A New Method for Stepped-frequency SAR Imaging
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Abstract

A coupling between subpulses and the 'fast' time is introduced when stepped-frequency pulse burst is applied to SAR to obtain ultra high range resolution, i.e. there is range migration among pulses in one burst. Traditional methods which simply combine synthetic HRR processing method and conventional SAR imaging algorithm can not remove the coupling which will bring phase discontinuities in adjacent subpulses and inevitable degrade SAR imaging, for the range migration within pulse bursts does not keep constant at all integration angles. This paper first concisely analyzes the causes, behaviors and main characteristics of the coupling mentioned above, and then studies a new method for stepped-frequency SAR imaging which is called the Subimage Coherent Accumulation Method (SCA). Compared with traditional ones, this method can accurately correct range migration of all subpulses by adjusting the sequence of signal processing and adding some simple operations. Stepped-frequency SAR image formed by SCA obviously has better imaging quality than that by traditional methods. This paper will give the detailed deduction and show some of the simulation results.

1 Introduction

The use of stepped-frequency chirp pulse burst in Synthetic Aperture Radar (SAR) to obtain ultra high range resolution widely spreads nowadays. A coupling between subpulses and the 'fast' time is introduced when stepped-frequency SAR works in synthetic bandwidth mode [1], i.e. there is range migration among pulses in one burst. Most traditional processing methods which simply combine synthetic high range resolution (HRR) processing methods and conventional SAR imaging algorithms, e.g. Synthetic Bandwidth Method (SB) [1], are all based on the processing of single chirp pulse. Firstly, they synthesize subpulses in one burst to a fictive pulse of very wide bandwidth, and then, all the fictive pulses are imaged as conventional SAR echoes. These methods can not remove the coupling between subpulses and the 'fast' time which will bring on phase discontinuities and inevitably degrade SAR imaging, especially when PRF is not much higher than the doppler bandwidth, for the range migration within pulse bursts does not keep constant at all integration angles, and it varies with targets’s positions in a scene as well.

The Subimage Coherent Accumulation Method (SCA) which will be studied in this paper is briefly described as follows: Firstly, pulses with different carrier frequencies are formed into images separately, so the phase discontinuities will be correctable after the image formation. Then all these coarse resolution subimages are time shifted in azimuth until overlap completely and are also frequency shifted in range to contain the total bandwidth, so we get a high range resolution image by coherently accumulating them all. Compared with traditional ones, this method can accurately correct range migration of all subpulses by only adjusting the sequence of signal processing and adding some simple operations. By using SCA, stepped-frequency SAR image has better imaging quality than that by traditional methods, which will be shown in the simulation results. Additionally, imaging process will fit the narrowband approximations better, data processing is easily parallelized, and the need for memory can be reduced to some extent.

The remains of this paper will be organized as follows: Section 2 will concisely analyze the causes, behaviors and main characteristics of the coupling mentioned above. Section 3 will give the deduction of the SCA method in detail. Section 4 will present some of the simulation results.

2 Coupling between Subpulses and Slant Range

2.1 Synthetic Bandwidth Mode

Stepped-frequency SAR operates at various radar modes to meet different demands. The most widespread mode is to scatter the subbands as in Fig.1, which is called synthetic bandwidth mode [2]. The left-hand side of the diagram shows the frequency allocation along the fast time axis (range direction) covering the transmit and receive period; on the right-hand side the instantaneous direction of
the illuminating beam is depicted along the same time axis. This mode is applied if the PRF demands are not too stringent [2]. It has been applied to PAMIR acting as spotlight SAR and ISAR, in which the concatenation of the sub-bands is done in a postprocessing step by combining all the subpulses into one single signal, as if only one pulse with very high bandwidth was transmitted. The processing method is based on method for synthetic HRR processing method [3]. In fact, subpulses in one burst are transmitted or received at different instantaneous directions along the flight path (azimuth) as depicted on the right-hand side in Fig.1, so they shouldn’t be simply processed as signals of synthetic high range resolution profile (HRRP).

