Space–time processing for multichannel synthetic aperture radar

by J. H. G. Ender

Synthetic aperture radar (SAR) provides high-resolution images of a non-moving ground scene, but fails to indicate the presence and position of moving objects. As in airborne MTI (moving-target indication) systems the solution to this problem is to use an array of antennas or subapertures and several receiving channels (‘MSAR’, or multichannel SAR), and to apply multichannel clutter suppression. One of the most efficient methods is adaptive space–time processing (STAP), which can be simplified to frequency-dependent spatial processing in the Doppler domain. In this paper, some of these techniques applied to SAR are reviewed and illustrated with data gathered by the German experimental multichannel SAR system ‘AER-II’.

1 Introduction

Moving-target indication (MTI) is a well-known and basic feature of ground-based surveillance radars. The implementation of this capability in airborne radar is more complex, since the radar platform also moves. In particular, the detection of objects which are moving at a lower speed than the radar (i.e. ground moving vehicles) is an especially difficult problem, since the clutter Doppler spectrum is spread over a frequency band and masks the target signal. SAR was originally developed for imaging non-moving scenes. Nevertheless, for many applications it is important to be able to detect moving targets and to derive information about their velocity, position, shape and kind from the raw or preprocessed SAR-data. In contrast to the classical airborne MTI situation, it is not possible to choose a nearly arbitrarily narrow beamwidth for SAR systems, since this would coarsen the azimuth resolution in the usual stripmap mode. Moreover, high range resolution and high data rates, even for moderate PRFs (pulse repetition frequencies), make the problem of simultaneous SAR and MTI rather involved.

The theory and practical implementation of detecting moving targets by synthetic aperture radar, estimating their velocity components and positions, and obtaining a focused image, have been addressed in several papers. In the classical case (only one antenna and one channel), MTI is restricted to targets moving at a speed sufficient to take them out of the clutter band, or having an RCS (radar cross-section) sufficiently large that the resulting high SCNR (signal-to-clutter-plus-noise ratio) will allow their detection irrespective of their motion.

The detection probability and the estimation accuracy can be increased considerably by use of an array of antennas; exploitation of the resulting vector-valued signals achieves sub-clutter visibility. One suitable technique is space–time adaptive processing (STAP), which is treated extensively in the literature for the general airborne radar case.

This is not the situation for imaging radars. There are very few SAR systems in the world which allow the luxury of more than a single or two antennas, mainly because of the immense data rates produced by a multichannel antenna. As a consequence, there are only a few published papers that address the special problems of STAP for SAR. Nevertheless, future SAR systems will be able to handle extremely high data rates, so the time has come to investigate multichannel SAR (‘MSAR’) theory and STAP processing applied to SAR.

There are two main differences compared to the conditions of non-imaging airborne radar. Firstly, the transmitted pulse trains are principally of infinite length, making filtering in the frequency domain feasible. Secondly, the number of spatial channels will be small in any case, which is of importance with regard to numerical manipulations of the spatial covariance matrix.

2 Basic SAR principles

We consider the usual SAR system operating in a stripmap mode. The platform moves along a straight line with velocity \( \mathbf{v} \), and the side-looking antenna illuminates a strip on the earth’s surface parallel to the flight path. An infinite sequence of pulses is transmitted with a repetition interval...
3 Moving targets

If we look at a SAR image of a motorway interchange, we notice that—despite the fine resolution of the stationary background—the streets seem to be empty; moving objects appear not to be displayed. This is because SAR is based on the trick of realising a spatially extended aperture by temporal sampling of the echoes while the radar is moving. The assumptions about the phase behaviour of a non-moving point scatterer signal are no longer valid for moving objects. Consequently, in most cases moving targets do not appear in the SAR image, and when they do they are smeared and mostly show up in the wrong azimuth position.

Moving target signals can be characterised as chirps with a certain Doppler offset due to the tranverse velocity component and with a different Doppler slope arising from the parallel velocity component. In an early publication, Raney has studied the different effects caused by target motion. These can be summarised as follows:

- **Slow tranverse motion** causes azimuthal displacement and coarsened resolution, a decreasing amplitude, and vanishing of the target image, if the Doppler frequencies fall outside the processed Doppler band.
- **Parallel motion** causes azimuthal and range defocusing, and amplitude reduction due to this defocusing.
- **Large radial velocities**, which induce Doppler frequencies larger than the half pulse repetition frequency, cause—in addition to the usual azimuthal displacement—incorrect azimuth positions due to Doppler aliasing and range defocusing.

