Airborne Interferometric SAR: Terrain Induced Phase Errors

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Abstract

Motion compensation is critical to successful topographic mapping with airborne Interferometric SAR (InSAR). The pixel dependent phase correction required depends upon the elevation of each scene patch, which is initially unknown, requiring a flat earth assumption. This leads to terrain induced phase errors in the compressed data, but accurate elevation estimates can still be obtained by interpreting the differential phase of the interferogram appropriately. The single reference track approach to motion compensation inherently compensates for this, while in the double reference track case a phase correction factor must be used.

Keywords: synthetic aperture radar interferometry, motion compensation, topographic mapping.

Introduction

Across-track airborne SAR interferometry (InSAR) has been shown to be an effective way to derive digital terrain models [1,2]. The terrain estimation accuracy depends, among other things, on the accurate knowledge of the phase centres of the two antennas used. This knowledge is necessary to compensate for turbulent aircraft motion and enable inter-channel image registration. In this paper, a systematic phase error caused by motion compensation to a reference track assuming flat terrain is characterized and a method to correct for it is presented.

The need for motion compensation to achieve accurate focusing of airborne SARs has long been understood. This typically involves using the displacement history of the antenna phase centre with respect to a selected reference track to calculate a range gate delay adjustment for each pulse and a slant range dependent phase correction for each range-compressed pixel. This compensated data then approximates the transmission and reception from an antenna travelling in a straight line along the reference track.

Accurate phase correction requires the elevation angle of each range-compressed pixel as well as the precise position of the antenna. The elevation angle requires a prior estimate of the terrain height. As height is initially unknown in the InSAR problem, the terrain is usually assumed to be flat at some reference level. When the terrain elevation varies from the reference level, this assumption causes phase errors along the aperture for a given scene patch, even when the antenna trajectory is parallel with the reference track.

The aircraft flight motion displacement from the reference track during an aperture synthesis can be described by a constant offset plus a component that varies due to across-track velocity, acceleration and higher order effects. Typically, the constant offset can be tens of meters while the changing component varies less than a meter over the aperture length. The offset leads to constant phase errors, due to the unknown terrain, which are predictable and this paper will indicate how to correct for them by interpreting the differential phase in a special way. This category of errors can be called terrain induced phase errors. The changing component of flight motion leads to phase errors that vary along the aperture, also due to the unknown terrain elevation. The effect of these are difficult to predict but are small for typical smooth flights. This second category of errors can be called flight motion induced phase errors because the flight path must be skewed with respect to the reference track in order for them to occur.

In this paper, we will consider the effect of the terrain induced phase errors for the cases of single and double reference track motion compensation with perfect antenna attitude data. We will assume:

- the range compressed channel B (receive only) data has been resampled to register pixel by pixel with channel A (transmit/ receive) assuming flat terrain at some reference level.
- the subsequent motion compensation operation is a phase correction only; there is no motion compensation resampling. (The errors resulting from only correcting the phase of the data for flight motion has been shown by the authors to be insignificant for typical flight motions.)
- the antenna is yaw steered to zero Doppler.

This type of processing is required in order to recover these terrain induced phase errors by the method described here.

Single Reference Track Approach

The single reference track approach to motion compensation was first used for InSAR by Gray [1]. Both channels are phase compensated to the same reference track assuming the terrain is flat at some reference level (Figure 1). The resulting differential phase in the interferogram must be interpreted differently from the conventional double reference track approach. The differential phase will be zero for scene patches that have the same elevation as the reference level.
because the motion compensation for these patches will have been done properly; the assumed and actual terrain elevation were the same. The differential phase will be non-zero for any scene patch whose elevation differs from the reference level. The value of this phase is the difference in the errors in the phase correction applied between the two channels. These errors depend upon the imaging geometry and can be related explicitly to the elevation of the scene patch relative to the reference level. No terrain elevation estimation errors result in the single reference track case due to the terrain induced phase errors because these phase errors are used as the expected differential phase signal.

