COVARIANCE MATCHING ESTIMATION OF OCEAN SURFACE VELOCITY AND COHERENCE TIME IN ATI-SAR SYSTEMS

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Abstract

Conventional along-track interferometric synthetic aperture radar (ATI-SAR) systems derive the ocean surface velocity from an estimate of the phase difference between the SAR echoes received from two displaced phase centres with a single time-lag. We propose an asymptotic maximum likelihood (ML) algorithm for jointly estimating the ocean surface velocity and ocean coherence time by using multibaseline ATI-SAR data collected from an array of phase centres with multiple time-lags. The method combines a covariance matching (COMET) algorithm with the use of the Extended Invariance Principle (EXIP). Simulated results show that the proposed technique is highly efficient in practical small sample regime, and provides better estimation accuracy than the conventional single-baseline system. Moreover, it produces unambiguous velocity retrieval and flexibility to varying ocean coherence time.

1. Introduction

In the last ten years it has been shown that along-track interferometric synthetic aperture radars (ATI-SARs) have the potential to measure ocean surface currents, ocean surface waves and internal waves, and other surface dynamical features such as turbulence phenomena [6,4]. While the related field of DPCA (displaced phase center antenna) and STAP (space-time adaptive processing) for target detection is quite mature [7,2,10], theory and experiments of interferometric ocean sensing are still evolving. ATI-SAR sensing is fundamentally a Doppler technique. It is based on the estimation of the phase difference between two complex SAR images acquired in identical geometry with a short time lag \( \tau \). This is obtained by placing two antenna phase centers separated by a baseline \( B \) along the flight axis \( x \) of a platform, as shown in Figure 1, and matching the pulse repetition time (PRT) to platform speed \( v \), which is a DPCA condition [7,10]. ATI-SAR measures the radial component of the surface velocity and can provide large scale high resolution information on currents and important velocity variations [6]. Existing and potential applications are environmental, scientific and commercial. It is worth noting that care must be taken in converting the Doppler measurements to a surface current image [14], in particular contributions associated with surface wave motion, depending on wind velocity, can be non negligible. In fact, it has been shown that along-track interferometry can be used to obtain ocean wavenumber spectra, as well [1]. Finally, estimation of scene coherence time is possible, it can provide important insight into many oceanographic phenomena [4].

However, accuracy of conventional single-baseline ATI is significantly degraded for low ocean coherence time or low signal to noise ratio [4]. Moreover, the single-baseline technique suffers from intrinsic limitations due to the unavoidable tradeoff in the selection of the optimum time-lag, in terms of interferometer sensitivity, signal decorrelation, Doppler unambiguous estimation range, and azimuth wavenumber bandwidth in the retrieval of sea wave spectra [4,8]. For a given coherence time, an increase of the time-lag (e.g., an increase of the baseline) produces higher velocity sensitivity of the along-track interferometer, but this is obtained at the cost of increased decorrelation and so larger phase noise. How the resulting velocity estimation accuracy is affected by the time-lag depends on the trade-off between the two effects, that is particularly important when the signal to noise ratio or the number of looks [4] in the SAR images are low. Also, a short lag is preferred to avoid ambiguity problems. As a contrast, a long baseline produces a high azimuthal cut-off wavenumber, and so improved imaging of ocean waves [1].

Currently, this technique is evolving into the multiple baseline domain. The availability of a set of different baselines (viz. time lags) together with a proper signal processing may allow to achieve all the above mentioned goals at the same time and improve accuracy, at the cost of system complexity. The use of more than two antennas in the ATI-SAR system has been mentioned as desirable for enhanced

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velocity and wave spectra estimation (see [5]). In [4, 11], dual baseline coherence time estimation for the Ku-band SAR from Norden Systems and for the L-band AIRSAR system from JPL has been presented. Multibaseline parametric estimation of the Doppler shift has been proposed in [9] for the case of known signal model.