In addition, the detection of moving targets is hindered by the platform-motion-induced spread of the clutter spectrum. A certain azimuth direction will be characterised by the directional cosine \( u \), which is the cosine of the angle between the flight and look directions. From \( u \) we get the Doppler frequency \( f = -u_0/\lambda \), so the clutter spectrum assumes the form of the two-way antenna pattern scaled to the frequency axis. It can be divided into three spectral areas:

- *(a) Mainbeam clutter:* Its bandwidth is given by \( B_{\text{main}} = 2\delta u_0/\lambda \) where \( \delta u_0 \) denotes the beamwidth of the two-way characteristics. Common SAR systems apply an azimuth sampling frequency (PRF) equal to or a little greater than \( B_{\text{main}} \). As a consequence, moving targets outside the clutter band are outside the unambiguous frequency interval causing only the aliased signal to appear. To avoid this, an MTT-capable SAR system should operate at a considerably higher PRF, including the expected moving-target Doppler frequencies.

- *(b) Sidelobe clutter:* The spectrum is sharply bounded by the condition \(-1 \leq u \leq 1\). So we have for the bandwidth of the whole clutter spectrum: \( B_{\text{clutter}} = 4u_0/\lambda \).

- *(c) The clutter-free region:* This is outside the bandwidth given in *(b).* For most SAR systems, it is impossible to cover all of the sidelobe clutter or even parts of...
the clutter-free region. This can be achieved only if 
\[ \Delta t > 1/B_{sw} - \lambda/(4u) \text{, i.e. the pulse repetition interval has to be shorter than the time needed by the platform to fly a distance of } \lambda/4. \] For gigahertz frequencies and normal airs speeds this would require a PRF larger than allowed to avoid range ambiguities. As a consequence there will be hardly any completely clutter-free region.

Many procedures have been proposed to detect moving targets by 'normal' (single-channel) SAR systems, e.g. using only frequencies outside the mainbeam clutter band\textsuperscript{1,2}, using the normal SAR processor with data prefiltered from other Doppler bands and subsequent subsampling\textsuperscript{3}, using change detection from different looks\textsuperscript{4}, or applying non-uniform sampling\textsuperscript{5}. A theoretically based detection theory is presented in Reference 6. For slowly moving targets whose Doppler frequencies fall completely into the clutter band, the effectiveness of these methods is in principle limited.

Some of the steps towards implementing a more sophisticated MTI-technique will be explained in the following sections.

4 Multichannel SAR

As indicated above, the main difficulties in detecting and estimating moving targets in the SAR environment lie in the motion-induced spread of the clutter Doppler spectrum covering the moving-target signals and the fact that motion-induced Doppler shifts are exchangeable for azimuth shifts. This ambiguity is inherent in the SAR principle and leads to the well-known azimuthal displacement of moving targets in the SAR image.

Both problems can be overcome by using a multi-channel SAR (MSAR), which solves the ambiguity problem and thus makes efficient clutter suppression possible. Moreover, target parameters, including the azimuth position, can be estimated correctly.

The key to the multichannel approach is the well-known idea of the displaced-phase centre antenna (DPCA). Imagine two identical antennas displaced in the flight direction (see Fig. 2). The active antenna is switched from pulse to pulse in such a way that the position of antenna 2 at the second pulse is the same as the position of antenna 1 at the first. So, the echoes received from stationary targets will be the same. If the two received signals are subtracted, the result should be zero apart from the noise contribution. Moving targets with a radial velocity component will produce a phase shift, so that the output of the two pulse canceller will not vanish and detection will be impossible.

One possible way of transferring the DPCA idea to SAR is \textit{along-track interferometry}\textsuperscript{7-8}, another concept is that of \textit{velocity image}\textsuperscript{9}. The approach closest to the optimum test for moving-target detection, but also with the highest computational load, is STAP. Various aspects of STAP in the MSAR-environment are contained in, for example, References 11-15.

