29.6     | 4.19            | -1.5        | -1.5        | 9 min           | 2.52  |
| stat9_7         | 8.7            | 12          | 29       | 2.63            | -1.5        | -1.5        | 8 min           | 2.27  |
| stat9_8         | 8.7            | 12          | 58.6     | 4.53            | -1.5        | -1.5        | 7 min           | 4.05  |
| stat9_9         | 8.7            | 12          | 10.2     | 2.03            | -1.5        | -1.5        | 46 min          | 0.51  |
| stat9_10*       | 8.7            | 12          | 4.9      | -0.13           | -1.5        | -1.5        | 22 min          | -0.06 |
| stat9_11        | 8.7            | 12          | 18.5     | 1.44            | -1.5        | -1.5        | 30 min          | 0.1   |
| stat9_12*       | 8.7            | 3           | 10.3     | 1.38            | -1.5        | -1.5        | 56 min          | 0.14  |
| stat9_13        | 8.7            | 3           | 5.5      | 1.13            | -1.5        | -1.5        | 102 min         | 0.11  |
| stat9_14        | 10             | 3           | 8.6      | 1.83            | -1.5        | -2          | 80 min          | 0.1   |
| stat9_15        | 10             | 30          | 8.5      | 1.9             | -1.5        | -2          | 92 min          | 0.66  |
| cycl. 1-6       | 10             | free        | 5        | (2L)            | 2           | -5          | 4 to 4          | 1d, (4.5h) |
| cycl. 8-12      | 10             | free        | 5        | (2L)            | 2           | -5          | 4 to 4          | 1d, (2.5h) |
| cycl. 13-20     | 10             | free        | 5        | (4L)            | 2           | -5          | 4 to 4          | 1d, (2.5h) |
| cycl. 21-26     | 10             | free        | 5        | (4L)            | 2           | -5          | 4 to 4          | 1d, (3.5h) |
| cycl. 28-32     | 10             | free        | 5        | (6L)            | 2           | -5          | 4 to 4          | 1d, (2.5h) |
| cycl. 33-38     | 10             | free        | 5        | (2L)            | 2           | -5          | 4 to 4          | 1d, (2.5h) |
| cycl. 39-42     | 10             | free        | 5        | (8L)            | 2           | -5          | 4 to 4          | 1d, (2.5h) |

* = runs are not considered for analysis because of irregularities in $Q$ or $T_{win}$

$dz/dt$ is calculated by averaging $z_2$, $z_3$, and $z_4$ to exclude disturbances from inlet and outlet.

$\alpha$ = free indicates that the ice surface is not levelled between cycles.

Figure S3: Ice level evolution of the numerical model with constant, pulsed (1:1 ratio) and cyclic 10 ± 8 W (sinusoidal) heat input with $T_{Trock}$ = -3 °C. The label of the pulsed heat input corresponds to the maximum value, hence pulsed10W has same average as const20W