The novel contribution of this paper is the derivation of a computationally simple and asymptotically (large number of looks) efficient multibaseline technique for jointly estimating the ocean surface Doppler shift and coherence time, so obtaining high quality estimates in an unknown and highly varying environment.

2. The data model

The multibaseline ATI-SAR statistical model introduced in [8, 9] is based on the classical two-scale electromagnetic model of the sea radar backscatter, taking account for the typical small τ values (50 ms) the large scale structure variation can be neglected. Consider a K antennas along-track array system which acquires N complex SAR images in identical geometries at K-1 time-lags \( l = \begin{pmatrix} (K - 1) 
\end{pmatrix}_{i=1}^{K-1} \). The PRF is matched to the platform speed so that \( nPRF = \tau / (K - 1) \), being \( \tau = B / v \) the overall time lag and \( n \) an integer \( [7, 10] \) (see Figure 2 for \( K=3 \) and \( n=4 \)).

We consider in each SAR image \( N \) independent looks of the given patch of sea. For each look the complex amplitudes of the pixels, corresponding to a same patch of sea, observed in the K SAR images are arranged in the vector \( y(n) = [y_1(n) \cdots y_K(n)]^T \), for \( n = \begin{pmatrix} 1 
\end{pmatrix}_{n=1}^{N} \). We assume that the \( N \) random vectors \( \{y(n)\}^N_{n=1} \) are independent and identically distributed and each one is modeled as

\[
y(n) = \Lambda(\phi_0) x(n) + \nu(n) \tag{1}
\]

where \( \Lambda(\phi) \) is the \( K \times K \) diagonal steering matrix

\[
\Lambda(\phi) = \text{diag} \left[ e^{j\phi_1}, e^{j\phi_2}, \cdots e^{j\phi_K} \right].
\]

\( \phi_0 = -\omega_0 \sigma \) is called the interferometric phase, with \( \omega_0 \) being the mean Doppler shift of the backscattered signal, which is related to the surface velocity given by currents, Bragg wave velocity, and long wave orbital motion [6, 4]. The vector \( x(n) \) represents the speckle and takes into account the random phase and amplitude changes of the backscattered signal, which has limited coherence time. In fact, scattering from Bragg waves dominates in the radar return for the typically large grazing angles employed in SAR [5, 14]. As a result, the Doppler spectrum has non-zero bandwidth associated with the distribution of the slant-range velocity within the SAR resolution cell [14]. The speckle vector \( x(n) \) represents a correlated multiplicative noise term and it is assumed to be complex Gaussian distributed with zero mean vector [13], variance \( \sigma_x^2 \), and normalized covariance matrix \( E\{x(n)x^H(n)\} = \sigma_x^2 \text{C}_x \), in shorthand notation we write \( x(n) \sim \text{CN}(0, \sigma_x^2 \text{C}_x) \). The Gaussian density assumption for the speckle relies on the large grazing angle SAR geometry. \( \nu(n) \) is complex white Gaussian thermal noise, \( \nu(n) \sim \text{CN}(0, \sigma_t^2 I) \). As a result \( y(n) \) is \( \text{CN}(0, R) \) with

\[
R = \sigma_x^2 \Lambda(\phi_0) \text{C}_x \Lambda(\phi_0)^H + \sigma_t^2 I \tag{2}
\]

Under the usual Gaussian autocorrelation assumption [14, 4], the elements of \( \text{C}_x \) are given by

\[
C_x(i, j) = \text{exp}\left\{-1j \left( (K - 1) \tilde{\tau} \right) (i - j) / K \right\}
\]

where \( \tilde{\tau} = \tau / \tau \) is the ocean coherence time, normalized to \( \tau \). Note that the thermal noise power \( \sigma_t^2 \) cannot be generally assumed to be known, since the contribution from the thermal emission of sea in the microwave band is highly varying and non-negligible compared to the receiver noise.

