Position Estimation of Moving Vehicles for Space-based Multi-channel SAR/MTI Systems

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Abstract

Ground moving target indication (GMTI) by a space-based radar system can be obtained by a multi-subaperture / multi-channel radar system or a satellite cluster. The simplest approach with the receiving antenna split into two halves - and most popular in the SAR community - is to use the technique of along-track interferometry (ATI) to recognize moving objects. Caused by the ambiguity between radial velocity and azimuth position, the SAR processor images the moving object at a wrong position. Re-positioning can be performed by the exploitation of the interferometric phase; but, if the clutter contribution from the affected resolution cell is not negligible compared to the signal power, a severe estimation error will result. A larger number of subapertures permits a better performance using space-time adaptive processing (STAP), but a larger number of receive channels is not attractive for space-based systems because of weight, power consumption and data rate. Nevertheless, since phased array antennas offer the possibility to switch the phases and amplitudes of the T/R modules from pulse to pulse within each subaperture, additional degrees of freedom can be introduced increasing the performance considerably. In this paper, space-based multi-aperture radar systems will be analysed with respect to the relocation error, including time-multiplexed aperture switching techniques. Moreover, an azimuth estimation algorithm for this mode is investigated.

1 Introduction

For a space-based sensitive detection of moving vehicles a spatial diversity of the antenna system is essential, either by multiple subapertures or by distributing the receive antennas over a satellite cluster [1, 2, 3]. As a special solution, this capability can be achieved by switching subapertures or aperture weightings from pulse to pulse. This temporal diversity has been proposed for SAR/MTI already in [4]. Recently, in [5] the MTI performance of such switched apertures was compared for a couple of spatial/temporal sampling strategies, and in [6] this method has been taken into account for the coming remote sensing system TerraSAR-X [7]. This paper investigates the azimuth position estimation for moving objects including the switched aperture approach. Without or with only small clutter interference ATI offers a basis for the simultaneous estimation of radial velocity and azimuth position. The statistics of interferograms has been analysed in depth in [8], some aspects concerning space-based MTI for TerraSAR-X can be found in [9]. For medium or strong clutter interference it is preferred to use directly the complex samples; a good performance can only be achieved with three or more simultaneous or time-multiplexed channels. Position estimation variances for multi-aperture systems were given in [10] based on the Cramér-Rao bounds (CRB), some details about CRB for STAP can be found in [11]. This approach will be extended in this paper to the switched aperture case. Moreover, as an implementable estimator the "adaptive monopulse estimator" (AME) [12] is analysed for comparison.

Table 1: Basic parameters of three space-based SAR-(MTI)-Systems

<table>
<thead>
<tr>
<th>Parameter</th>
<th>TERRA-SAR-X</th>
<th>RADARSA T-II</th>
<th>COSMO-Skymed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center frequ.</td>
<td>9.65 GHz</td>
<td>5.40 GHz</td>
<td>9.60 GHz</td>
</tr>
<tr>
<td>Antenna length</td>
<td>4.78 m</td>
<td>15.00 m</td>
<td>5.70 m</td>
</tr>
<tr>
<td>Antenna height</td>
<td>0.75 m</td>
<td>1.40 m</td>
<td>1.40 m</td>
</tr>
<tr>
<td>Beamwidth (azim)</td>
<td>6.5 mrad</td>
<td>3.7 mrad</td>
<td>5.5 mrad</td>
</tr>
<tr>
<td>Velocity</td>
<td>7600 m/s</td>
<td>7600 m/s</td>
<td>7600 m/s</td>
</tr>
<tr>
<td>Incidence angle</td>
<td>45 deg</td>
<td>45 deg</td>
<td>45 deg</td>
</tr>
<tr>
<td>Center range</td>
<td>700 km</td>
<td>700 km</td>
<td>700 km</td>
</tr>
</tbody>
</table>

2 ATI versus STAP

In this section, we will discuss the possibility to estimate the vehicle’s position by means of either ATI or STAP techniques in a heuristical manner. Figure 1 shows some situations in the Gaussian plane where target and clutter signals are superposed. Remember, that in general the interfering clutter comes from another azimuth position than that of the vehicle. It may be e. g. the echo of a large building with a very large cross section.
For an along-track array with signals (after Doppler filtering or even SAR focusing) which are motion-compensated from antenna to antenna, the clutter return will always stay at the same complex point (if no temporal change of the clutter reflectivity is taken into account), indicated by the vector $c$. A moving object will produce a vector $s$, rotating with a phase increment according to the actual radial velocity. The vector sum runs along a circle with center $c$. Since there is an ambiguity between radial velocity and azimuth position, the accuracy of the position estimate is directly coupled to that of the azimuth direction.

