INTERFEROMETRIC SYNTHETIC APERTURE RADAR (InSAR) SUMMARY
(Sandwell and Price, JGR, v.103, p.30283-30208, 1998, Appendix A and B)

Forming an Interferogram

Single Look Complex (SLC) Image:

\[ C(x) = A(x) \ e^{i f(x)} \]

Align two images by cross correlation of amplitude \( A(x) \).
Multiply complex SLC images to form interferogram.

\[ C_1 C_2^* = A_1 A_2 e^{i (f_1 - f_2)} = R(x) + i I(x) \]

Apply earth flattening [and topographic phase] to unfiltered interferogram.

Phase of interferogram:

\[ \frac{I}{R} = \tan^{-1} \left( \frac{I}{R} \right) \]

Contributions to Phase

phase = earth curvature (almost a plane, known) +
topographic phase (broad spectrum) +
surface deformation (broad spectrum, unknown) +
orbit error (almost a plane, largely known) +
ionosphere delay (almost a plane?) +
troposphere delay (power law, unknown) +
phase noise (white spectrum, unknown)
Phase due to Earth Curvature

The geometry of repeat-pass interferometry is shown in the following figure.

\[ r \] - range from reference track to reflector (~830 km for ERS)
\[ B \] - total baseline distance between reference and repeat track
\[ \theta \] - look angle (~20° for ERS)
\[ \alpha \] - angle between the baseline vector and the tangent plane

For discussion purposes one can divide the baseline into parallel and perpendicular components given by

\[ B_{\parallel} = B \sin (\theta - \alpha) \]  \hspace{1cm} \text{(B5)}
\[ B_{\perp} = B \cos (\theta - \alpha) \]  \hspace{1cm} \text{(B6)}

(Note since the look angle \( \theta \) changes across the swath, this terminology is not useful for quantitative calculations.)

The phase difference \( \phi \) to a point on the ground is related to the range difference \( d \).

\[ \phi = \frac{4 \pi}{\lambda} \sqrt{d^2 + 2d r \cos \theta - r^2} \]  \hspace{1cm} \text{(A1)}
where \( \lambda \) is the wavelength of the radar. The law of cosines provides the relationship among the repeat-pass range, the reference-pass range, the baseline length \( B \) and the baseline orientation \( \alpha \).

\[
(r + dr)^2 = r^2 + B^2 - 2B \sin(\alpha - \theta) \tag{A2}
\]

Since \( dr \ll r \), we have

\[
r = \frac{B^2}{2r} - B \sin(\alpha - \theta) \tag{A3}
\]

and since \( B \ll \lambda \), the parallel ray approximation yields.

\[
\theta = \frac{4B}{\theta} \sin(\alpha - \theta) \tag{A4}
\]

The phase difference depends on the parallel component of the baseline. The derivative of the phase with respect to range is

\[
\frac{\partial \theta}{\partial r} = \frac{4B \cos(\alpha - \theta)}{\theta} \frac{\partial \theta}{\partial \theta} \tag{A5}
\]

This phase gradient depends on two terms, the perpendicular component of the baseline \( B_\perp = B \cos(\theta) \) and the derivative of look angle with respect to range \( \frac{\partial \theta}{\partial \theta} \). The perpendicular baseline varies slightly with look angle across a typical SAR image. The change in look angle usually increases with range so \( \frac{\partial \theta}{\partial \theta} > 0 \). However, when the local terrain slope exceeds the look angle (actually the incidence angle), an increase in look angle does not produce a corresponding increase in range. This is the layover geometry where \( \frac{\partial \theta}{\partial \theta} \leq 0 \).