2.2 Coupling between Subpulses and Slant Range

Stepped-frequency SAR working in synthetic bandwidth mode transmits subpulse as:

\[ s_p(t) = r(\frac{t}{D_p}) e^{j\pi f_c t^2} e^{j2\pi f_p t} \]  

(1)

with pulse duration \( D_p \), bandwidth \( \Delta f \), chirp rate \( \gamma \), and stepped carrier frequency \( f_p = f_c + \Delta f_p = f_c + \gamma \theta_p \), where \( f_c \) is the center frequency of pulse burst, \( \theta_p = (p+1/2-N/2)D_p/2 \) is the time offset of subpulse, and \( p \) from 0 to N-1 denotes N subpulses which are transmitted at time interval \( T_r \). Assumming point target \( P_0 \) with distance \( Y_0 \) to flight path and posited at \( X_0 \) in azimuth on an azimuth-range plane. When the first subpulse in a burst is transmitted and received (considering the assumption of ‘stop-and-go’ is satisfied), the phase center of sensor posits at \( X_i \). The received signal after being demodulated by \( f_p \) becomes

\[ b_p(t) = r(\frac{t - \tau_p}{D_p}) e^{j\pi \gamma (t - \tau_p)^2} e^{-j2\pi f_p \tau_p} \]  

(2)

where \( \tau_p = \frac{2}{c} \left( \frac{(X_i + p\Delta x - X_0)^2 + Y_0^2}{c} \right) \) is the two way time delay, \( v \) is the velocity of sensor’s phase center, and \( c \) is speed of light. Obviously \( \tau_p \) varies with the increase of \( p \). Considering combining all the subpulses of one burst into a single signal with bandwidth of \( N\Delta f \), first, they should be frequency shifted and phase corrected to \( \hat{x}_p(t) \) (see equation(5) in [3]):

\[ \hat{x}_p(t) = b_p(t)e^{j2\pi f_c \tau_p + j2\pi f_p \Delta \tau_p + j2\pi f_c \Delta \tau_p / \tau} \]

(3)

where \( \tau_{\Delta \tau_p} = \tau_p - \tau_0 \), and \( \tau_0 = 2\sqrt{(X_i - X_0)^2 + Y_0^2}/c \) is the two way time delay while the sensor was at \( X_i \), i.e. \( \tau_{\Delta \tau_p} \) is the variation between time delay of the \( j \)th and the \( 1 \)th subpulse, and it is \( \tau_{\Delta \tau_p} \) that brings on the phase discontinuities and range migration in the burst. Secondly, all \( \hat{x}_p(t) \) are time shifted until they compose a new longer pulse and never overlap each other. Finally, these complex data are accumulated. The sum is the ultra high bandwidth chirp pulse:

\[ s_b(t) = \sum_{p=0}^{N-1} \hat{x}_p(t - \tau_p + \tau_{\Delta \tau_p}) \]

(4)

\[ = r(\frac{t - \tau_0}{D}) e^{j\pi \gamma (t - \tau_0)^2} e^{-j2\pi f_c \tau_0} \]

with the same chirp rate, but the total pulse duration \( D = ND_p \), which is called fictive pulse in [3]. In fact, it is impossible to correct the phase discontinuity error or the range migration that depends on \( \tau_{\Delta \tau_p} \) in (3) and (4) for all targets in a scene, because \( \tau_{\Delta \tau_p} \) has characteristics as follows:

i. \( \tau_{\Delta \tau_p} \) varies with change of \( p \) in a burst for one target;

ii. \( \tau_{\Delta \tau_p} \) with identical \( p \) at different integration angles don’t keep constant;

iii. For echoes of the \( j \)th subpulse from different targets, their \( \tau_{\Delta \tau_p} \) are different.

This makes a trouble for stepped-frequency SAR imaging, because it is not an easy task to correct the range migration in a burst by using traditional method e.g. the SB method unless PRF is high enough so that the discontinuities are negligible.

3 Subimage Coherent Accumulation Method

If we take the pulses of stepped-frequency SAR as signals of conventional SAR systems with different carrier frequencies and form images separately. Range migration will have been corrected after subimage formation and phase discontinuities will be correctable at the moment. The SCA method is basing on this idea. In order to keep the discussion and the formulas as concise as possible, continuous variables will be used below both in range and azimuth.
3.1 An Assumption in the Processing

It must satisfy an assumption while applying the SCA method to stepped-frequency SAR imaging, that is, the time-bandwidth product for each subpulse should be large enough, usually larger than several hundreds. The requirement of the time-bandwidth product for stepped-frequency chirp pulse train can be interpreted by the theory in [4] using the autocorrelation function. For the sake of integrity and clarity, we make a further explain from the processing of the SCA method, although it seems to be an inconsiderable requirement for ultra high resolution SAR.