The situation b) in the figure shows the circumstances if only the interferometric phase $\phi$ is used. Depending on the relative phase between clutter and target signal, the interferometric phase does not necessarily change from pulse to pulse; so the vehicle in many cases will not be detected, least of all will its position be retraceable. The additional use of the interferogram’s amplitude doesn’t solve the problem, since only the product of the two single amplitudes is available and amplitude changes cannot be observed. Situation c) shows, that only two samples in the Gaussian plane are not yet sufficient for estimation: the same two points can result from ambiguous constellations between target and clutter signals. In situation a) it becomes clear, that at least three samples are required to reconstruct the circle, to subtract the clutter contribution and to estimate the phase increment unambiguously.

The amount of information lost by application of a mapping like $(z_1, z_2) \rightarrow z_1^* z_2$ - describing the formation of an interferogram - can be calculated by the Fisher information reduction indicated in section 5, Eq. 7.

### 3 Analysis results

The MTI detection and estimation performance has been analysed for various constellations and aperture switching strategies. The underlying statistical model and the analysis methods (optimum signal-to-clutter-plus-noise ratio (SCNR) achievable by STAP, CRB and AME performance) are described in the following sections.

In the following we will present only some examples. Figure 2 compares the MTI performances of RADARSAT-II for a partitioning of the receive aperture from one to four equal subapertures. It can be seen that the SCNR gain is drastically improved by the step from one to two subapertures whereas the additional gain by the use of further channels is only marginal. The CRB-plots as lower bounds for the RMS estimation error start at two subapertures. Here, the step from two to three subapertures is decisive. The plots show a nearly optimum behaviour close to velocity zero. This seems to be contradictory; nevertheless a deeper look into the mathematics exhibits that within the notch of the clutter filter indeed the amplitude of the useful signal is reduced; on the other hand the sensitivity to direction deviations is increased by the same value so that the two effects cancel out.

In Figure 3 different sampling strategies are compared for the system parameters of TerraSAR-X. Four subapertures of equal size are assumed, the performance of four parallel receive channels is compared to toggeling between the combinations of subaperture 1 & 2 and 3 & 4, or 1 & 4 and 2 & 3, respectively. Also the switching over four pulses in the order 1, 2, 3, 4 is taken into account. In the latter case, at first the performance is poor because of undersampling with the effective PRF (four times smaller than the original). If the PRF is increased by a factor of two, the performance becomes acceptable again.

Figure 4 again relates to TerraSAR-X. Now three apertures are considered, where the centre subaperture is varied in
size and the centre and the outer apertures are toggled. In addition to the CRB the standard deviation of a real estimator - the AME - is plotted. It can be seen that the good performance close to the clutter notch is preserved by the AME. The standard deviation of the direction estimate can be converted to the standard deviation of the azimuth position.

Obviously, the switching concept can achieve results close to those for pure parallel channels, provided that the effective PRF is high enough. Nevertheless, for the assumed parameters the error is in the order of 200 m for TerraSAR-X in the clutter free region and more than 1 km for vehicles with a radial speed of about 10 m/s. So, even for optimum processing, none of the regarded spaceborne systems is able to relocate moving vehicles to the street without additional context information. The intrinsic reason is in all cases the limited aperture compared to the wavelength; This disadvantage can be overcome only by an extended aperture for instance by a satellite cluster. Moreover, it is shown in [3] that configurations like the Cartwheel concept are able to cover also a satellite cluster. Moreover, it is shown in [3] that configurations like the Cartwheel concept are able to cover also a satellite cluster.

4 Statistical space-time model of clutter and target

In the following, the mathematical background will be sketched. We take over the signal model given in [5]. If the coherent integration time is long enough to result in a Doppler resolution patch on the earth surface much smaller than the antenna footprint, and short enough to ensure that the instantaneous Doppler frequency does not leave the Doppler resolution cell during the time of integration, a target signal with radial velocity \( v \) at range \( R \) is interfered by clutter patches in the directions

\[
\mathbf{u}_{\mu\nu} = f^{-1}\left( (R + \mu \Delta R, v_r + \nu \Delta v)^T \right)
\]

with \((\mu, \nu)\) running through all index pairs with non negligible clutter contributions. Here, \( \Delta R = c/(2PRF) \) and \( \Delta v = \frac{c}{2}PRF \) (\( \lambda = \) wavelength) denote the range and velocity ambiguities introduced by the effective pulse repetition frequency \( PRF \), and \( f \) is the function assigning to a direction vector \( \mathbf{u} \) the corresponding range and radial velocity of the clutter.