The problem considered herein is to estimate

\[
\theta = [\sigma_x^2, \tilde{\tau}, \phi_0]^T = [\tilde{\Omega}, \phi_0]^T
\]

from a set of \( N \) looks \( \{y(n)\}^N_{n=1} \). The nuisance parameters \( \sigma_x^2 \) and \( \tilde{\tau} \) are estimated jointly with the useful parameters \( \phi_0 \), \( \tilde{\Omega} \), following a modern adaptive approach. It is worth noting that azimuth blurring due to the radial motion of the scatterers is not taken in account in this model. Blurring effects are not crucial for the present work, and the complex amplitude of each SAR image pixel is treated as if the image were formed with a real aperture radar with the SAR system nominal resolution. Non-blurred and accurate Doppler shift and coherence fields may be obtained by pixel-by-pixel Fourier transform of the \( K \) images along the image index, proper shift of the resulting \( K \) velocity images [5], inverse Fourier transform, and subsequent processing of the \( K \) re-located image data as presented in the sequel. This extension of the velocity SAR (VSAR) technique in [5], when merged with the COMET-EXIP method presented in the following section, could produce a very good exploitation of the information content of multibaseline ATI-SAR data.

3. The COMET-EXIP estimator

Our approach utilizes covariance matching estimators (COMET) [12]. The method amounts to minimize the (properly weighted) squared norm of the error between the true covariance matrix and its sample estimate \( \tilde{R} \). COMET estimates are obtained as

\[
\hat{\theta}_{\text{COMET}} = \arg \min_{\theta} \left\| W^{1/2} \left( \tilde{R} - R(\theta) \right) W^{1/2} \right\|^2
\]

where \( W \) is a positive definite weighting matrix. With a suitable choice of \( W \), namely \( W = \tilde{R}^{-1} \), (3) is asymptotically equivalent to the maximum likelihood (ML) estimator (which is computationally unfeasible, involving a 4-D maximization problem). The criterion in (3) is quadratic in

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\(\sigma^2_\epsilon\) and \(\sigma^2_\zeta\); hence, it can be compressed, leaving out a 2-D minimization problem over the parameters \(\phi_0\) and \(\bar{\tau}_c\). In order to reduce computational load, we propose to use the Extended Invariance Principle (EXIP) [15] which enables to decouple estimation of \(\phi_0\) from estimation of \(\bar{\tau}_c\); hence, the 2-D minimization problem is replaced by two successive 1-D problems. This is achieved through a two-step procedure. At a first stage, a less detailed (or unstructured) model for the covariance matrix \(R\) is used with the aim of simplifying the minimization problem in (3). More exactly, \(R\) is re-parametrized in terms of another parameter vector \(\eta = g(\theta)\). Once the estimates \(\hat{\eta}\) of \(\eta\) in the unstructured model are obtained, a simple least-squares fit between \(\hat{\eta}\) and \(g(\theta)\) is required to estimate the original parameter vector \(\theta\).

We now describe in detail the mentioned procedure when applied to the case of ATI-SAR data. First, note that \(R = \Lambda(\phi_0)Q(\bar{\theta})\Lambda^H(\phi_0)\) where \(Q(\bar{\theta}) = \sigma^2_\zeta C_\zeta(\bar{\tau}_c) + \sigma^2_\epsilon I\). It is straightforward to verify that \(Q(\bar{\theta})\) is a real-valued, symmetric Toeplitz matrix; hence, it is completely characterized by its first column, say \(\xi\). Consequently, the first step of our procedure consists of using the following less detailed model for \(R\)

\[
R = \Lambda(\phi_0)T(\xi)\Lambda^H(\phi_0)
\]

(4)

where \(T(\xi)\) denotes the real-valued symmetric Toeplitz matrix whose first column is \(\xi = [\xi_1, \xi_2, \ldots, \xi_K]^T\). The parameter vector which describes the unstructured model (4) is then \(\eta = [\xi^T, \phi_0]^T\). The COMET estimator in this unstructured model is then given by

\[
\hat{\eta} = \arg \min_{\eta} \|I - \Lambda(\phi_0)T(\xi)\Lambda^H(\phi_0)\hat{R}^{-1}\|^2
\]

(5)

We now show how these estimates can be obtained. Only the main results are stated, the details can be found in [3].

