A SIMULATION STUDY OF THE $\omega - k$ SAR ALGORITHM FOR THE HIGHLY SQUINTED CASE WITH APPLICATION TO RUNWAY IMAGING

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ABSTRACT

The wavenumber-domain SAR algorithm is studied for the case of a high squint angle with particular application to runway imaging from an approaching aircraft. The feasibility of using the algorithm for this application is shown by point target simulations which indicate that aberrations due to the high squint angle can be eliminated with finer interpolation in the processing.

1. INTRODUCTION

Synthetic Aperture Radar (SAR) is used to produce high-resolution radar imagery [1]. Conventional SAR methods typically employ the plane wave assumption. However, in some cases, including in runway imaging, wavefront curvature must be taken into account [2]. Recently proposed so-called wavenumber-domain methods (also known as $\omega - k$ or k-domain algorithms) increase the resolution by modeling the actual spherical wave [3, 4].

The $\omega - k$ algorithm has been suggested for imaging a runway from an approaching aircraft during landing in order to determine whether there is any object either on or near the runway that would make the landing unsafe [5].

In runway imaging, the radar looks almost straight ahead. For such a high-squint-angle case, the $\omega - k$ algorithm shows some aberrations depending on a number of factors such as the location of the target in the scene, accuracy of the interpolation, etc. Here, we will determine the aberrations that occur when the squint angle is very high and give simulations demonstrating the feasibility of using this method for runway imaging.

2. WAVENUMBER DOMAIN PROCESSING

Figure 1 describes the geometry of the data collection scenario in the ground plane. The radar platform is an aircraft approaching a runway for landing. The runway lies along the $y$ axis straight ahead of the aircraft. The elevation of the aircraft is omitted in the following but the generalization of the data model to slant range geometry is straightforward. The radar collects data along its flight path from $y = -\frac{L}{2}$ to $y = \frac{L}{2}$, where $L$ is the synthetic aperture length. At regular spatial intervals the radar transmits a pulse and collects the return. The transmitted signal is a real passband signal $\Re \{ s_r(t) \}$ where $s_r(t)$ is a linear FM waveform given as $s_r(t) = s_0(t) \exp(j\omega_0 t)$ where $\omega_0 = 2\pi f_0$ and $s_0(t) = \exp(j\pi \alpha t^2)$ for $|t| \leq T/2$ and zero otherwise. $f_0$ is the center frequency of the chirp signal and $\alpha$ is the chirp rate. The squint angle $\gamma_0$ of the scene center $(X_0, Y_0)$ with respect to the origin of the coordinate system, is given by $\arctan(Y_0/X_0)$. The return signal from a scene with reflectivity $g(x, y)$ is the collection of returns from all infinitesimal scatterers in the scene. Note that due to the finite antenna footprint, $g(x, y)$ vanishes outside the antenna beam, that is, the antenna gain is actually incorporated in $g(x, y)$. Let the range function be denoted as $r(x', y' - y) = [x'^2 + (y' - y)^2]^{1/2}$ when the radar is at location $y$ on the flight path and the coordinates of a particular scatterer are given by $(x', y')$. Depending on the location of the scatterer and the position of the radar on the flight path, $r(x', y - y')$ takes values in $[r_{\text{min}}, r_{\text{max}}]$. Let us define $R_0$ to be the distance from the origin of the coordinate system to the center $(X_0, Y_0)$ of the target scene. Then $\tau_0 = \frac{2R_0}{c}$. Also define $\omega = \omega_0 + 2\pi \alpha (t - \tau_0)$. The wavenumber is given by $k = \omega/c$. If the transmitted signal is the linear FM pulse given before, then passing the return signal through a quadrature demodulator we obtain the return signal in complex baseband as

$$s_n(t, y) = \int_{-\infty}^{\infty} g(x', y') \exp \left(-\frac{j4\pi \alpha}{c^2} (r(x', y - y') - R_0)^2\right) \exp(-j2kR_0) \exp(j2kr(x', y - y')) \, dx' \, dy' \quad (1)$$