In [5] a set of \( M \) receiving apertures with the complex antenna characteristics \( D_{\mu m}(\mathbf{u}) \), \( m = 1, \ldots, M \) were defined which are distributed in time and space over subsequences each of \( N \leq M \) pulses which are repeated infinitely. So, some of the subapertures are sampled in parallel, some by switching e. g. the T/R modules from pulse to pulse. This results in an effective pulse repetition frequency \( N \) times lower than the original one.

For a certain direction \( \mathbf{u} \) at Doppler \( F \) a generalized DOA-vector \( \text{DOA} = \text{direction of arrival} \) can be defined by

\[
d(\mathbf{u}, F) = D(t)(\mathbf{u})D_{\mu m}(\mathbf{u})\exp(-j2\pi n_m F \Delta T)\big|_{m=1,\ldots,M}.
\]

In this equation, the complex transmit antenna characteristics is denoted by \( D(t)(\mathbf{u}) \), the time shift between the samplings has been accounted for in the exponential term, where \( n_m \) is the index of the subpulse at which the individual subaperture is activated and \( \Delta T \) is the original pulse repetition interval.

\[ \text{Figure 3: TerraSAR-X, 4 subapertures, different sampling strategies, } PRF = 6332 \text{ Hz (last plot: } 12664 \text{ Hz), SNR = 20 dB, CNR = 30 dB; beamwidth } \equiv 4548 \text{ m} \]

The whole measured signal is expressed by the random vector

\[
\mathbf{Z} = a \mathbf{d}(\mathbf{u}_s, F) + \mathbf{W},
\]

where \( \mathbf{W} \) is the clutter-plus-noise interference from all of the previous mentioned clutter directions with covariance matrix \( \mathbf{R}_W \).

The resulting detection performance is a function of the signal-to-clutter-plus-noise ratio after optimum processing \( \text{SCNR} \):

\[
\text{SCNR} = |a|^2 \mathbf{d}^H(\mathbf{u}_s, F)\mathbf{R}_W^{-1}(R, F)\mathbf{d}(\mathbf{u}_s, F).
\]

5 Cramér-Rao bounds

The variance of any unbiased estimator is larger or equal to the Cramér-Rao-Bound (CRB). For a model \( \mathbf{Z} = \mathbf{s}(\vartheta) + \mathbf{W} \) with an unknown parameter vector \( \vartheta \) and Gaussian interference \( \mathbf{W} \) with covariance matrix \( \mathbf{R}_W \), the elements
of the Fisher information matrix $J$ for the estimation of $\vartheta$ are given by

$$J_{kl} = E \left[ \frac{\partial}{\partial \vartheta_k} \ln p_\vartheta^Z(Z) \frac{\partial}{\partial \vartheta_l} \ln p_\vartheta^Z(Z) \right] = 2 \text{Re} \left\{ s_k^* R_w^{-1} s_l \right\} ,$$

where $p_\vartheta^Z(z)$ is the probability density function, and an index $k$ means the derivation with respect to the component $\vartheta_k$. In our case, we set $\vartheta = (u, \alpha, \varphi)$ and $s(\vartheta) = \alpha e^{j\varphi} d(u)$. Then, the diagonal coefficient of the inverse of the Fisher matrix corresponding to $u$ results in the CRB for the variance of an estimator $\hat{u}$ of $u$.

If the estimation is based on the values of a mapping $T$ applied to the vector $Z$ (for instance an interferogram), the Fisher information is reduced to

$$J_{kl} = E \left[ F_k(T(Z)) F_l(T(Z)) \right] \quad (6)$$

with

$$F_k(t) = E \left[ \frac{\partial}{\partial \vartheta_k} \ln p_\vartheta^Z(Z)|T(Z) = t \right]$$

$$= 2 \text{Re} \left\{ s_k^* R_w^{-1} E \left[ W|T(W + s) = t \right] \right\} .$$

In this way, the loss of information by application of the mapping $T$ can be calculated.

### 6 Adaptive monopulse error as performance measure

The question arises if a real direction estimator can be found with a mean square error close to the CRB. We propose to use as a performance measure the error of the AME which can be implemented in real time and is one of the best algorithms available. The algorithm and the analysis of its performance have been treated in depth in [12]. In the most cases this error is very close to the CRB.

### References