\textbf{Experimental set-up}

To investigate the multichannel environment experimentally, a four channel X-band SAR with an active phased array antenna, called 'AER' (airborne experimental radar), has been constructed at FGAN, see Fig. 3. A detailed description can be found in References 16 and 17. All of the image examples contained in this paper have been obtained with this system.

\textit{Along-track interferometry}

In a similar way to the DPCA-technique, along-track interferometry uses two displaced antennas, connected to parallel receiving channels. For each channel a SAR image can be generated. The time delay between the azimuth signals can be compensated during the azimuth compression using two different reference signals, incorporating the azimuth chirps generated in the two channels by a common point scatterer. If the first image is multiplied by the complex conjugate of the second, the remaining phase is zero for stationary objects and non-zero otherwise.

If the two receiving antennas are separated in azimuth by the distance \( d \), the interferometric phase is approximated by \( \Delta \phi = -2\pi d \lambda \rho_\rho \), where \( \rho_\rho \) is the radial velocity of the target. Table 1 shows the sensitivity of this arrangement to slow target motions for realistic parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antenna separation, ( d )</td>
<td>1 m</td>
</tr>
<tr>
<td>Platform velocity, ( v_\rho )</td>
<td>100 m/s</td>
</tr>
<tr>
<td>Wavelength, ( \lambda )</td>
<td>3 cm</td>
</tr>
<tr>
<td>Radial velocity, ( v_\rho )</td>
<td>1 m/s</td>
</tr>
<tr>
<td>Interferometric phase, ( \Delta \phi )</td>
<td>120 deg</td>
</tr>
</tbody>
</table>

In the open literature, along-track interferometry is mainly treated in the context of measurements of the ocean surface\textsuperscript{7-8}, though the capability for sensitive moving-target detection is obvious\textsuperscript{9}.

Fig. 4 shows along-track results generated with data collected by the AER-II system. Fig. 4a shows a normal SAR image of a harbour scene. On closer examination the shape of at least one ship can be recognised, but—due to the azimuth shift—it is displaced from the path of the water. The image in Fig. 4b was derived by
the interferogram and shows only pixels with a differential phase exceeding a threshold. Obviously only the moving objects remain visible. The image in Fig. 4c is complementary to the moving-target image and shows only the stationary scene.

One shortcoming of along-track interferometry is similar to the one of limited efficiency of the two-pulse canceller: to get a high sensitivity, the two antennas have to be widely separated, but this leads to a comb of blind velocities $\nu_\text{blind} = \nu_0 \pm k \lambda/d$, where $k$ is an integer, which thin out the velocity range of interest. Moreover, the above-mentioned distortions in the SAR image remain. In a similar way to the progression from a two-pulse canceller to a Doppler filter bank, this problem can be generalised to a linear array of identical antennas. For each channel a complex SAR image is calculated. A Fourier transform along the physical aperture (i.e. the channel number) is applied to each pixel, and this corresponds to a multiple beamformer. For each Fourier cell the related image shows the scene for a certain range of radial velocities (velocity image\cite{12}). Neither method was originally designed to suppress clutter, since no attempt is made to subtract signals. The requirement to cancel out all the echoes from stationary scatterers leads directly to adaptive space–time filtering.

**MSAR fundamentals**

The techniques described above do not make full use of the information contained in the multichannel SAR data. They require identical antennas. Moreover, the moving target signals must not be suppressed by the SAR imaging processor, which often happens, since the azimuth focusing filter doesn't match the signal. In the following a statistical model of useful and interfering signals arising from a moving array is established without imposing further restrictions on the positions of the phase centre or equality of the characteristics of the component antennas or subapertures. The formalism is based on that presented by Ender\cite{13}.

**Geometry and signal model:** For simplicity, the following considerations are restricted to a two-dimensional geometry in the essential co-ordinates of azimuth and slant range. Moreover, range walk problems are excluded. Though they will have to be taken into consideration in the processor implementation, the correction details are of minor importance.

As illustrated in Fig. 5, we consider an array comprising $N$ sensors at positions $r_n, n = 1 \ldots N$, measured with respect to the array-fixed co-ordinate system $(x, y)$. The origin of the co-ordinates is placed at the transmitter's phase centre.

Let $\mathbf{u}$ denote the unit vector in a certain direction and $D_n(\mathbf{u})$ the two-way characteristics of the $n$th transmit/receive pair.

