A Moving Target in a SAR Image Analysed with Time-Frequency Methods
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Abstract
Moving targets within SAR scenes are distorted depending on the particulars of the target motion. For small motions, the uncompensated phase of a point reflector is a sum of a term proportional to the range component of the target motion, and a more complicated term depending on the azimuth motion. The resulting phase may be analysed with time-frequency techniques since the motion effect may alternatively be seen as a time dependent Doppler frequency. An experiment performed with a moving target within a scene collected by the German E-SAR system gave a signature that agrees well with theoretical predictions. Time-frequency analysis, using the smoothed pseudo Wigner-Ville method, gave a linear chirp with superimposed oscillations as predicted from the theory and the target motion.

1 Introduction
Synthetic aperture radar (SAR) is a useful radar technique to generate images of a scene with fairly high resolution from standoff ranges. SAR imaging of stationary scenes is well understood theoretically [1], and the major issue is the speed and accuracy of the SAR processor. For a moving target within the scene, the situation is more complicated. On the one hand, it is easier to detect the target using techniques different from SAR [2]. On the other hand, target motion results in image distortion in the SAR image of the target itself. A well known example is the azimuth displacement of a target with a small constant range rate. More complicated motion leads to other effects. Vibrations and rotations cause micro-Doppler [3], and azimuth smearing may result [4].

We analyse the effect of general target motion on the SAR signal phase. For reasonably small motions, the residual uncompensated phase of a moving point target after the SAR processor consists of two terms, one proportional to the range component of the motion, and one containing the product of the platform motion and the target azimuth motion. This phase can be complicated, and time-frequency methods are useful for the analysis.

An experiment was performed in Lillestrøm, Norway, using the DLR E-SAR from Germany as SAR platform. A moving target with a known motion was present at the time of data collection. The motion was an approximately constant range rate, with an oscillation in the range direction superimposed. Results obtained with time frequency methods agreed well with theoretical predictions.

2 Moving Targets in SAR Images
SAR processors usually assume that the scattering centres within the SAR scene are stationary during the time of data collection. Each point within the scene is then characterized by a unique phase history, and the SAR processor exploits the uniqueness to place the point within the scene.

2.1 General Motion
We consider a SAR platform moving along a straight line with constant platform velocity \( v_p \). We define the straight line as the x-axis in a cylindrical coordinate system as in [1]. At the time \( t = 0 \) the radar antenna phase centre is at the origin. At the same time, a target is exactly broadside at the range \( d = r_0 \). The target has a general motion described by its cylindrical coordinates \( s(t) \) as given in fig. 1.

Fig. 1 Geometry of a SAR system with a moving target.
Accordingly, the range from the radar to the target at a
general time is given by
\[ d(t) = \sqrt{(v_p t - x(t))^2 + r^2(t)}. \] (1)
The usual hyperbolic expression for the point target
range history is obtained for \( x(t) = 0 \) and \( r(t) = r_0 \).
Note that the cylindrical angle \( \theta \) does not enter the
equations as it does not affect the range when the
platform trajectory is a straight line. The phase history
of the target may then be obtained from
\[ \phi(t) = 2kd(t) = \frac{4\pi}{\lambda} d(t). \] (2)
Ideally, the SAR processor focuses the target using
the phase history
\[ \phi_0(t) = \frac{4\pi}{\lambda} d_0(t) = \frac{4\pi}{\lambda} \sqrt{v_p^2 t^2 + r_0^2}. \] (3)
Accordingly, the uncompensated phase may be found
as
\[ \Delta \phi(t) = \phi(t) - \phi_0(t). \] (4)
Depending on the particulars of the motion, the
uncompensated phase may result in a wide range of
phenomena, from a simple shift in position of the
target to a smearing making the target impossible to
see.

2.2 Small Motions

Equations (1) to (4) give the general results, but are
difficult to analyse directly. If the synthetic aperture
is sufficiently short as compared to the distance to the
scene, and the target motion is slow as compared to
the platform velocity, the square roots may be
approximated by parabolas in the usual way [1]. The
results are
\[ d(t) \approx r_0 + \frac{v_p^2 t^2}{2r_0} - \frac{x(t)v_p t}{r_0} + \Delta r(t) \] (5)
\[ d_0(t) \approx r_0 + \frac{v_p^2 t^2}{2r_0} \]
Here, \( \Delta r(t) = r(t) - r_0 \), which is much smaller than
\( r_0 \) under the stated conditions. Accordingly, the phase
residual is given by
\[ \Delta \phi(t) \approx \frac{4\pi}{\lambda} \Delta r(t) - \frac{4\pi x(t)v_p t}{\lambda r_0}. \] (6)
Note that for \( x(t) = x_0 \), a constant, the expression
represents the azimuth shift linear phase ramp to
move the reflector to the new position when
\( \Delta r(t) = 0 \). On the other hand, when neglecting the
second term and setting \( \Delta r(t) = v_c t \), we get an
azimuth phase ramp corresponding to the well known
azimuth shift of a target with a moderate constant
range rate \( v_c \). A target moving with a constant
azimuth rate will however introduce a quadratic phase
term, resulting in smearing of the target.