for $-\frac{x}{2} + \frac{x}{2} \leq t \leq \frac{x}{2} + \frac{x}{2}$. The second exponential in the above expression is known and the first exponential term can be corrected afterwards or totally omitted if the time-bandwidth product of the radar signal is large enough [6]. Let the imaging kernel be denoted by $f(x', y; \omega) = \exp(j2kr(x', y))$ and write the return signal, after omitting the first and second exponential terms, as

$$s(\omega, y) = \int_{-\infty}^{\infty} g(x', y') f(x', y - y'; \omega) \, dx' \, dy'. \quad (2)$$

Denote the two-dimensional Fourier transform of the imaging kernel by $F(k_x', k_y'; \omega)$. Inserting this into (2)

$$s(\omega, y) = \int_{-\infty}^{\infty} F(k_x', k_y'; \omega) G(-k_x', k_y) \frac{\exp(jk_y y)}{4\pi^2} \, dk_x' \, dk_y \quad (3)$$

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where \( G(k_x', k_y) \) is the two-dimensional Fourier transform of \( g(x, y) \). F(k_x', k_y; \omega) \) is approximated as \( \omega \),

\[
F(k_x', k_y; \omega) \approx \begin{cases} A \delta(k_x' - \sqrt{4k^2 - k_x^2}) & |k_y| < 2k \\ 0 & |k_y| > 2k \end{cases} \quad (4)
\]

where \( A \) is a complex constant. Then,

\[
S(\omega, k_y) \approx \frac{A}{2\pi} G(-\sqrt{4k^2 - k_x^2}, k_y)
\]

where \( S(\omega, k_y) \) is the one-dimensional Fourier transform of \( s(\omega, y) \). Let \( k_x = -f \sqrt{4k^2 - k_y^2} \). Then taking the one-dimensional Fourier transform of the collected data \( s(\omega, y) \) in the \( y \) dimension, we obtain \( A \frac{f}{2\pi} G(k_x, k_y) \). Thus, taking the two-dimensional inverse Fourier transform gives the reflectivity function. However, as indicated by the relation between \( k_x \) and \( k_y \), the Fourier data lies on an irregular grid. Thus we need to interpolate the data onto a Cartesian grid before we can take a two-dimensional fast Fourier transform. The image formation algorithm is summarized in Figure 2. In the first phase multiplication, \( k_x' \) is the center frequency of the data in the \( y \) dimension. This phase shift in the spatial domain causes the \( k_y \) spectrum of the data to be shifted to baseband. The second phase multiplication in the wavenumber domain shifts the spatial-domain data such that the center of the reconstructed image is the scene center.

\[
\begin{align*}
\text{return signal} & \xrightarrow{\text{1-D FFT in } y} jk_y y' \\
\text{image} & \xrightarrow{\text{2-D IFFT}} \text{INTERPOLATION}
\end{align*}
\]

Figure 2: Block diagram of \( \omega - k \) algorithm.

The wavenumber in the \( y \) dimension is given as \( k_y' = -2f \sin \gamma_y' \) where \( \gamma_y' \) is the squint angle with respect to broadside between the radar at position \( y \) and a point scatterer at \( (x', y') \). As the radar moves along the flight path, and for point scatterers at various locations in the scene, the angle \( \gamma_y' \) takes values in \([\gamma_{y\text{min}}, \gamma_{y\text{max}}]\). The temporal frequency is in \([\omega_{\text{min}}, \omega_{\text{max}}]\). Hence, the Fourier data \( S(\omega, k_y) \) lies in an area determined by the extremes of squint angle and frequency in the \( \omega - k_y \) plane. Equivalently, this data can be represented in the \( k_x - k_y \) plane: The wavenumber in the \( x \) direction is \( k_x = -2f \cos \gamma_y' or k_x^2 + k_y^2 = 4f^2 \) which is the equation of a circle with radius \( 2f \) in the \( k_x - k_y \) plane. Thus, the Fourier data lies on arcs which are parts of concentric circles between radial lines determined by the extremes of the squint angles. This is the data to be interpolated onto a Cartesian grid before inverse Fourier transformation. Let us denote equally spaced values of \( \omega \) as vectors \( \hat{\omega} \) and \( \hat{\omega}_y \) of size \( N_\omega \) and \( N_y \) respectively. Then for a fixed \( k_y \) value, say \( k_y^j \), that is the \( j \)-th component of \( \hat{\omega}_y \), we can write the following,