**Proposition 1.**

The COMET estimates in the unstructured model (4) are given by

\[
\hat{\phi}_0^* = \arg \max \text{vec}(A(\phi))^T J^T [J^T (A^T(\phi) \otimes A(\phi)) J]^{-1} J^T \text{vec}(A(\phi))
\]

\[
\hat{\xi} = (J^T (A^T(\hat{\phi}_0^*) \otimes A(\hat{\phi}_0^*)) J)^{-1} J^T \text{vec}(A(\hat{\phi}_0^*))
\]

(6)

where \(A(\phi) = \Lambda^H(\phi)\hat{R}^{-1}\Lambda(\phi)\). \(J\) is the \(K^2 \times K\) matrix such that \(J^T = \text{vec}(T(\xi))\) and vec is the operator which consists of stacking the columns of the matrix between parenthesis.

**Proposition 2.**

If \(\phi_0 \in [0, (K-1)\pi]\), \(\hat{\eta}\) is a consistent estimate of \(\eta\), its asymptotic covariance matrix is derived in [3] and the asymptotic variance of \(\hat{\phi}_0^*\) coincides with the Cramer-Rao Bound (CRB) for estimation of \(\phi_0\). The non-ambiguous estimation range is half that of the clairvoyant maximum likelihood (CML) estimator [9]. However, it can be raised to the full range \([- (K-1)\pi, (K-1)\pi]\) in a simple way, as described in [3].

Once the unstructured estimates are obtained, the EXIP principle is invoked to obtain estimates asymptotically equivalent to \(\hat{\theta}^{\text{COMET}}\) as described below. Let \(f: \hat{\theta} \mapsto \xi\) be the mapping between \(\xi\) and \(\hat{\theta}\). Then,

**Proposition 3.**

Asymptotically efficient estimates can be obtained as

\[
\hat{\phi}_0^* = \hat{\phi}_0^* ; \hat{\theta}^* = \arg \min_{\phi_0} \|\xi - f(\hat{\theta})\|^2 C_{\xi}^{-1}[\xi - f(\hat{\theta})]
\]

(7)

where \(C_{\xi}^{-1}\) is the Fisher Information Matrix for estimation of \(\xi\) [3].

Since the criterion in (7) is quadratic in \(\sigma^2_\zeta\) and \(\sigma^2_\epsilon\), it can be concentrated w.r.t. these two variables, leaving out a 1-D problem with respect to \(\bar{\tau}_c\) only.

**4. Numerical results**

In this section, we investigate the performance of the proposed COMET-EXIP estimator (henceforth EXIP) by means of Monte-Carlo simulations and compare it with the CRB. Additionally, for comparison purposes, we also illustrate the performance of the CML estimator of \(\phi_0\) [9], in order to evaluate the loss induced by the fact that the covariance matrix of the speckle and the noise power are unknown. Finally, because it has been the method used so far, the ML estimator of \(\phi_0\) with \(K=2\) [6,4] is also analyzed so as to quantify the improvement that can be achieved when using more than two antennas. Unless otherwise stated the value of \(\phi_0\) is selected as \(\phi_0 = 0\) (it is without loss of generality because estimation accuracy does not depend on the actual value of \(\phi_0\)) and \(\bar{\tau}_c = 2\). The signal to noise ratio (SNR) is set to 24 dB. The latter two values corresponds to typical system and signal parameters of the JPL AIRSAR system [6,4]. 5000 Monte-Carlo simulations were run in order to estimate the mean-square error (MSE) and bias of the estimators.