3 Time-Frequency Analysis

The phase residual as calculated in the previous
section, given in eq. (6), may alternatively be seen as
a Doppler shift. The basis is to define the
corresponding Doppler frequency as
\[ f_d = \frac{1}{2\pi} \frac{d\phi}{dt}. \] (7)
We see then that a linear phase ramp becomes a
constant frequency, while the quadratic phase
corresponding to constant azimuth rate becomes a
linear chirp. For complicated frequency dependencies,
time-frequency methods can be used for analysis.

3.1 Quadratic Time-Frequency
Methods

There is a large number of different time-frequency
methods that may be applied to a particular problem,
and it is not always obvious which one to choose. The
Cohen’s class of quadratic time-frequency methods
[5] offers some attractive alternatives due to the
potential for high resolution. This must be balanced
against the interference between signal components
inherent in such methods. The class is described by
\[ C(t, f_d) = \int W(u, v) \Psi(u-t, v-f_d) du dv, \] (8)
with \( W \) the baseline Wigner-Ville distribution given by
\[ W(t, f_d) = \int \int \Psi_{\psi}(t + \tau/2, v - f_d / 2) \exp(-j2\pi f_d \tau) d\tau d\tau. \] (8)
Various instances of Cohen’s class are generated by
choices of the kernel function \( \Psi \).

We use two methods, the smoothed pseudo Wigner-
Ville (SPWV) [6] and the adaptive optimal kernel
(AOK) [7]. Both methods use a kernel that is
essentially a low-pass filter, smoothing away high
frequency interference, while retaining low frequency
signal content. The SPWV is fairly simple and
computationally efficient due to the simple separable
kernel. The AOK method is more sophisticated as it
adapts the kernel to some extent to the underlying signal.

3.2 Alternative Methods

Other methods include the affine class and related methods. We often find the spectrogram and the instantaneous frequency useful. The spectrogram is a sliding window FFT for practical work. This is a method that is fast and gives little interference. The spectrogram is useful for a quick look. The instantaneous frequency can be defined from the analytic signal [8], and gives high resolution results for single component signals. In a practical situation, we typically apply several methods to examine the signal from various angles [9].

4 Experimental Results

A test was performed at Lillestrøm, Norway, 2 Jun 2003, using the German E-SAR system.

4.1 Description of Test

SAR scenes were collected over the test area at X-, L- and C-band. Several experiments were set up at the time. For the present study, a car was used as a moving target. A corner reflector was placed on the roof of the car as shown in fig. 2. The corner reflector was oriented to give maximum reflection in the general direction of the radar.

Fig. 2 A moving target: A car with a corner reflector on the roof.

The position of the car was measured using a GPS receiver. The car path was chosen as a small road nearly in the azimuth direction. The road was in the middle of a uniform farming field to minimize clutter problems for the relevant data area. The car path was a fairly slow azimuth motion, about 5 m/s, combined with a side-to-side oscillation covering the width of the road. The aim was a linear motion in azimuth combined with a sinusoidal motion in range.

4.2 SAR Results

Fig. 3 shows a part of an X-band image collected over Lillestrøm. The image is processed by DLR e.V., Germany. Clutter is reduced using multi-look processing and the resolution is approximately 2m. Azimuth is along x-axis and range along y-axis.

Fig. 3 Part of X-band E-SAR image of the Lillestrøm area. Processing by DLR e.V., Germany. The white rectangle shows the area containing the moving target signature.

The bright parts in the middle of the image correspond to buildings at the site of the Norwegian Defence Establishment (NDRE) and some other institutions. The uniform part to the right corresponds to farming fields. The white rectangle shows the location of the bright signature resulting from the moving target.

Fig. 4 shows a close-up of the area within the rectangle of fig. 3.
Fig. 4 Close up of moving target signature, shown as a rectangle in fig. 3.

The signature is seen to be a smear nearly in the azimuth direction as discussed in section 2.2. We see that the signature stands out clearly against a fairly uniform speckle background. The background is as we would expect form a framing field.

Taking an azimuth slice through the single-look complex representation of the signature and inverse Fourier transforming, we obtain a complex time series that may be analysed with time-frequency methods. A time frequency signature, calculated with the SPWV method is shown in fig. 5.

Fig. 5 Time-frequency signature of the moving target.

Since the ground target motion was approximately linear in azimuth, with a range oscillation superimposed, we expect the signature to be a linear chirp with sinusoidal modulations. The signature in fig. 5 seems to agree well.

5 Conclusions

Moving targets introduce phase modulation in addition to usual point target phase history. Depending on the particulars of the motion, various effects may be seen in the SAR image, range form simple azimuth shift, to complicated smearing. For small motions, the motion related phase consists of two terms: One related to the range motion, and one to the azimuth motion. The phase may be analysed using time-frequency methods since the effect may alternatively be seen as a time variable Doppler frequency.

An experiment was carried out in Lillestrøm, Norway, where a car with a corner reflector was used as a moving target. The signature of the target was clearly seen in the DLR E-SAR image taken at the time. Preliminary analysis of the signature with the SPWV method shows a linear chirp with approximately sinusoidal modulation in agreement with the theoretical predictions. Accordingly, time-frequency methods may give valuable information on moving targets within a SAR image, particularly when the motion is complicated.

Literature