Considering the pulse compression, match filter is widely used. The high range resolution profile can be depicted as the convolution of $s_b(t)$ and its transfer function $h(t)$:

$$RP(t) = \mathcal{F}^{-1}\left\{\mathcal{F}_x\{s_b(t)\} \times \mathcal{F}_t\{h(t)\}\right\}$$

(5)

where $\mathcal{F}\{\cdot\}$ and $\mathcal{F}^{-1}\{\cdot\}$ denote Fourier transform and the inverse one, subscript $y$ is the variable. We cut both $s_b(t)$ and $h(t)$ into several sections by the way which is a little different from that in (4) as:

$$s_b(t) = \sum_{p=0}^{N-1} x_p(t - \tau_0)$$

$$= \sum_{p=0}^{N-1} rect\left(\frac{t - \theta_p - \tau_0}{\Delta f}\right)e^{j2\pi(t - \tau_0)\Delta f}e^{-j2\pi f_c \tau_0}$$

(6)

where there is no migration among different sections. Sections with identical band domain in $s_b(t)$ and $h(t)$ correspond to each other, and every pair of them are conjugated in Fourier domain despite of a linear phase brought on by the transmitting time delay. If the time-bandwidth product for each section is large enough, sequence of processing could be made a little change as:

$$RP(t) = \mathcal{F}^{-1}\left\{\sum_{p=0}^{N-1} \mathcal{F}_x\{x_p(t - \tau_0)\} \times \mathcal{F}_f\{x_p(t)\}\right\}$$

$$\approx \sum_{p=0}^{N-1} \mathcal{F}^{-1}\{rect\left(\frac{f - f_{an}}{\Delta f}\right)\mathcal{F}_f\{x_p(t - \tau_0)\} \times \mathcal{F}_f\{x_p(t)\}\}$$

$$\approx \sum_{p=0}^{N-1} \mathcal{F}^{-1}\{\mathcal{F}_f\{b_p(t - \tau_0)\} \times \mathcal{F}_f\{b_p(t)\}\} \times e^{j2\pi f_{an} t}$$

(7)

where $*$ denotes the conjugate complex. By compressing each section of high bandwidth signal separately and summing up them but with no need to time shift by $\theta_p$ in fact, we can get an ultra high range resolution profile as well.

3.2 Subimage Coherent Accumulation Method

Firstly, it is assumed that all subpulses have already been coherently demodulated and range compressed. Two dimensional SAR data for point target $P_0$ carried by $f_p$ can be expressed as:

$$d_p(x,t) = A_1w(x - X_0 + \nu T_0, Y_0)e^{-j2\pi f_p R_p(x - X_0, Y_0)}$$

$$sin[\pi \gamma D_p(t - \tau_p(x - X_0, Y_0))]$$

$$\pi \gamma D_p(t - \tau_p(x - X_0, Y_0))$$

(8)

where $A_1$ is a constant caused by range compression, $\tau_p(x,y) = 2R_p(x,y)/c$ is the time two delay, $R_p(x,y) = \sqrt{(x + \nu T)^2 + y^2}$, and $W(x,y) = rect(x/(2y\tan(\beta/2)))$ denotes antenna radiation pattern, $\beta$ is the antenna illuminating angle. If it is compressed by two dimensional transfer function $h_p(x,t)$ whose Fourier domain expression is:

$$\mathcal{F}\{h_p(x,t)\} = \mathcal{F}^*\{W(x,Y_0)\delta(t - \Delta R(x,Y_0))\}$$

$$e^{-j2\pi f_p \Delta R(x,Y_0)/c}$$

(9)

where $\Delta R(x,y) = \sqrt{x^2 + y^2} - y is the range migration for targets at $y$ in slant range. We have $N$ coarse resolution subimages for point target $P_0$, in order to simplify analyses, azimuth resolution is not considered below:

$$u_p(x,t) = A_2 \delta(x - X_0 + \nu T_0)e^{-j2\pi f_p \frac{2\nu}{c}}$$

$$sin[\pi \gamma D_p(t - 2Y_0/c)]$$

$$\pi \gamma D_p(t - 2Y_0/c)$$

(10)