Under these assumptions, the sensor output voltages for a point scatterer in the direction $\mathbf{u}$ is, apart from a complex constant, described by the $N$-dimensional vector:

\[
\mathbf{a}(\mathbf{u}) = \left( D_1(\mathbf{u}) e^{j \varphi_{r_1}} \right) \ldots \left( D_N(\mathbf{u}) e^{j \varphi_{r_N}} \right)
\]  

\[\text{(1)}\]
The scalar products $u_r$ represent the distance differences from the sensors to the scatterer. This vector, which depends only on the direction to the probe, will be called the DOA-vector (direction of arrival) in accordance with usage in the array-processing literature.

Now we look at a point scatterer moving with constant velocity $v_r = (v_x, v_y)$ which passes at time $t = 0$ through the point $\xi = (\xi_x, \xi_y)$. Whereas the parameter $\xi_y$ is regarded as known (due to the range resolution of the system), the other three parameters $v_x$, $v_y$, and $\xi_y$ are unknown for a moving target; they are collected in the parameter vector $\vartheta = (\xi_y, v_x, v_y)^T$.

Let $R(t)$ and $u(t)$ denote the time histories of distance and direction, which of course depend on $\vartheta$. These functions can be calculated by simple geometrical considerations, depending only on the target positions $r_{uk}(t) = (x(t), y(t))^T$ measured in sensor coordinates and the corresponding relative velocity $v_{uk}(t) = (v_x - v_{uk}, v_y)^T$.

Assuming far-field conditions, we get the following signals:

$$ s(t) = e^{-j2\pi R(t)} u(t) $$

The exponential term in eqn. 2 represents the azimuth chirp as measured by an omnidirectional sensor at the origin of the array coordinate system; the multiplication by the DOA vector accounts for the direction-dependent phase and amplitude modulations by the individual two-way characteristics.

This model signal can serve three purposes:

(a) If the scatterer velocity is set to zero, we get a reference function for SAR imaging of the scene.
(b) The clutter can be modelled by the superposition of time-shifted fixed-target signals weighted by the corresponding reflectivity distribution.
(c) For $v_i \neq 0$ we get a reference function for moving targets.

$R(t)$ and the $x$-component, $u_x(t)$, of the vector $u(t)$ may be approximated by quadratic and linear time functions in the vicinity of the main beam direction $u_0$. The Doppler frequency can be written as a function of the directional cosine $u$. The trajectories are parts of ellipses whose exact shapes are given in Reference 15, but may be approximated by straight lines in the vicinity of the broadside direction.

Fig. 6 shows the trajectories of the direction–Doppler pair for four target motion examples. Scatterers without motion lead to an exact straight line passing through the origin (Doppler = 0 in the broadside direction). Across-track motion causes a Doppler shift. Parallel motion leads to a decrease of the Doppler rate, the chirp always remaining within the clutter band; antiparallel motion results in an
increase in the Doppler rate with an interval at the beginning and the end of the visible path, while the clutter band is left.

Response to the azimuth reflectivity: We consider a strip parallel to the flight path with a width given by the range resolution. The scattering from this strip can be characterised by an azimuthal reflectivity \( p(t) \) scaled to the pulse-to-pulse time domain.

The moving antenna array operates like a time-invariant multidimensional linear filter with input \( p(t) \) and impulse response \( s_\delta(t) \), which is given by the signal of eqn. 2 for a stationary probe at \( \xi_0 = 0 \). The response, \( c(t) \), of the system is calculated by the convolution:

\[
e(t) = (s_\delta * p)(t)
\]  

The Fourier transform \( S_\delta(f) \) of this impulse response can be regarded as the vector-valued transfer function of the system, the azimuthal reflectivity of the scene as the input signal. As a consequence, the Fourier transforms of the reflectivity \( P(f) \) and of the system response \( C(f) \) are related by:

\[
C(f) = S_\delta(f) \cdot P(f)
\]

It is worthwhile having a closer look at this system transfer function. The Fourier transform is given by:

\[
S_\delta(f) = \int e^{j2\pi f} \exp(-j2\beta R(t)) a(u(t)) dt
\]

Since the DOA-term is slowly varying, the different coefficients of \( S_\delta(f) \) basically represent the Fourier transforms of the azimuth chirp windowed by the corresponding two-way patterns. Even more important is the vector-structure of \( S_\delta(f) \). If the time–bandwidth product of the azimuth signal is large enough, the Fourier transform at frequency \( f \) 'picks out' the signal at that point of time when the azimuth chirp assumes the instantaneous frequency \( f \) (chirp selection theorem, see Reference 18):

\[
S_\delta(f) = a(f) a(u(f))
\]

in which \( a(f) \) is a frequency-dependent complex multiplier and \( u(f) \) denotes the direction in which the instantaneous Doppler frequency \( f \) appears. In simpler terms, because of the dependence of Doppler on direction, the Doppler filter selects the corresponding direction.