From Figure 1 the resulting single track differential phase $\Phi_s$ can be shown to be:

$$\Phi_s = \frac{-2\pi}{\lambda} (R_{aw} - R_{ak})$$  \hspace{1cm} (1)$$

where $\lambda$ is the radar wavelength. The off-nadir angle $\theta_{ak}$ measured from the instantaneous antenna position $A_k$ can be obtained in terms of known and measured parameters:

$$\theta_{ak} = \cos^{-1}\left(\frac{(\frac{-\Phi_s}{2\pi} + R_{aw})^2 - b^2 - R_{ak}^2}{2bR_{aw}}\right) - \alpha_{ab}$$ \hspace{1cm} (2)$$

where the scene patch elevation is:

$$h = Y_a - R_{aw} \cos \theta_{ak}$$ \hspace{1cm} (3)$$

By truncating a binomial series for $R_{ak}$ and $R_{aw}$ we get a simpler expression that is accurate over the useful part of the range swath (away from nadir):

$$\theta_{ak} \approx \cos^{-1}\left(\frac{1}{2b} \Phi_s + \cos \left(\alpha_{ab} + \cos^{-1}\left(\frac{Y_a}{R_{aw}}\right)\right)\right) - \alpha_{ab}$$ \hspace{1cm} (4)$$

Double Reference Track Approach

In the double reference track case each channel is motion compensated to its own reference track assuming flat terrain at some reference level. This approach has been used by Madsen [2] but with resampling used in the motion compensation stage. Conventionally, the effects of the flat earth assumption are ignored and it is assumed that the data has been adequately corrected to the two reference tracks. The resulting interpretation of the differential phase from Figure 2 is:

$$\Phi_t = \frac{-2\pi}{\lambda} (R_{aw} - R_{ak})$$ \hspace{1cm} (5)$$

The off-nadir angle $\theta_t$ from the reference track $RT_t$ can be solved explicitly, yielding a height estimate.

What is missing in this interpretation is consideration of the phase errors in each channel that result from motion compensation assuming flat terrain. When these systematic terrain induced phase errors are included in Equation 5 we obtain a new double reference track approach:

$$\Phi_t = \frac{-2\pi}{\lambda} (R_{aw} - R_{ak} + R_{uw} - R_{ub})$$ \hspace{1cm} (6)$$

The off-nadir angle $\theta_{ak}$ can be explicitly related to the measured differential phase and other known parameters:

$$\theta_{ak} + \alpha_{ab} = \cos^{-1}\left(\frac{1}{2b} \Phi_t + R_{aw} - R_{ab} + R_{ub} \right)^2 - b^2 - R_{ak}^2)$$ \hspace{1cm} (7)$$

By truncating a binomial series for $R_{ak}$ and $R_{aw}$ we get a simpler expression that is accurate over the useful part of the
range swath (away from nadir):
\[ \theta_{\text{st}} + \sigma_{\text{st}} \approx \cos^{-1} \left( \frac{1}{2} \left( \frac{\Delta \Phi}{2\pi} + R_{\text{ref}} - R_{\text{th}} \right) + \cos (\sigma_{\text{st}} + \theta_{\text{st}}) \right) \]
Equation 3.

The terrain elevation can then be estimated from Equation 3.

**Terrain Induced Height Errors**

If these terrain induced phase errors are not considered then systematic errors will result in the differential phase which will lead to height estimation errors. The differential phase error, inherent to the conventional double reference track approach, is the difference in the motion compensation phase errors between the two channels. This can be shown to be:

\[ \Delta \Phi = \Phi - \Phi_1 = -\frac{2\pi}{\lambda} \left( R_{\text{ref}} - R_{\text{th}} + R_{\text{st}} - R_{\text{sh}} + R_{\text{th}} - R_{\text{ref}} \right) \]
Equation 9.