$A_2$ is a constant caused by two dimension convolution. One difference for the $N$ subimages is that the same target is formed at different position in azimuth. If all subimages are time shifted along azimuth separately until they are totally overlap in the scene, then we coherently accumulate these complex images after they are frequency shifted to contain the total bandwidth, the range resolution will be enhanced to $1/N$ of the subimage. The process can be depicted as:

$$u(x,y) = \sum_{p=0}^{N-1} u_p(x - \nu T_0, 2y/c) e^{j2\pi f_{an} \frac{2\nu}{c}}$$

$$sin[2\pi N \Delta f(y - Y_0/c)]$$

$$2\pi N \Delta f(y - Y_0/c)$$

(11)

The term $e^{-j2\pi f_{an} \frac{2\nu}{c}}$ indicates the information that borne by carrier frequency. The image above is just as same as the one done with conventional SAR of the same total bandwidth, while all the time shifts in azimuth or the frequency shifts in range are independent of targets.

The first step of SCA is to form the coarse range resolution subimages using echoes with identical carrier frequency, this step can truly be taken as narrow bandwidth signal processing. Secondly, subimages are time shifted in azimuth and frequency shifted in range dimension, all the subimages are coherently accumulated into one image with ultra high range resolution.
4 Simulation Results

For lack of real data, only simulation results can be shown here. The SAR in the simulation works at $K_a$ band, which uses the synthetic bandwidth mode to obtain a total bandwidth of $1.08GHz$ with three subbands and $360MHz$ as the frequency step. A chirp signal of $380MHz$ was utilized to avoid bandwidth gap, waveform length amounts to $3.6\mu s$, antenna aperture length of $0.28m$, and PRF of about 1.4 times the doppler bandwidth. As in a conventional SAR geometry perspective with azimuth-range plane, $x- y$ axes of the 2D coordinate often denote flight path (azimuth) direction and slant range direction. Setting only one point scatter $P_0$ at $(0m, 1km)$ in the plane.

![Image Contour Plot by SB Method](image1.png)

![Image Contour Plot by SCA Method](image2.png)

**Figure 2:** Contour plots of impulse responses (simulated signals): (a) the SB Method, (b) the SCA Method.

Fig.2 is an image domain contour plot of impulse response for $P_0$. For purposes of comparison, the SB method is also used, which can not resolve the subpulse-range-coupling problem especially for envisaged stepped-frequency SAR. Fig.2 show that the imaging quality by the SCA method is superior to that of the SB method.

The corresponding profiles for both azimuth and slant range after two dimension compression (red solid line: the SB method; blue solid line: the SCA method) are depicted together with the theoretical profile of conventional SAR impulse response of the same theoretical space resolution (green dash-dotted line) in Fig.3.

In comparison to the theoretical impulse response, the azimuth profile of the SB method obviously shifts about $0.04m$ from the real position, while the one of the SCA method is identical to the ideal profile, which is shown in Fig.3(a). In Fig.3(b), the range profiles of the two methods are shown. Concerning the SB method, the peak sidelobe ratio degrades by $6dB$, much worse is that the sidelobe energy is rather high and asymmetry as the red solid line depicted. As to the SCA method, the mainlobe width and the peak sidelobe ratio are identical to the ideal one, but the sidelobe energy far from the mainlobe is some higher than the latter which may behave as a long light tail in the real SAR image. In all cases for Fig.3, the two dimension compression was performed with Hamming weighting in both the doppler domain and the frequency domain.

There are many simulations having been done with varying the frequency step and the partition of the total bandwidth, or changing the ratio of PRF to doppler bandwidth, but only a few results are shown in this paper. Mathematics deduction and simulation validating show that the imperfection on the image formed by the SB method, which brought on by the coupling of subpulse and slant range highly depends on the number of subbands and the ratio of PRF to doppler bandwidth, i.e. the more subbands or the lower ratio, the more serious the imperfection is.

![Figure 3: Profiles comparison for impulse responses in Fig.2](image3.png)

**Figure 3:** Profiles comparison for impulse responses in Fig.2: (a) Azimuth profiles, (b) Range profiles.

5 Conclusion and Future Work

Simulation results show that the stepped-frequency SAR image formed by the SCA method obviously has better imaging quality than that by traditional one, but higher range sidelobe energy than the theoretical one. Additionally, data processing is easily parallelized, and memory need can be reduced to some extent.

Future studies will be carried out as follows: (1) The design of waveform and system parameters is an art for stepped-frequency SAR, and its accurate relationship with the SCA method is studied in the next step. (2) A new idea for suppressing the high sidelobe energy of the SCA method will be validated soon. (3) Real data experiment would be done.

References