**MSAR azimuth focusing:** The purpose of azimuth focusing is to get a reconstruction \( \hat{p}(t) \) of the reflectivity \( p(t) \) based on the measured azimuth signal:

\[
z(t) = c(t) + n(t)
\]

consisting of the useful signal \( c(t) \) and receiver noise \( n(t) \). For a single scatterer at \( t = \tau \) with reflectivity \( p(\tau) \) the measured signal takes the form:

\[
z(t) = p(\tau)s_\delta(t-\tau) + n(t)
\]

The reconstruction of \( p(\tau) \) is performed by correlation with a reference signal \( s_\tau(t) \):

\[
\hat{p}(\tau) = \int \overline{s_\tau(t)} z(t) dt
\]

If the noise is white, the signal-to-noise ratio is optimised by using the matched filter \( s_\tau(t) = s_\delta(t) \). For practical purposes the ideal matched filter is replaced by a windowed signal \( s_\tau(t) = w(t)s_\delta(t) \) or by use of a fixed beamformer \( b \) in the main beam direction: \( s_\tau(t) = w(t) bb^H s_\delta(t) \).

**Statistical description of clutter:** In the following considerations the azimuth signal of the stationary scene is taken as interference and is called 'clutter' which can be described by its statistical properties.

We assume that the clutter distribution is spatially stationary, i.e. that the statistical properties are invariant with azimuthal shifts. In this case the clutter distribution is characterised by the covariance function

\[
R_x(\tau) = E[p(t) p^H(t+\tau)]
\]

where \( E[.\] denotes the expectation operator. This function depends on the spatial behaviour of the clutter; it reflects its smoothness or rapid changes in the reflectivity. In most cases it will be appropriate to assume spatially white clutter, especially if many scatterers with independent random phases are contained in each resolution cell.

For the purpose of gaining an insight into space–time adaptive filtering it is important to calculate the cross-covariance matrix of the clutter signals, i.e. that of the system response to \( p(t) \):

\[
R_x(\tau) = E[e(t) e^H(t+\tau)]
\]

If the clutter is spatially white, the expectation integral reduces to the one-dimensional integral:

\[
R_x(\tau) = \sigma^2 \int s_\delta(t) s_\delta^H(t+\tau) dt
\]

where \( \sigma^2 \) denotes the clutter intensity. The relations
appear simpler in the Fourier domain. The Fourier transform of the cross-covariance matrix is called the spectral density matrix and corresponds to the covariance matrix of the Fourier transformed signals. The spectral density matrix $R_c(f)$ of the system’s clutter response can be calculated by the spectral density $R_p(f)$ of the reflectivity:

$$R_c(f) = S_c(f)C_p(f)S_p^H(f)$$  \hspace{1cm} (13)

Note that this matrix has the rank 1, which means that the clutter returns are collected in a one-dimensional subspace, the clutter subspace:

$$CSS(f) = \text{span}(S_c(f) = \text{span}(\mathbf{a} \mathbf{u}(f)))$$  \hspace{1cm} (14)

This is a fundamental property, because it implies that space–time filtering can remove the clutter completely by projection to a subspace orthogonal to the clutter subspace. This is valid only for an infinite time base and for Nyquist sampling in the azimuth direction; otherwise the dimension of the clutter subspace will increase, see Reference 12.

If the noise is modelled by a stationary vector process $n(t)$ with spectral density matrix $R_n(f)$, which is independent of the clutter process, then the complete description of clutter + noise data, $q(t) = c(t) + n(t)$ is again given by a stationary vector process with spectral matrix:

$$R_q(f) = S_c(f)R_p(f)S_p^H(f) + R_n(f)$$  \hspace{1cm} (15)

If the receiver noise components are identical and independent, the eigenspectrum of this matrix consists of $N–1$ small noise-eigenvalues and one large eigenvalue representing the complete clutter contribution plus the $N$th noise term.