Firstly, we analyze the validity of the theoretical results concerning EXIP estimation. Towards this end, Figures 3 and 4 display the mse on estimation of \(\phi_0\) and of the (normalized) coherence time \(\bar{\tau}_c\) as a function of \(N\), for \(K=5\) and \(K=3\), respectively. The estimators of \(\phi_0\) are unbiased modulo \((K-1)2\pi\). From inspection of these fig-
ures, it can be observed that EXIP estimates have a variance which comes close to the CRB for large N, as predicted by the theory. However, this “asymptotic regime” is achieved for a rather small number of looks, typically N=16 for K=3 (not shown in Figure 3) and N=32 for K=5. This is an interesting feature of the method since it is efficient for a number of looks which is commonly encountered in practice [4]. Secondly, we can see that the CML estimator of $\hat{\phi}_0$ has a variance which is approximately equal to the CRB for unknown $\hat{\theta}$. Finally, we note that a significant improvement w.r.t. the conventional K=2 case is always achieved when K is increased, provided that N is sufficiently high. A loss arises for the case K=5 compared to the conventional case with K=2 when N=8, this point will be commented in the sequel. For the $\hat{\tau}_e$ estimation, the K=2 performance cannot be employed as a reference since estimation of $\hat{\tau}_e$ is not possible with just one baseline when the noise power is unknown (the problem is not identifiable and the CRB is infinite).

In a second set of simulations, we investigate the performances of the estimates when the (normalized) coherence time is varied (cf. Figures 5, 6, 7 and 8). Note that when $\hat{\tau}_e$ is small, the autocorrelation of the speckle decreases very rapidly making estimation more challenging. It is also observed that the mae on estimation of $\hat{\phi}_0$ decreases when $\hat{\tau}_e$ increases. In contrast, the mae of $\hat{\tau}_e$ is large for very small values of $\hat{\tau}_e$, reaches a minimum and then increases (which is not surprising since the parameter to be estimated increases itself). The EXIP estimator of $\hat{\phi}_0$ and $\hat{\tau}_e$ still remain close to the CRB. Some discrepancy of the CRB is observed for very small $\hat{\tau}_e$ when estimating $\hat{\tau}_e$. The floor effect which can be observed may be due to the fact that the estimator exploits knowledge that the coherence time is always greater than zero while the CRB does not take this constraint into account. Observe that in this region the CRB is very large, indicating that results should not be accurate anyway. The absolute value of the bias for $\hat{\tau}_e$ is shown in Figure 8, it also sheds some light on the discrepancy of the CRB for very small values of $\hat{\tau}_e$, since in this region the estimator of $\hat{\tau}_e$ is highly biased (upward), while the CRB is for unbiased estimators. To understand the floor effect in Figures 5 and 6, it is necessary to remember that the operational capability of the K-channel system is extended down to $\hat{\tau}_e=0.5/(K-1)$, beyond which the resulting error distribution is practically uniform over the non-ambiguous estimation range and the interferometric phase does not contain any information on the sensed scene. Analogously, the discrepancy of the CRB on $\hat{\phi}_0$ for very small values of $\hat{\tau}_e$ has to be attributed to the fact that the CRB does not take into account the circularity of $\hat{\phi}_0$, that is defined modulo $(K-1)2\pi$. It can also be observed that the performance of the EXIP $\phi_0$ estimator closely approaches that of the CML (although it does not require knowledge of $\tau_e$, $\sigma^2_e$ and $\sigma^2_x$). Moreover, for large $\hat{\tau}_e$ the conventional ML method for estimation of $\phi_0$ with K=2 performs as well as the other methods. This seems meaningful since, when $\hat{\tau}_e$ is large, the vector $\hat{x}$ is nearly constant, and hence estimation is not really improved when K is increased. The slight loss of EXIP for K=5 can be explained as follows. For fixed N, when K increases, the estimation of the covariance matrix tends to be less accurate. Hence, the weighting matrix $W = \hat{R}^{-1}$ departs further from the optimal choice, i.e., $W = R^{-1}$, which, in turn, results in poorer weighting, hence in less accurate estimates. This phenomenon has already been observed in Figure 3; it means that increasing K without increasing N accordingly does not necessarily results in better performance. As a contrast to the high $\hat{\tau}_e$ region, multibaseline processing exhibit a significant accuracy gain for low $\hat{\tau}_e$. It is expected that this should also impact on the design of multibaseline ATI-SAR systems. For increasing K, best performance should be presumably obtained by adopting an increasing overall time lag $\tau$, e.g., a larger baseline B, as far as both Doppler shift and coherence time estimation accuracy are concerned (a few results in this sense have been obtained in [8] for the CML estimator of $\omega_0 = -\phi_0 / \tau$).