\[
\hat{\omega}_y^j = \sqrt{\frac{\omega^2}{c^2} - (\hat{\omega}_x^j)^2}, \quad 0 \leq j \leq N_y - 1
\]

where \( \hat{\omega}_x^j \) denotes the vector of unevenly spaced \( k_y \) values, of size \( N_y \).

Now define the vector \( \hat{\omega}_x \) of size \( N_x \) to be the vector of evenly spaced \( k_x \) values. Then, \( \hat{\omega}_x \) and \( \hat{\omega}_y \) define a Cartesian grid and this corresponds to a matrix \( \hat{\omega} \) of size \( N_y \times N_y \) which consists of unevenly spaced \( \omega \) values

\[
\hat{\omega} = \frac{c}{2} \sqrt{(\hat{\omega}_x^j)^2 + (\hat{\omega}_y^j)^2}
\]

Let us define the matrices \( \hat{G} \) of size \( N_y \times N_y \) and \( \hat{G}_x \times N_x \), respectively, such that

\[
\hat{G}_x = G(k_x^1, k_y^1) \quad \text{and} \quad \hat{G}_y = G(k_x^j, k_y^j).
\]

From this notation, it is clear that interpolation maps \( \hat{G}_x \) to \( \hat{G}_y \). We use a Hamming-windowed sinc kernel \( h(t, N_y) \) of size \( N_y \) for the interpolation:

\[
\hat{\hat{G}}_y^j = \begin{cases} \sum_{t \in L_{ij}} \hat{G}_x^j h(\frac{1}{N_y} |\hat{\omega}_y^j - \hat{\omega}_y^j|, N_y) & \text{if } L_{ij} \neq \emptyset \\ 0 & \text{otherwise} \end{cases}
\]

where \( L_{ij} = \{ t : \frac{1}{N_y} |\hat{\omega}_y^j - \hat{\omega}_y^j| \leq N_y \} \). Note that the interpolation from \( \hat{G}_x \) to \( \hat{G}_y \) is actually done in a one-dimensional manner between \( \hat{\omega}_y \) and \( \hat{\omega}_y \). Having denser samples in \( \hat{\omega}_y \) or larger \( N_y \) makes the interpolation more accurate. Also notice that as the squint angle increases, the irregularity in \( \hat{\omega}_y \) increases. For given \( k_y \), \( \omega \) and \( k_x \) are related by \( k_x = k_y \cot(\arcsin(-k_y/2f)) \), which is more nonlinear when the squint angle relating \( k_y \) to \( k_x \) is large. Hence, if the squint angle is large we require more accurate one-dimensional interpolation.

The instantaneous frequency of the signal \( S(\omega, k_y) \) in the \( \omega \) dimension is \( \nu_\omega = \frac{f}{2} (x' - X_0) \cos(\gamma_y') \). Then \( \sigma_\nu = \max |\nu_\omega| \) and the temporal frequency sampling interval must satisfy

\[
\Delta \omega \leq \frac{\pi}{\sigma_\nu}.
\]

Notice that \( \Delta \omega \) depends on the offset \( |x' - X_0| \) of a target in the scene. The farther a target is from the center of the scene in the cross-track direction, the smaller should be
the temporal frequency sampling interval. The cosine term indicates that for a high squint angle, a smaller \( \Delta \omega \) is necessary. Notice also that the bound for \( \Delta \omega \) is determined by the Nyquist criterion which holds for evenly spaced samples that are infinite in extent. However, the interpolation step, which uses a finite set of samples separated by a distance of \( \Delta \omega \), may require a smaller \( \Delta \omega \) than dictated by (9) for accurate reconstruction.