It should be pointed out that the structure of this matrix is range-independent if the array is arranged along-track and the elements have a common elevation pattern.

5 Space–time clutter suppression

The optimum space–time processing for the detection of moving targets can be derived analytically. Using the clutter model derived in the preceding section, the test problem can be formulated in a classic manner (see below). We shall derive an optimum solution in the frequency domain and transform it back to the time domain. This results in an infinite impulse response filter. We shall then consider FIR filter solutions based on the space–time covariance matrix. In the following subsection we shall return to the Doppler domain and consider ambiguities.

Test problem

The task of detecting a moving target against a clutter background can be formulated as a statistical test problem. We consider a measurement of the random time series $z(t) = a s(t, \theta) + q(t)$, where $a$ is an unknown complex constant, $s(t, \theta)$ is the moving-target signal, and $q(t)$ comprises statistically independent clutter and noise. The alternative $H: a \neq 0$, that a signal is present, must be tested against the hypothesis $H: 0$, that the measurement reflects only clutter and noise.

Since the interference term represents a coloured random variable, there is no simple ad hoc solution. We find it in the Doppler domain, using the Fourier transform $Z(f)$ of the random time series:

$$Z(f) = aS(f, \theta) + Q(f)$$  \hspace{1cm} (16)

where capital letters denote transforms of the corresponding time series. Now the random variables are asymptotically independent; thus the statistics of the interference are described by eqn. 15.

Optimum solution

The optimum test statistics $T$ for multichannel detection of moving targets in a clutter-plus-noise environment can be derived from the test problem described above.

The Fourier transforms of a stationary process at fixed frequencies are asymptotically independent if the time base tends to infinity. So the different Doppler contributions are asymptotically decoupled and the complete density ratio can be factorised into the contributions from the different frequencies.

The log likelihood ratio tends to the integral over the single contributions. We get the following result:

$$T(Z, \theta) = \left| \mathbf{s}^H(f, \theta)R_q^{-1}(f)Z(f)df \right|^2$$  \hspace{1cm} (17)

The Fourier-transformed data $Z(f)$ are first filtered for each $f$ by the corresponding inverse interference spectral density matrix (clutter suppression). The product with the complex-conjugated Fourier transform of the expected moving-target signal is then formed and the result integrated over all Doppler frequencies (target match).

![Fig. 7 Optimum processor in Doppler domain](image)
magnitude of the result has to be compared to a threshold.

In contrast to the FIR (finite impulse response) filter algorithms described later, this approach can compensate for arbitrarily differing sensor patterns, since \( R_q(f) \) can be written as:

\[
R_q(f) = \gamma(f) a(u(f))a^H(u(f)) + R_N(f)
\] (18)

To each Doppler frequency \( f \) there corresponds a clutter direction \( u(f) \). The application of \( R_q^{-1}(f) \) to the data vector \( Z(f) \) results in the creation of a spatial null to \( u(f) \), see Fig. 7. For optimum processing, the maximum achievable SCNR (signal-to-clutter-plus-noise ratio) is calculated as:

\[
SCNR(\delta) = \int S^*(f, \delta) R_q^{-1}(f) S(f, \delta) df
\] (19)

This expression is suitable for testing different antenna configurations according to their principal MTI-performance.

**STAP in the time domain**

*Time behaviour of the optimum clutter filter.* We can get some insight into the behaviour of a STAP filter operating in the time domain if the optimum filter in eqn. 17 is retransformed into the time domain. As pointed out in Reference 12, assuming equispaced, identical elements, a rectangular clutter spectrum and white receiver noise, the inverse Fourier transform

\[
H_m = \int R_q^{-1}(f) e^{j2\pi fm \Delta f} df
\] (20)

can be written in the form \( H_m = \delta(m) I - P(m) \), where \( \delta \) denotes the Kronecker function and \( I \) is the unit matrix.

The matrix \( P \) represents a (sinx)/x interpolator operating on a one dimensional rearranged sequence of all the temporal and spatial samples, see Fig. 8. The interpolated values are subtracted from the actual sample vector. The operation of this filter can be recognised as a kind of motion compensation.