Finally, results for varying SNR, not reported here, show that the hierarchy between the methods remains approximately the same when SNR decreases. This reveals that the proposed technique keeps good accuracy gain in critical conditions (smooth ocean and/or high range) in addition to good flexibility for varying coherence time.

5. Conclusions

The contribution of this work is the derivation of an asymptotic ML data fusion algorithm for joint Doppler shift and coherence time estimation of ocean backscatter in multibaseline ATI-SAR. The proposed method extends the works in [11,4], which deal with dual baseline coherence time estimation, and [9], which presents the multibaseline clairvoyant ML estimator of Doppler shift. The method combines a COMET algorithm with the use of the EXIP. It is computationally simple since it splits the original 4-D maximization problem into two successive 1-D maximizations and one 2-D linear problem. Both analytical and simulated performance analysis have been carried out. Results show that a novel advanced processing technique such as COMET-EXIP can be successfully applied to multibaseline ATI-SAR ocean sensing. The proposed technique offers significant improvement in estimation accuracy and extended operational capability in critical conditions when compared to conventional single baseline systems. Advan-
tages in terms of statistical accuracy of the Doppler shift and coherence time maps can be also regarded as a spatial resolution gain, through possible relaxing of the number of required looks. Currently, the number of systems with ATI mapping capabilities is increasing. This growing interest stems from the fact that ATI can provide rich information about ocean and tidal currents, wave spectra and various surface features, which are of considerable interest in oceanographic investigations, hydrology, environmental management, coastal protection, off-shore industry, ship transportation. The technique proposed in this paper contributes to the evolution of along-track interferometry into the multiple baseline domain, and it may be effective both for existing airborne and planned spaceborne ATI-SAR systems.

Work is planned on multibaseline ATI-SAR system design issues, on further enhancement of small sample statistical performance and on estimation of parameters of possible non-unimodal spectra in cross-wind geometries [14]. Clearly, there is a need for validating the proposed technique with both ocean electromagnetic simulators and recorded real data.

References


Fig. 3. CRB and Mean-Square Errors of $\phi_0$ estimates versus $N$. $\phi_0=0$, $\bar{r}_c=2$ and SNR=24 dB.

Fig. 4. CRB and Mean-Square Errors of $\bar{r}_c$ estimates versus $N$. $\phi_0=0$, $\bar{r}_c=2$ and SNR=24 dB.

Fig. 5. CRB and Mean-Square Errors of $\phi_0$ estimates versus $\bar{r}_c$. $\phi_0=0$, $N=32$ and SNR=24 dB.

Fig. 6. CRB and Mean-Square Errors of $\phi_0$ estimates versus $\bar{r}_c$. $\phi_0=0$, $N=32$ and SNR=24 dB.

Fig. 7. CRB and Mean-Square Errors of $\bar{r}_c$ estimates versus $\bar{r}_c$. $\phi_0=0$, $N=32$ and SNR=24 dB.

Fig. 8. Bias (absolute value) of $\bar{r}_c$ estimates versus $\bar{r}_c$. $\phi_0=0$, $N=32$ and SNR=24 dB.