The cross-track resolution is

\[
\delta_x = \frac{c \pi}{\omega_{\text{max}} \cos \gamma'_{\text{max}} - \omega_{\text{min}} \cos \gamma'_{\text{min}}} 
\]  

where the cosine terms correspond to minimum and maximum values of \( k_x \). Notice that resolution in the cross-track direction is dependent on the range of target angles as seen by the radar as it moves along the flight path. Thus, objects closer to the runway will be imaged with less resolution.

3. SIMULATIONS AND DISCUSSION

To evaluate the feasibility of \( \omega - k \) processing for runway imaging, we simulated several example scenarios. In these examples, the temporal center frequency of the radar signal was 10 GHz and the bandwidth was 32 MHz. The radar traveled \( L_x = 500 \) m along track and illuminated a scene area of \( L_x = 120 \) m by \( L_y = 500 \) m.

In Figure 3 we see the results of a simulation where targets are at \((160, 3100), (180, 3200), (200, 3300), (220, 3400)\); all numbers are in meters. The scene center is at \((200, 3200)\), hence the squint angle \( \gamma_0 = 86.42^\circ \). In Figure 3, images of targets are reconstructed at their correct locations. In Figure 4, without changing the positions of the targets relative to the scene center, we moved the scene center to \((60, 3200)\), which corresponds to a squint angle of \( \gamma_0 = 88.93^\circ \) and the new locations of the targets are \((20, 3100), (40, 3200), (60, 3300), (80, 3400)\). As the squint angle approaches 90° for targets nearer the runway center, the targets appear
smeared out with energy smeared toward the center of the scene in the reconstructed image. The dashed line at $x = 76.2$ m is the edge of the FAA zone. For these simulations the number of temporal samples, $N_x$, was held constant at 1024. Notice that the condition (9) was satisfied with $N_u = 1024$ for all targets except the target at (20,1000). To satisfy that condition we need $N_u = 1536$. The result of the simulation with that many samples is shown in Figure 5. Although the condition (9) has been satisfied, there are still certain aberrations. This suggests that the interpolation is not fine enough and that as we increase $N_u$, the interpolation will become more accurate as described in the previous section. The computational cost of interpolation increases only linearly with increasing $N_u$. In Figures 6 and 7, $N_u$ was increased to 3072 and 6400 respectively. There is some improvement in the reconstruction because of the finer interpolation, but beyond some point finer interpolation may not help because as the target is located closer to the runway, the range of observation angles becomes very narrow. As an extreme case, for a point target at the center of the runway, that range is zero since that target is observed only at $\gamma = 90^\circ$. To see what the reconstruction might look like for such a target, another simulation was performed. We added a point target at (0,3000) to the previous set of targets and used $N_u = 6400$. The result is shown in Figure 8, where it is seen that the target is not reconstructed exactly but its energy is distributed all across the cross-track dimension. This is predicted by the resolution formula in (10). We expect a similar reconstruction for targets that are anywhere near the center of the runway. This is not a problem for the runway imaging application, however, because targets on or very close to the runway need not be imaged with high resolution. An indication of their existence is sufficient, which occurs in Figure 8. On the other hand, targets should not appear outside the FAA zone if they are actually inside. Our simulations, above, show that with accurate interpolation this is not a problem. Targets outside the FAA zone should not appear inside and cause a false alarm, which, of course, is a less hazardous case. From the simulations, it is seen that targets that are inside the FAA zone but not close to the runway center are accurately imaged. Objects outside the FAA safety zone will be imaged with even higher resolution, eliminating the possibility of a false alarm.

4. REFERENCES