A simple non-adaptive finite impulse response space-time filter could be realised by truncating the infinite series and taking only \( M \) of the matrices in eqn. 20.

**Space-time covariance matrix:** STAP in the time domain means application of a matrix-valued filter with adaptively calculated weights to a finite number, \( M \), of consecutive sample vectors and thus forms only a suboptimum solution to the clutter suppression problem. In view of the stationary nature of the SAR sampling and the potentially infinite sequence of sample values a time sequential FIR filter operation is appropriate, as described in Reference 19. From the two-dimensional data array \( z_m = z(m \Delta t) \) we take a sequence of space–time vectors:

\[
\check{z}_m = \begin{pmatrix} z_{m-M+1} \\ \vdots \\ z_m \end{pmatrix}
\] (21)

comprising \( M \) consecutive sample vectors, so that \( \check{z}_m \) has the dimension \( M \times N \). The covariance matrix of the interference part \( \check{q}_m \) of these vectors is given by the \( MN \times MN \) space–time covariance matrix \( \check{R}_q = E[\check{q}_m \check{q}_m^H] \).

**Filter schemes:** There are different approaches to deriving filter schemes from the space–time covariance matrix.

The first approach is the optimum processing for a known signal. If \( \check{w}_m \) denotes the space–time model signal, the optimum (time-dependent) weights are given by

\[
\check{w}_m = \check{R}_q^{-1} \check{w}_m \text{ and the filtered sequence takes the form}
\]

\[
\check{z}_m = \check{w}_m \check{w}_m \check{z}_m.
\]
This is impractical for SAR purposes, since the weight vector is time dependent and suitable only for a single model signal. An equivalent method is filtering with the $MN \times MN$ filter matrix $\mathbf{H} = \mathbf{R}^{-1}$ followed by signal matching.

Another approach has been proposed by Barbarossa and Farina. No assumptions are made about the useful signal: it is modelled as white noise. The ratio between the expected signal power and the expected interference power is maximised by choosing as weight vector the eigenvector corresponding to the smallest eigenvalue. This approach has the advantage that a constant weight vector produces a one-dimensional sequence of filtered samples.

A third method is addressed by Klemm and Enders. We search for a matrix-valued FIR filter $\mathbf{H}_m$ working on the spatial vector data:

$$\mathbf{z}_m = \sum_{i=0}^{M-1} \mathbf{H}_m \mathbf{z}_{m+i}$$

A suitable filter consists of any block row of the inverse space–time covariance matrix which can be partitioned into $M \times M$ block matrices, each of dimension $N \times N$:

$$\mathbf{R}_z^{-1} = \begin{pmatrix} G_{1,1} & \ldots & G_{1,M} \\ \vdots & \ddots & \vdots \\ G_{M,1} & \ldots & G_{M,M} \end{pmatrix}$$

The filter matrices $\mathbf{H}_m$ are chosen as the $k$th block-row of this matrix, and combined to form the $N \times NM$ matrix $\mathbf{H} = (\mathbf{H}_1, \ldots, \mathbf{H}_M)$.

The filtered sequence is of dimension $M$ and can be treated analogously to eqn. 9 for estimating the moving-target reflectivity $\hat{\rho}(t)$:

$$\hat{\rho}(t) = \frac{\sum_{m} s^H(\mathbf{m} \Delta t - \tau, \vartheta) \mathbf{H} \mathbf{z}_m}{\sum_{m} s^H(\mathbf{m} \Delta t - \tau, \vartheta) \mathbf{H} \mathbf{s}(\mathbf{m} \Delta t - \tau, \vartheta)}$$

If fixed-beam processing is preferred, the beamformer $\mathbf{b}$ can be integrated in the filter, resulting in $\mathbf{w} = \mathbf{H} \mathbf{b}$.

**Estimation of the covariance matrix:** The space–time covariance matrix can be estimated by forming the empirical covariance matrix, using a training set $\mathbf{q}_1, \ldots, \mathbf{q}_K$ of space–time vectors, which should not contain moving-target signals, and averaging over the index $k$:

$$\mathbf{R}_z = \frac{1}{K} \sum_{k=1}^{K} \mathbf{q}_k \mathbf{q}_k^H$$

The averaging can be performed over the range domain if an along-track array is used. Normally, in SAR applications there are a large number of range bins to get an estimate with low variance: although the clutter-to-noise ratio will vary rapidly along the range dimension for real data, the structure of the underlying clutter subspace remains unchanged. Strong moving targets contained in the training set can deform the space–time covariance matrix in such a way that they are later suppressed by the adaptive filter; to prevent this, the averaging should be extended also along subsequent azimuth intervals.

