IFSAR Correlation Improvement Through Local Slope Correction

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ABSTRACT

We have developed a robust algorithm to help compensate for the phenomena of local slope decorrelation in SAR interferometric pairs. A standard maximum-likelihood algorithm assumes that the underlying terrain elevation within the computation window is constant. An improved maximum-likelihood correlation measure makes the assumption that the underlying terrain elevation within the moving window box is not constant, but is instead planar. Estimating the planar phase slopes is equivalent to measuring the instantaneous frequency of the interferogram in the moving window box of interest. The maximum-likelihood estimate of the planar phase is equivalent to finding the location of the peak of the Fourier transform of the interferogram data within the local computation window. Once estimated, the terrain can be “flattened” and the correlation computed on the “flattened” terrain, hence compensating for local slope decorrelation. Wrapped phase estimates are also significantly improved. This algorithm has also been extended to allow “space-variant aperture trimming” which offers additional improvement in the correlation estimates at the expense of space-variant resolution.

1. INTRODUCTION

Typically, the local complex correlation between two registered complex SAR images requires a complex correlation computation over a small moving window box (i.e., 5 x 5, 7 x 7 pixels, etc.). The computed complex correlation coefficient is assigned to the center sample of the small box. The box is then moved over by one sample and the process is repeated until the entire SAR interferogram is processed. The magnitude of the complex correlation coefficient is sometimes referred to as the “coherence” whereas the phase (once unwrapped) is related to topography. The quality of the estimate of the complex correlation critically depends on the robustness of the computation algorithm and whether it adequately models the physical phenomena.

The correlation between two interferometric SAR images is subject to several degrading influences. The overall correlation $\mu_{\text{total}}$ can be represented as a product of individual correlation factors,

$$\mu_{\text{total}} = \mu_{\text{noise}}^2 \mu_{\text{temporal}}^2 \mu_{\text{slope}}^2 = \frac{\langle s_1 s_2^* \rangle}{\sqrt{\langle |s_1|^2 \rangle \langle |s_2|^2 \rangle}},$$

where $s_1$ and $s_2$ are the complex SAR images comprising the interferometric pair and $\langle \ldots \rangle$ refers to a spatial averaging over a small local region. Here, $\mu_{\text{noise}}$ is given by

$$\mu_{\text{noise}} = \frac{1}{1 + 1/\text{SNR}},$$

where SNR refers to the thermal (receiver) signal-to-noise ratio. Note that $\mu_{\text{noise}}$ can be estimated everywhere in the interferogram and that it sets an upper limit to the achievable correlation between two IFSAR images.

Let us assume for the purpose of this discussion that there are no temporal decorrelation effects, therefore $\mu_{\text{temporal}} = 1$. The terms that remain, $\mu_{\text{noise}}$ and $\mu_{\text{slope}}$, will be considered the dominant indicators of performance. The term $\mu_{\text{slope}}$ refers to the correlation as a function of local terrain slope. It is well known that local terrain slopes cause a fundamental loss of correlation [1]. Furthermore, the computational means by which the correlation is computed is corrupted by the same local terrain slope. In other words, local terrain slope causes a loss of correlation but the estimate of this correlation is further degraded by the fact that there is a local terrain slope. The inability to accurately compute the correlation coefficient of steeply sloped terrain can severely affect the confidence of “downstream” processing that requires accurate correlation estimates.

Another way of visualizing the slope decorrelation phenomenon is to imagine computing the Fourier transforms of two corresponding resolution cells in the registered SAR images. Each resolution cell is synthesized from a piece of Fourier space that spans a finite dimension in both the range and cross-range spatial frequency dimensions [2, 3]. If the terrain across the resolution cell is sloped, the two Fourier patches do not overlap completely. Only the fraction of the two apertures that are in common contribute to correlated signals. The parts of the apertures not in common contribute...
noise which results in a decrease in correlation between those two resolution cells considered.

Fig. 1 depicts the aperture shifts and the potential loss of correlation resulting therefrom. In the figure, the dimensions $M$ and $N$ refer to the number of samples spanning the Fourier bandwidths, while $m_0$ and $n_0$ refer to the respective offsets of the two apertures. Thus, the shaded region labeled $A_1$ is common to both apertures and contributes the signal component to the interferometric correlation, while the non-common area labeled $A-A_1$ -impacts noise. The slope correlation (magnitude) is simply the ratio of the common area to the total area.