Consider the normal phase gradient due to the local curvature of the Earth. In the SAR Summary we derived the following formula for the relationship between look angle and range.

\[
\theta = \cos \theta = \frac{b^2 + \theta^2 r_z^2}{2b} \tag{A6}
\]

After a little algebra and using (A5), we find an expression for the phase gradient.

\[
\frac{\partial \theta}{\partial r} = \frac{4B \cos(\theta) \sin \theta}{\theta} \frac{\partial \theta}{\partial \theta} \tag{A7}
\]

After a little more algebra one arrives at an expression for the phase gradient in terms of the range that has slightly faster execution on a computer.
\[
\frac{\partial \theta}{\partial r} = \frac{4B}{(1 - \frac{h}{h'})^{1/2}} \cos \theta + \sin \theta \frac{h - r}{b} \quad \text{(A8)}
\]

where \( \theta \) is given in (A6). Using (A4), one can also derive an expression for total phase versus range.

\[
\theta = -\frac{4B}{\sqrt{h'}} \left( 1 - \frac{h}{h'} \right)^{1/2} \cos \theta + \sin \theta \left( h - r \right) \quad \text{(A9)}
\]

Equation (A9) is used to form the earth flattening correction.

\textit{Critical baseline} - If the fringe rate across the interferogram due to earth curvature exceeds \( 2\pi \) radians per range cell then the reference and repeat images will be completely decorrelated. Consider a single range pixel of length \( \sqrt{h'} = C/2 \). The phase of this pixel in the reference image is the vector sum of the phase from all of the scatterers in the pixel. The repeat image will have the same scatters but at if the baseline is critical, there will be an additional \( 2\pi \) phase delay across the range cell that will cause the sum of the scatterers to be randomly different from the reference image. This is called baseline decorrelation and the length at which complete decorrelation occurs is the \textit{critical baseline}.

For this calculation, we start with equation (A7) and make a flat-earth approximation \( \sqrt{h'/b} = 0 \). To avoid baseline decorrelation, change in phase with range must be less than

\[
\frac{\partial \theta}{\partial r} = \frac{4B}{\sqrt{h'}} \frac{\cos \theta}{\sin \theta} < 2\pi
\]

The critical baseline is

\[
B_c = \frac{\sqrt{h'}}{C} \tan \theta
\]

This is 1030 m for the parameters of the ERS satellite. For topographic recovery, a baseline of 150 m is optimal. Of course for change detection, a zero baseline is optimal but not usually available.

\textbf{Phase due to Topography}

One can use this formulation to relate earth-flattened phase to topography. The actual radius of the earth, \( r \), is usually greater than the radius of the spheroid \( r_e \) and this difference is geometric elevation. The phase due to the actual topography can be expanded in a Taylor Series about \( r_e \).
Using equations A4 and A6 one can calculate the first two derivatives. It turns out that the second derivative is about \((r - r_e)/r\) times the first derivative (i.e., 2.7/6371 for our area) so we only need to keep the first two terms in the series. The first term is \(A9\) while the second term is

\[
\frac{\partial \mathcal{F}}{\partial r}(r_e) = \frac{4b}{b} \frac{B \cos(\mathcal{F})}{\sin(\mathcal{F})} \cos(\mathcal{F})
\] (A11)

where \(\mathcal{F}\) is the look angle to the spheroid (A6). The mapping of total unwrapped phase into elevation as a function of range is

\[
(r \cdot r_e) = \frac{\mathcal{F} \sin(\mathcal{F})}{b} \frac{B \cos(\mathcal{F})}{\sin(\mathcal{F})} \cos(\mathcal{F})
\] (A12)

One should remember that the unwrapped interferogram does not provide the complete phase difference \(\mathcal{F} - \mathcal{F}_k\) since there is an unknown constant of integration. Since the mapping from phase to topography varies significantly with range, an appropriate constant should be added to the unwrapped phase or a more accurate solution is to set the local earth radius \(r_e\) to the average radius of the topography in the frame.

**Altitude of ambiguity** - The fringe rate due to topography can be reformulated to provide the error in the elevation model that will produce one fringe \((2\mathcal{F})\) error in the interferogram. Again we’ll use a flat-earth geometry so \(b/r_e = 1\). The altitude of ambiguity is

\[
h_a = \frac{\mathcal{F} \sin(\mathcal{F})}{2B}
\]

For the case of ERS with a perpendicular baseline of 100 m, this altitude of ambiguity is about 90 m. For change detection, a higher number is better.

**Phase due to Surface Deformation**
(see student term papers)