This expression is equal to zero when the target is at the reference level, as expected, and varies with the positions of the reference tracks relative to the antennas. To understand this further, we can make the following approximations \( \frac{R_{\text{ref}}}{R_{\text{st}}} < 1, \frac{R_{\text{ref}}}{R_{\text{sh}}} < 1 \) resulting in:

\[ \Delta \Phi \approx \frac{2\pi}{\lambda} \left( \cos (\theta_{\text{st}} + \alpha_{\text{RT}}) - \cos (\theta_{\text{sh}} + \alpha_{\text{RT}}) \right) - \frac{2\pi}{\lambda} \left( \cos (\theta_{\text{st}} + \alpha_{\text{RT}}) - \cos (\theta_{\text{sh}} + \alpha_{\text{RT}}) \right) \]
Equation 10.

It is clear that this error can be minimized by making the reference track baseline and the antenna baseline the same length and angle (\( b_{\text{RT}} = b = B, \alpha_{\text{RT}} = \alpha_{\text{sh}} = \alpha \)). The remaining error in this case is due to the scene patch elevation (\( \theta_{\text{st}} \neq \theta_{\text{sh}}, \theta_{\text{sh}} \neq \theta_{\text{sh}} \)) and the displacement of the antennas from the reference tracks (\( \theta_{\text{st}} \neq \theta_{\text{st}}, \theta_{\text{sh}} \neq \theta_{\text{sh}} \)) characterized by \( b_{\text{st}} \) and \( \gamma \) in Figure 2. With further approximations for this particular case yields:

\[ \Delta \Phi \approx \frac{2\pi b_{\text{st}}}{\lambda R_{\text{st}}} \left( \sin (\theta_{\text{st}} + \theta_{\text{sh}}) \sin (\theta_{\text{st}} + \gamma) - \sin (\theta_{\text{sh}} + \theta_{\text{sh}}) \sin (\theta_{\text{sh}} + \gamma) \right) \]
Equation 11.

In summary, for a given displacement \( b_{\text{st}} \) of the reference track, \( \Delta \Phi \) of Equation 9 has an offset (Equation 10 with \( \theta_{\text{st}} = \theta_{\text{st}}, \theta_{\text{sh}} = \theta_{\text{sh}} \)) plus a component which varies sinusoidally with \( \gamma \) (Equation 11).

During typical flights the baseline lengths can be kept the same (\( b_{\text{RT}} = b \)) but the aircraft may have several degrees of roll (\( \alpha_{\text{RT}} = \alpha_{\text{sh}} \)). Equation 9 was evaluated for a typical CCRS imaging situation using the parameters of Table 1. Table 2 shows the resulting offset (or mean) of the differential phase error and the peak to peak value, for \( \gamma \) varying over [\( -\pi, \pi \)]. The offset error is independent of the antenna to reference track displacement (\( b_{\text{st}} \)) but depends approximately linearly on the scene patch elevation (\( h \)) and the difference between the reference track and antenna baseline angles (\( \alpha_{\text{RT}} - \alpha_{\text{sh}} \)) caused by aircraft roll. The peak to peak value of the sinusoidal variation about the offset, as \( \gamma \) varies over \( [\mathbf{-\pi, \pi}] \), depends approximately linearly on \( h \) and \( b_{\text{st}} \), but is independent of \( \alpha_{\text{RT}} - \alpha_{\text{sh}} \).

It should be noted that a differential phase error of \( 25 \text{ mrad} \) for the parameters of Table 1 causes a height estimation error of about \( 1 \text{ m} \). It is therefore evident that ignoring this effect can lead to significant systematic height estimation errors for rugged terrain, typical aircraft roll, and typical deviations of the flight path from the reference tracks.

**Conclusions**

It has been shown that systematic errors can occur in the differential phase due to motion compensation assuming flat terrain. These terrain induced errors can occur even when the flight path is parallel to the reference track. The errors are inherent to the conventional double track method (equation 5) but are recovered in the single track (equation 1) and new double track (equation 6) approaches.

**References**