![Figure 1. Fourier domain apertures showing the relative displacements caused by locally sloped terrain.](image)

The shift theorem [4] of Fourier transforms implies that displaced (shifted) pieces of Fourier space impart a linear (two-dimensional, in this case) phase shift across the corresponding image. This two-dimensional phase shift is nothing more than a two-dimensional phase plane whose slope is related to the aperture mismatch which is related to the local terrain slope. It is important to note that this phenomenon of aperture misalignment from locally sloped terrain occurs independently of a global phase plane across the interferogram that could be caused by imprecise knowledge of the interferometric baseline orientation relative to the imaged terrain [1]. **Global phase flattening should not be confused with the process for local slope correction. It is not possible to correct for local slope decorrelation by a single global phase plane removal or, equivalently, by a single aperture trimming process.**

### 2. IMPROVED MAXIMUM LIKELIHOOD METHOD FOR LOCAL SLOPE CORRECTION

The maximum likelihood estimate of the local planar phase is equivalent to finding the location of the peak of the Fourier transform of the interferogram data within the moving window box [5-7]. There are three stages in the algorithm to compensate for local slope decorrelation:

1. Estimate the local terrain slope within a small moving window box over a coarse set of grid points covering the IFSAR conjugate product imagery $(s_1, s_2^*)$.

2. Filter the coarse estimates and interpolate the coarse grid estimates to a fine grid consistent with the spatial grid of the original SAR images.

3. Compute the correlation coefficient between the complex SAR images with use of the local slope estimates to “flatten” the terrain over the moving window estimation box.

Space restrictions prevent detailed discussion of the above processing steps. Some details are summarized briefly.

#### 2.1 Instantaneous Frequency Estimation.

The local instantaneous frequencies are proportional to the phase partial derivatives (slopes) in the $x$ and $y$ dimensions. We estimate those slopes with the use of the FFT and a method for multidimensional function minimization based on the Numerical Recipes algorithm AMOEBA [8], for estimating the fractional sample locations of the maximum spectrum magnitude as well as the complex value at that location. For efficiency reasons, the instantaneous frequencies are estimated on a coarse grid spanning the interferogram.

#### 2.2 Filtering and Interpolation.

We now assume that we have the two coarsely gridded arrays containing samples of $\phi_1(x, y)$ and $\phi_2(x, y)$. Some samples may be corrupted due in part to noise, low signal values (i.e., shadows, etc.), or because the planar phase assumption was violated within some FFT windows (i.e., superposition from layover, volume scattering, etc.). Fortunately, it is possible to perform a rather efficient culling based on computed signal-to-noise. Next, we interpolate the remaining partial derivative estimates onto a regular grid. We have chosen to use a triangular irregular network (TIN) for the grid points and a cubic spline interpolating function that has $C^1$ continuity. In other words, we will use the coarse grid points to form a TIN, then the spline function will equal the values exactly at the grid points and have continuous first partial derivatives there. Therefore, this procedure produces a two-dimensional surface approximation to the instantaneous frequencies. Furthermore, this surface can be sampled anywhere desired to obtain an interpolated value. In practice, we sample the surface at regular intervals corresponding to the pixel
2.3 Phase Flattening and Correlation Computation.

We compute the maximum likelihood complex correlation with local slope compensation by the following formula:

$$
\mu_{bc}(k, l) = \frac{\langle s_1(m, n) s_2^*(m, n) e^{-j(\alpha + \beta)} \rangle}{\sqrt{\left\langle |s_1(m, n)|^2 \right\rangle \left\langle |s_2(m, n)|^2 \right\rangle}}.
$$

Application of (3) for a given sample location \((k, l)\) yields a single value for the complex correlation at that sample index. The indices \((k, l)\) are sequentially (raster) scanned and (3) is computed for each location until the entire interferogram is covered. The magnitude and phase of \(\mu_{bc}\) are then extracted for downstream processing for coherent change detection or for topographic mapping through phase unwrapping [9].

3. EXAMPLES

A few examples are shown below. All correlation computations were performed within a 7 x 7-pixel moving window. Slope estimation used 8 x 8 FFTs.

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Figure 2. Single-look magnitude (left), and noisy pixel-by-pixel wrapped phase (right), of interferogram \(s_1s_2^*\).

Figure 3. Correlation magnitude (left), and estimated wrapped phase (right), without local slope correction. (7 x 7 box.)

Figure 4. Correlation magnitude (left), and estimated wrapped phase (right), with local slope correction. (7 x 7 box.)

Careful comparison of Figs. 3 and 4 show an overall improvement in correlation, especially in the steeply sloped regions. In addition, the extracted wrapped phase appears smoother, more continuous in some regions, and overall less noisy than the uncompensated result.

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4. REFERENCES