**STAP in Doppler domain**

We can apply an adaptive version, derived by the shape of the optimum filter in the Doppler domain, in the following way. A finite sequence $\mathbf{z}(\Delta f), \ldots, \mathbf{z}(M \Delta f)$ of sample vectors is Fourier transformed to $\mathbf{Z}(\Delta f), \ldots, \mathbf{Z}(M \Delta f)$. In the same way we get a training set of vectors in the Doppler domain, for which the empirical spectral density matrix can be formed analogously to eqn. 25. The filter matrices are derived, for example, by matrix inversion (SMI = sample matrix inversion) or by eigenspace decomposition. After application to the data $\mathbf{Z}(\omega \Delta f)$, target matching and back transformation are carried out.

The length of the time base needed for the Fourier transform depends on the correlation time of the clutter process. This is related to the time needed by the platform to fly a distance equal to the complete real aperture. The rank of the spectral matrix approaches if the DOA vector can be assumed constant over the width of the synthetic beam, i.e., that portion which is cut out of the pattern by the Doppler transform.

**Ambiguities:** The effects of azimuth and real-aperture undersampling are studied in Reference 12. In the case of azimuth undersampling (low PRF), the rank of the clutter spectral density matrix can—dependent on the Doppler frequency—be greater than one. The reason for this is that for a given Doppler frequency there are clutter contributions from more than one direction, since the directions associated with a Doppler frequency increased or decreased by a multiple of the PRF have to be taken into account too. On the other hand, real-aperture undersampling (too large spacing of the subaperture phase centres) has no effect on the clutter space dimension, but may generate 'blind directions'.

**Experimental SAR/MTI**

In the processing of the experimental multichannel data gathered by the AER system, clutter suppression and
target detection are followed by high-resolution azimuth position estimation and short-time target tracking. SAR-image and MTI-information are subsequently combined. Fig. 9 is a SAR/MTI result which shows a motorway exit and the automatically determined positions of many detected vehicles.

6 Conclusions

This paper has discussed different approaches to the suppression of clutter interference and to the detection and estimation of moving targets using a SAR system equipped with two or more antennas or subapertures. Simple deterministic schemes such as along-track interferometry and the ‘velocity image’ can be improved by using adaptive processing.

The optimum processor in the frequency domain, from which suitable adaptive algorithms can be derived, has been determined. Time domain application is possible but, in the author’s opinion, using the spectral matrix as a basis for filters has proven to be the most suitable approach for SAR/MTI.

Time-space filtering in the time domain suffers from unequal subaperture characteristics. The filter length has to be chosen sufficiently large to achieve acceptable clutter suppression. In the Fourier domain, each Doppler frequency corresponds to a particular direction described by the DOA-vector including the different subaperture characteristics corresponding to that specified direction. If a narrowband-like interference suppression is carried out for each Doppler cell (=direction), these variations are equalised.

The most interesting property of Doppler-domain-processing is that the clutter energy is concentrated in a one-dimensional subspace (only for the Nyquist PRF). Thus, the clutter can be cancelled nearly perfectly, while an (N-1)-dimensional subspace is left for the useful signal. This is very important for SAR echoes which have a large dynamic range and strong hard target returns.

The detection of moving targets is useless if their positions are not accurately estimated. The spectral density matrix again offers a tool for measurement of the array manifold and subsequent high-resolution angular estimation.

Real multichannel SAR-data has been exploited at FGAN with encouraging results.

Acknowledgments

I should like to thank all the people who have supported the theoretical and experimental work described in this paper and in particular Dr. Richard Klemm, with whom I have had many inspiring discussions on STAP over the last decade, and who stimulated the idea of realising the Airborne Experimental Radar. I should also like to thank Ursula Skupin for her considerable work on SAR processing, and Dr. Ludwig Rössing, Parick Berens, Claus Kirchner and Winfried Jansen for fruitful discussions and for checking the manuscript.

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Received 14th December 1998.

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